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# Locality *vs.* nonlocality of $(2+1)$-dimensional light-induced space-charge field in photorefractive crystals

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**Abstract.** – A novel analytical description of the nonlocal anisotropic space charge field, induced by $(2+1)$D localized light beams in photorefractive crystals, is introduced. Closed-form expressions for the space-charge field induced by circularly symmetric beams are obtained. Also, on the basis of our formulation, numerical simulations of the beam dynamics are performed and compared with those corresponding to a local modeling of the space-charge field.

Photorefractivity is an optical nonlinearity occurring at low light intensity levels that offers an excellent scenario for the investigation of a diversity of nonlinear physical phenomena. Among these, the study of propagation of self-focused optical beams and spatial solitons in photorefractive (PR) crystals has attracted an intense research effort over the last decade [1]. At variance with nonlinearities based on electronic polarizability, such as the Kerr nonlinearity, that lead to local nonlinear refractive index variations $\Delta n(I)$ dependent on the light intensity $I$, the PR nonlinearity exhibits a much more complicated dependence on $I$ because it involves a light-induced charge transport mechanism which is characterized by both spatial and temporal nonlocalities. This transport mechanism gives rise to the development of a space-charge field $E_{\mathrm{sc}}$ that, through the electro-optic effect, produces the nonlinear refractive index change $\Delta n(I)$.

From a theoretical point of view, the complexity of the PR effect makes the description of nonlinear beam propagation rather challenging, especially in more than one spatial dimension (1D). Although a general theoretical framework, initially developed by Kukhtharev *et al.* [2], is available to deal with PR phenomena, even in its time-independent version it involves the nontrivial and generally formidable interplay between a transverse electrostatic problem and a coupled parabolic optical wave equation. When the drift mechanism is the most important contribution to charge transport, one can provide a relatively simple and yet effective local description (disregarding small nonlocal effects) in the 1D geometry of the space-charge field [3,4]:

$$E_{\rm sc} = E_0 \frac{1}{1+I}\,, \tag{1}$$

where $I$ is the total light intensity of the propagating beam (in units of the saturation or background intensity) and $E_0$ is the transverse dc electric field applied to the PR crystal. Equation (1) displays one of the most important features of the PR effect: *saturation*. This is a property of $E_{\rm sc}$ that occurs in higher dimensions as well, and this fact has been exploited in a number of recent works dealing with $(2+1)$D spatial screening solitons [5] by using a local model for $E_{\rm sc}$ of the form given by (1). Under certain conditions, such a representation constitutes a convenient approximation that allows the application of the results obtained for the conventional saturable local response, for which much information is available [6]. However, higher dimensionality introduces one key ingredient: *anisotropy*. In addition, the fact that nonlocality becomes much more important than in 1D, requires that both, anisotropy and nonlocality, be taken into account in order to predict most of the relevant features of beam propagation in PR crystals, as revealed in a number of works [7–10]. An illustrative example is the nonexistence of stable localized optical vortex solitons (*i.e.* finite-size self-guided beams with an internal vorticity and a phase dislocation) in PR media. Experimental results show that they experience an immediate transverse instability during propagation, whereas in *purely* local saturable self-focusing media much longer distances are required for their breakup.

The complete description of the PR response in more than 1D has been obtained only in limited cases through numerical calculations. The purpose of the present paper is to develop a satisfactory analytical 2D formulation of the PR nonlinearity, including both local and nonlocal anisotropic contributions, that would allow for an adequate analysis of the light-induced space-charge field generated by localized beams and spatial solitons. The results obtained are valid for light intensities up to the moderate saturation regime ($I \gtrsim 1$), which is the most relevant regime for the propagation of PR spatial solitons [8]. The expression found for the 2D light-induced space-charge field can be split into local and nonlocal contributions. The local part exhibits a new intensity dependence that accounts for the saturable behavior and considerably improves on the commonly used expression given by eq. (1). Moreover, closed-form solutions are obtained for the very important case of circularly symmetric localized beams. Also, to illustrate the validity of our approach, we have used the expression obtained for the 2D light-induced space-charge field and performed numerical simulations of the nonlinear beam propagation for bright rings and vortices showing a remarkable agreement with the experimental results found in the literature.

