



Optical solitons in $(2 + 1)$ -Dimensions with Kundu–Mukherjee–Naskar equation by extended trial function scheme

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ABSTRACT

This paper addresses Kundu–Mukherjee–Naskar equation by the aid of extended trial function method to recover optical soliton solutions in $(2 + 1)$ -dimensions. The integration algorithm revealed doubly periodic functions. Upon taking the limiting values of the modulus of ellipticity, bright and singular solitons as well as singular periodic solutions emerge. Additional solutions such as plane waves also fall out of the scheme.

1. Introduction

The dynamics of soliton propagation through optical fibers is typically studied in $(1 + 1)$ -dimensions. In fact, there is a plethora of mathematical models and plentiful mathematical techniques that are available to study these soliton dynamics in the context of nonlinear optics and other areas of mathematical physics [1–25]. It was during 2014, a new model was proposed to address rogue waves in deep sea by three elite physicists namely Kundu et al. [13]. In the same work, it was suggested that this model is applicable to study soliton pulses in $(2 + 1)$ -dimensions [12]. This paper therefore addresses the dynamics of two-dimensional soliton propagation through an optical fiber with Kundu–Mukherjee–Naskar (KMN) model.

It must be noted that soliton dynamics has been studied in the past by a variety of authors using a wide range of powerful mathematical techniques for a plethora of physical systems [1–6,16–25]. These studies are from nonlinear optics [1,2,4–6,9,10,16,21,22,24,25], plasma physics [18], fluid dynamics [7,17,19], liquid crystals [8], theoretical physics [14], condensed matter physics [23] and many more. In particular, solitons in $(2 + 1)$ -dimensions have been addressed in the past by several authors, but notably during 2005 [16] and 2009 [21]. The integration scheme adopted in this paper is the extended trial function method. It will reveal bright and singular optical solitons as well as several other solutions written in terms of Jacobi's elliptic function. In the limiting case of the modulus of ellipticity, these solutions converge to optical solitons. The details are explored in the

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rest of the paper.

1.1. Governing model

The dimensionless form of KMN equation is given by [12,13,17,20]

$$iq_t + aq_{xy} + ibq(qq_x^* - q^*q_x) = 0. \tag{1}$$

In Eq. (1), the spatial variables are x and y while t gives the temporal variable and the dependent variable is $q(x, y, t)$ that represents nonlinear wave envelope. The first term in (1) stands for temporal evolution of the wave followed by the dispersion term that is given by the coefficient of a . Finally, the nonlinear term is the coefficient of b that is different from the conventional Kerr law nonlinearity or any known non-Kerr law media. In (1), this nonlinear term accounts for “current-like” nonlinearity that stems from chirality. This model was first proposed in 2014 to address oceanic rogue waves as well as hole waves. It was also proposed during 2013 that this model can be applied to address optical wave propagation through coherently excited resonant waveguides that is doped with Erbium atoms [5]. In particular, it was indicated that the model can study the phenomena of bending of light beams.

1.2. Mathematical analysis

In order to get started, the following hypothesis is selected:

$$q(x, y, t) = P(\xi)e^{i\phi(x,y,t)}, \tag{2}$$

where $P(\xi)$ represents the amplitude portion and

$$\xi = B_1x + B_2y - vt, \tag{3}$$

and the phase portion of the soliton is defined as

$$\phi(x, y, t) = -\kappa_1x - \kappa_2y + \omega t + \theta. \tag{4}$$

Here, κ_1 and κ_2 are the frequencies of the soliton in the x - and y -directions respectively while ω is the wave number of the soliton and finally θ is the phase constant. Also, the parameters B_1 and B_2 in (3) represent the inverse width of the soliton along x - and y -directions respectively, while v represents the velocity of the soliton. Inserting (2) along with (3) and (4) into (1) and decomposing into real and imaginary parts, the following pair of equations, respectively yield

$$aB_1B_2P'' - (\omega + a\kappa_1\kappa_2)P - 2b\kappa_1P^3 = 0, \tag{5}$$

$$v = -a(\kappa_1B_2 + \kappa_2B_1). \tag{6}$$

Although equation (5) will be handled by extended trial function approach in the subsequent section, it must be noted that this equation was already handled by a variety of authors in the past and their research findings are reported all across the journal spectrum [2,22,25].