*General model.* – Our analysis starts from the standard Kukhtarev formulation [2] describing the PR effect. In the steady state, the material equations are given by

$$S(1+I)\big(N_{\rm D} - N_{\rm D}^{+}\big) - \gamma_{\rm r} N_{\rm e} N_{\rm D}^{+} = 0, \tag{2a}$$

$$-\epsilon_0\epsilon_{\rm r}\nabla_\perp^2\phi = q\big(N_{\rm D}^{+} - N_{\rm A} - N_{\rm e}\big), \tag{2b}$$

$$\nabla_\perp \cdot \boldsymbol{j} = 0. \tag{2c}$$

Here $N_{\rm D}$, $N_{\rm D}^{+}$, $N_{\rm e}$, and $N_{\rm A}$ represent the density of donors, ionized donors, free electrons, and acceptors, respectively, $S$ and $\gamma_{\rm r}$ are the photoexcitation and recombination constants, $\epsilon_0\epsilon_{\rm r}$ the scalar dielectric constant of the material, $q$ the elementary charge, $\boldsymbol{j} = \mu q N_{\rm e}(\boldsymbol{E}_0 - \nabla_\perp\phi) + \mu k_{\rm B} T \nabla_\perp N_{\rm e}$ the current density, $\mu$ the electron mobility, $k_{\rm B}$ Boltzmann's constant, $T$ the absolute temperature, and $\nabla_\perp$ the gradient in the transverse $(x,y)$-plane.

The electrostatic potential $\phi$ gives rise to the screened space-charge field, $\boldsymbol{E}_{\rm sc} = \boldsymbol{E}_0 - \nabla_\perp \phi$. To find an equation for $\phi$, one substitutes the expression for $N_{\rm e}$ from eq. (2a) into eq. (2c):

$$\nabla_\perp^2 \phi + \nabla_\perp \ln(1+I) \cdot \nabla \phi - E_0 \partial_x \ln(1+I) = \frac{k_{\rm B}T}{q} \Big\{ \nabla_\perp^2 \ln(1+I) + \big[\nabla_\perp \ln(1+I)\big]^2 \Big\}, \quad (3)$$

where it has been assumed that $N_{\rm D}^+ \approx N_{\rm A} \gg N_{\rm e}$ (possible trap saturation effects are neglected [11]), and that $\boldsymbol{E}_0$ is applied along the $x$-axis. Equation (3) has already been used in numerical studies of $(2+1)$D propagation of solitons [9] and it has shown a remarkable agreement with experiments [8, 10]. It is interesting to note that it can be solved exactly in the 1D case ($x$ coordinate only). The solution is given by

$$E_{\rm sc}(x) = E_0 \frac{1 + I_\infty}{1+I} - \frac{k_{\rm B}T}{q} \frac{{\rm d} \ln(1+I)}{{\rm d}x}\,, \quad (4)$$

where $I_\infty$ denotes the intensity when $|x| \to \infty$. Equation (4) consists of the above-mentioned saturable intensity dependence term, and a diffusion contribution, which is responsible for the self-bending of the beam.

In the 2D case, eq. (3) cannot be solved in closed form. Let us introduce a transformation of the potential,

$$\phi(x,y) = E_0 \frac{u(x,y)}{\sqrt{1+I}} + \frac{k_{\rm B}T}{q} \ln(1+I), \quad (5)$$

which brings eq. (3) to the canonical elliptic form that is more analytically amenable:

$$\nabla_\perp^2 u - \left\{ \nabla_\perp^2 \ln \sqrt{1+I} + \left[ \nabla_\perp \ln \sqrt{1+I} \right]^2 \right\} u = 2 \partial_x \sqrt{1+I}\,. \quad (6)$$

Owing to the transformation (5), the contribution of the second term on the left-hand side of eq. (6) has decreased relative to the other two, as compared to eq. (3). Also, the fact that $|u(x,y)| < 1$ for all $I(x,y)$ causes the second term on the left-hand side to be small in the case of beams localized in the two transverse directions. In addition, since typical soliton solutions and localized beams of nonlinear propagation equations have exponentially decaying asymptotics at infinity, a good approximation to the solution of eq. (6) can be obtained by neglecting the second term on the left-hand side. The validity of this approximation (which can be shown rigorously by discretizing eq. (6)) will be confirmed *a posteriori* by comparing the found analytical solution for $E_{\rm sc}(x,y)$ to the corresponding full numerical solution obtained from eq. (6), for several representative beam profiles. In particular, such an approximation yields an excellent agreement in the important case of nearly radially symmetric beams. Hence, we restrict ourselves to solving the following Poisson's equation:

$$\nabla_\perp^2 u = 2 \partial_x \sqrt{1+I} \quad (7)$$

in terms of the 2D Green's function:

$$u(x,y) = \frac{1}{\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ \frac{\partial}{\partial \xi} \sqrt{1 + I(\xi,\eta)} \right] \ln \sqrt{(x-\xi)^2 + (y-\eta)^2}\, {\rm d}\xi\, {\rm d}\eta. \quad (8)$$

By introducing polar coordinates,

$$\rho \cos\theta = \xi - x, \qquad \rho \sin\theta = \eta - y, \quad (9)$$

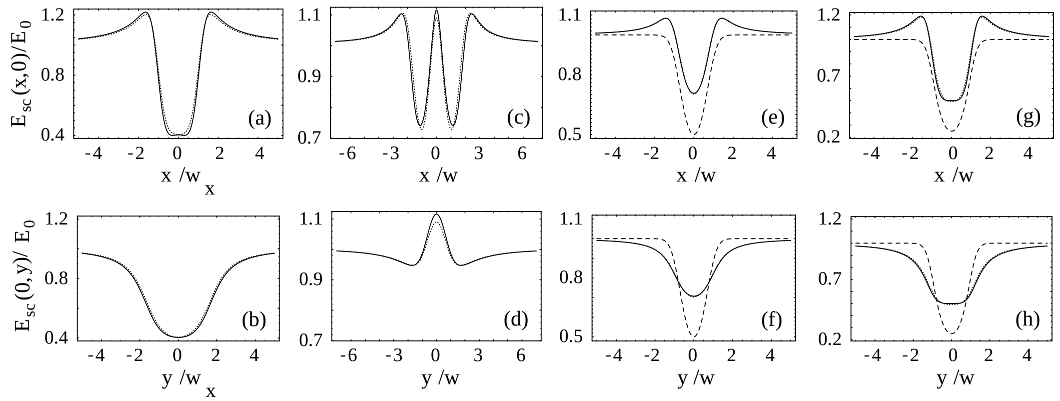



Fig. 1 – Space-charge field induced by an elliptic (a) and (b), two circular (c) and (d), and one circular (e)-(h) Gaussian beams; (a), (c), (e), and (g) along $x$-axis, (b), (d), (f), and (h) along $y$-axis. (a) and (b) correspond to $w_y/w_x = 1.5$ and peak intensity $I = 3$; (c) and (d) to peak intensity $I = 1$ and $d = 2w$; (e) and (f) to $I_0 = 1$, whereas (g) and (h) to $I_0 = 3$ (with $I_\infty = 0$). Full numerical solutions are represented by the dotted lines. In (a)-(d) solid lines correspond to eq. (11). In (e)-(h) the solid line corresponds to eq. (13), whereas the dashed line corresponds to eq. (4). Note that in (e)-(h) the dotted and solid lines are almost indistinguishable.

eq. (8) can be integrated by parts, to yield

$$u(x,y) = -\frac{1}{\pi}\int_0^\infty \int_0^{2\pi} \cos\theta\sqrt{1+I(x+\rho\cos\theta, y+\rho\sin\theta)}\,\mathrm{d}\rho\,\mathrm{d}\theta. \tag{10}$$

Combining eqs. (5) and (10), we finally obtain the $x$-component of the space-charge field:

$$\begin{aligned} E_{\rm sc}(x,y) &= -\frac{k_{\rm B}T}{q}\partial_x \ln(1+I) + E_0\Bigg\{\sqrt{\frac{1+I_\infty}{1+I}}\times \\ &\times \left[1+\frac{1}{\pi}\int_0^\infty\int_0^{2\pi}\frac{\cos 2\theta}{\rho}\sqrt{1+I(x+\rho\cos\theta,y+\rho\sin\theta)}\,\mathrm{d}\rho\,\mathrm{d}\theta\right] - \\ &- \frac{\partial_x \ln(1+I)}{2\pi\sqrt{1+I}}\int_0^\infty\int_0^{2\pi}\cos\theta\sqrt{1+I(x+\rho\cos\theta,y+\rho\sin\theta)}\,\mathrm{d}\rho\,\mathrm{d}\theta\Bigg\}, \end{aligned} \tag{11}$$

where $I_\infty$ denotes the intensity when $|x|, |y| \to \infty$. Expression (11) includes both the local and nonlocal contributions. Aside from the diffusion term (henceforth we disregard this term), the two integrals in eq. (11) represent both, nonlocality and anisotropy of the space-charge field. Figures 1(a)-(d) depict a comparison of the space-charge fields calculated from eq. (11) and the corresponding numerical solution obtained from the complete eq. (6), for an elliptic Gaussian beam of widths $w_x$ and $w_y$ along the $x$ and $y$ axes, respectively (figs. 1(a), (b)); and two circular Gaussian beams of width $w$ along the $x$-axis at a distance $d = 2w$ (figs. 1(c), (d)).

*Circularly symmetric beams.* – For radially symmetric beams ($I = I(r)$), eq. (11) can be further simplified and yields (leaving aside the diffusion contribution)

$$\begin{aligned} E_{\rm sc}(r,\varphi) &= E_0\sqrt{\frac{1+I_\infty}{1+I}}\Bigg\{2\cos^2\varphi - \cos 2\varphi\sqrt{\frac{1+I}{1+I_\infty}} - \\ &-\frac{2}{r^2}\left[\cos 2\varphi + r\cos^2\varphi\frac{\mathrm{d}\ln\sqrt{1+I}}{\mathrm{d}r}\right]\int_0^r \rho\left[1-\sqrt{\frac{1+I(\rho)}{1+I_\infty}}\right]\mathrm{d}\rho\Bigg\}. \end{aligned} \tag{12}$$

Notice that $E_{\rm sc}(0,\varphi) = E_0\sqrt{(1+I_\infty)/(1+I(0))}$, meaning that the space-charge field at the center of the beam is purely local. There are some other interesting features of $E_{\rm sc}(r,\varphi)$; owing to the anisotropy ($\varphi$-dependence), along the direction of $\varphi = \pi/4$, $E_{\rm sc}(r,\pi/4)$ exhibits an almost local behavior. Also, for the case of bright beams ($I_\infty = 0$) of width $w$ having $I \ll 1$, and when $r \gg w$, $E_{\rm sc}(r,\varphi) \simeq E_0(1 + P\cos 2\varphi/2\pi r^2)$, where $P = 2\pi\int_0^\infty \rho I(\rho){\rm d}\rho$ is the power of the beam. This form of the space-charge field leads to a refractive index change that resembles the Snyder and Mitchell model of high nonlocality created by circular solitons [12], and suggests that the interaction between PR solitons, for large distances, is of dipolar nature.

Using eq. (12) it is possible to obtain closed-form formulas for several beam profiles (either bright or dark). The most simple one corresponds to the case of a cylindrical light rod of radius $R$ with $I(r) = I_0$ for $r < R$ and $I(r) = I_\infty$ for $r > R$. For such a profile, it is straightforward to find the expression of $E_{\rm sc}(r,\varphi)$ from eq. (12). We give the result for a *grey* Gaussian beam $I_{\rm G}(r) = I_\infty + I_0\exp[-2r^2/w^2]$ (*bright* and *dark* Gaussian beams correspond to the cases where $I_\infty = 0$ and $I_0 = -I_\infty$, respectively):

$$\begin{aligned} E_{\rm sc}(r,\varphi) &= E_0\sqrt{\frac{1+I_\infty}{1+I_{\rm G}}}\left\{2\cos^2\varphi - \cos 2\varphi\sqrt{\frac{1+I_{\rm G}}{1+I_\infty}} + \frac{w^2}{r^2}\left[\cos 2\varphi + r\cos^2\varphi\frac{{\rm d}\ln\sqrt{1+I_{\rm G}}}{{\rm d}r}\right]\times\right.\\ &\quad\left.\times\left[\frac{\sqrt{1+I_0+I_\infty}-\sqrt{1+I_{\rm G}}}{\sqrt{1+I_\infty}} + \ln\left(\frac{\sqrt{1+I_\infty}+\sqrt{1+I_{\rm G}}}{\sqrt{1+I_\infty}+\sqrt{1+I_0+I_\infty}}\right)\right]\right\}. \end{aligned} \tag{13}$$

Figures 1(e)-(h) show a comparison of the space-charge fields induced by bright Gaussian beams as calculated from eqs. (4), (13), and the full numerical solution obtained from eq. (6).

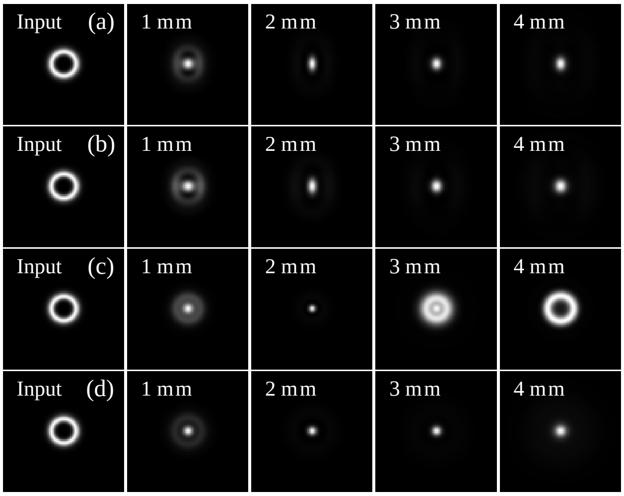
As shown above, at the central part of the beam, where the induced change in the space-charge field is the largest, the contribution of the two integrals in (11) is small for nearly radially symmetric beams. Then, within a local approximation to describe $E_{\rm sc}$, one can use

$$E_{\rm sc}(x,y) = E_0\sqrt{\frac{1+I_\infty}{1+I}}\,. \tag{14}$$

This square-root intensity dependence represents accurately the most relevant contribution of the space-charge field and constitutes an improved model to be used for describing the propagation of (2+1)D spatial screening solitons in comparison to the straightforward generalization of the 1D formula used by many authors. Interestingly enough, this type of square-root intensity dependence has appeared as a saturable model for the light-induced refractive index in the context of several (1+1)D nonlinear wave propagation problems. Representative examples are soliton propagation in plasmas [13] and strontium barium niobate waveguides [14].

It is important to emphasize that both expressions (11) and (14), at variance with the analytical approach discussed by Zozulya *et al.* [8] for elliptical beams, are not restricted to low saturation intensities ($I \ll 1$). For strongly elongated beams (*i.e.* those for which the ratio $w_y/w_x$ is either $\gg 1$ or $\ll 1$) the deviation between the exact numerical solution of $E_{\rm sc}$ and eq. (11) tends to increase whereas the 1D solution given by (4) is progressively recovered. Therefore, one can consider eq. (11) as an excellent approximation to the space-charge field induced by $(2+1)$D localized beams, up to the moderate saturation regime ($I \gtrsim 1$).

*Nonlinear beam propagation.* – To illustrate the predictions on nonlinear beam propagation of the different models for the space-charge field given above, we launch several initial beams into the crystal and compare the influence of local and nonlocal anisotropic models. Two local models are utilized, the inverse intensity formula, eq. (4), and the square root formula, eq. (14). For the nonlocal models we use both the full solution obtained from eq. (6)



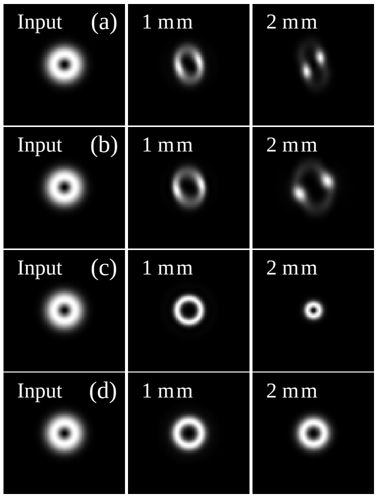




Fig. 2 – Intensity contour profiles of input bright rings (left) and charge +1 vortices (right), for different propagation distances. The rows (a) correspond to the full numerical case, rows (b) to the case where use of eq. (11) was made, whereas rows (c) and (d) correspond to the local cases represented by eqs. (4) and (14), respectively. The input peak intensity of both beams is $I \simeq 1$. Also, we used $E_0 = 3.6\,\mathrm{kVcm}^{-1}$. The $x$ and $y$ axes are oriented along the horizontal and vertical directions of each of the shown windows, respectively. The sides of the windows are $104\,\mu$m.

and eq. (11). The slowly varying envelope $A$ obeys the nonlinear paraxial wave equation

$$2ikn_0\partial_z A + \nabla_\perp^2 A = -k^2 n_0^4 r_{\mathrm{eff}} E_{\mathrm{sc}} A, \tag{15}$$

where $k$ is the free-space wave vector, $n_0$ the bulk refractive index, and $r_{\mathrm{eff}}$ is the effective component of the electro-optic tensor. The beam propagates in the $z$-direction, and is polarized along the $x$-direction, which is also the direction of the crystalline $c$-axis. The used numerical parameters are wavelength $\lambda = 2\pi/k = 532\,\mathrm{nm}$, $n_0 = 2.35$ and $r_{\mathrm{eff}} = 300\,\mathrm{pmV}^{-1}$. These correspond to typical values in experiments with strontium barium niobate crystals [10].

Launching Gaussian beams leads to subtle differences in the profiles and similar behavior in all the models (the local models preserve the initial beam symmetry). Thus, we choose to launch ring and vortex-mode solitons, whose behavior is much more model dependent. The results of our calculations are displayed in fig. 2. When the input beam is a bright ring (first set of images), there is a strong self-focusing in all cases caused by the high applied field ($E_0 = 3.6\,\mathrm{kVcm}^{-1}$). However, the inverse-intensity formula gives rise to large-width oscillations of the beam whereas in the other cases there is a quasi-stationary propagation. This example clearly shows the advantage of using the square-root formula instead of the inverse-intensity formula. For the case of an input vortex beam of topological charge +1 (second set of images in fig. 2), the predictions of local and nonlocal models are quite dissimilar. At variance with linear and self-defocusing media, where vortices are stable, in saturable self-focusing nonlinear media vortices are always unstable against azimuthal perturbations. However, the development of such instabilities is immediate for anisotropic models. The refractive index distribution induces the fission and decay of the vortex into a dipole-mode soliton that partially rotates in both cases (the dipole rotation of the full numerical model is slightly larger than the one predicted by eq. (11)). On the other hand, neither of the two local models predict the immediate breakup of the beam, which is caused by the anisotropy of the PR media. The much longer distances ($\sim 50\,\text{cm}$) which are required to observe the vortex fission indicate that for the propagation of beams with a topological charge the local models are not applicable.

*Conclusions.* – We have obtained new analytical expressions for the space-charge field induced by $(2+1)$D localized and self-trapped optical beams in PR crystals. An accurate description of the local and nonlocal anisotropic contributions of the space-charge field is provided. In particular, we have found an improved and closed-form representation of the space-charge field induced by radially symmetric light beams and a novel representation for the local contribution of the space-charge field that is more appropriate for the $(2+1)$D spatial soliton propagation problem in PR media. Also, we have tested and compared the predictions of the different models by simulating the complete nonlinear beam propagation problem. Apart from their conceptual importance, the results presented in this paper may provide potential usefulness towards nonlinear waveguide design based on $(2+1)$D PR screening solitons [15].

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