2. Extended trial function method

This section will employ extended trial function technique [7–10,14,15] to construct soliton and other solutions to the KMN equation. To initiate the extraction of solutions to (5), the following assumption for the soliton structure is established:

$$P = \sum_{i=0}^{\zeta} \tau_i \Psi^i, \tag{7}$$

where

$$(\Psi')^2 = \mathcal{U}(\Psi) = \frac{\Phi(\Psi)}{Y(\Psi)} = \frac{\mu_\sigma \Psi^\sigma + \dots + \mu_1 \Psi + \mu_0}{\chi_\rho \Psi^\rho + \dots + \chi_1 \Psi + \chi_0}. \tag{8}$$

Here, in (7) and (8), $\tau_0, \dots, \tau_\zeta; \mu_0, \dots, \mu_\sigma$ and χ_0, \dots, χ_ρ are constants to be determined later. Integral form of Eq. (8) is given by

$$\pm(\xi - \xi_0) = \int \frac{d\Psi}{\sqrt{\mathcal{U}(\Psi)}} = \int \sqrt{\frac{Y(\Psi)}{\Phi(\Psi)}} d\Psi. \tag{9}$$

Balancing the orders of P'' and P^3 in (5) gives

$$\sigma = \rho + 2\zeta + 2. \tag{10}$$

Let $\sigma = 4, \rho = 0$ and $\zeta = 1$ in Eq. (10). Then, the adopted approach proposes the use of

$$P = \tau_0 + \tau_1 \Psi, \tag{11}$$

where τ_0 and τ_1 are all constants such that $\tau_1 \neq 0$, and Ψ satisfies Eq. (8). Putting (11) into (5), and solving the resulting system, the following set of solutions is derived:

$$\begin{aligned} \mu_0 &= \mu_0, \quad \mu_1 = \mu_1, \quad \mu_2 = \mu_2, \quad \tau_0 = \tau_0, \quad \tau_1 = \tau_1, \\ \chi_0 &= \frac{aB_1B_2(2\mu_2\tau_0 - \mu_1\tau_1)}{8b\kappa_1\tau_0^3}, \\ \mu_3 &= \frac{\tau_1(2\mu_2\tau_0 - \mu_1\tau_1)}{2\tau_0^2}, \\ \mu_4 &= \frac{\tau_1^2(2\mu_2\tau_0 - \mu_1\tau_1)}{8\tau_0^3}, \\ \omega &= -\frac{\kappa_1[2b\tau_0^2(2\mu_2\tau_0 - 3\mu_1\tau_1) + a\kappa_2(2\mu_2\tau_0 - \mu_1\tau_1)]}{2\mu_2\tau_0 - \mu_1\tau_1}. \end{aligned} \tag{12}$$

Inserting these results into (8) and (9) yields

$$\pm(\xi - \xi_0) = \Omega \int \frac{d\Psi}{\sqrt{\mathfrak{U}(\Psi)}}, \tag{13}$$

where

$$\Omega = \sqrt{\frac{\chi_0}{\mu_4}}, \tag{14}$$

$$\mathfrak{U}(\Psi) = \Psi^4 + \frac{\mu_3}{\mu_4}\Psi^3 + \frac{\mu_2}{\mu_4}\Psi^2 + \frac{\mu_1}{\mu_4}\Psi + \frac{\mu_0}{\mu_4}. \tag{15}$$

Eventually the exact traveling wave solutions for the model (1) are recovered as below:

For $\mathfrak{U}(\Psi) = (\Psi - \epsilon_1)^4$,

$$\begin{aligned} q(x, y, t) &= \left\{ \tau_0 + \tau_1\epsilon_1 \pm \frac{\tau_1\Omega}{B_1x + B_2y + a(\kappa_1B_2 + \kappa_2B_1)t - \xi_0} \right\} \\ &\times \exp \left[i \left\{ -\kappa_1x - \kappa_2y - \left(\frac{\kappa_1[2b\tau_0^2(2\mu_2\tau_0 - 3\mu_1\tau_1) + a\kappa_2(2\mu_2\tau_0 - \mu_1\tau_1)]}{2\mu_2\tau_0 - \mu_1\tau_1} \right) t + \theta \right\} \right]. \end{aligned} \tag{16}$$

If $\mathfrak{U}(\Psi) = (\Psi - \epsilon_1)^3(\Psi - \epsilon_2)$ and $\epsilon_2 > \epsilon_1$,

$$\begin{aligned} q(x, y, t) &= \left\{ \tau_0 + \tau_1\epsilon_1 + \frac{4\tau_1\Omega^2(\epsilon_2 - \epsilon_1)}{4\Omega^2 - [(\epsilon_1 - \epsilon_2)(B_1x + B_2y + a(\kappa_1B_2 + \kappa_2B_1)t - \xi_0)]^2} \right\} \\ &\times \exp \left[i \left\{ -\kappa_1x - \kappa_2y - \left(\frac{\kappa_1[2b\tau_0^2(2\mu_2\tau_0 - 3\mu_1\tau_1) + a\kappa_2(2\mu_2\tau_0 - \mu_1\tau_1)]}{2\mu_2\tau_0 - \mu_1\tau_1} \right) t + \theta \right\} \right]. \end{aligned} \tag{17}$$

However, when $\mathfrak{U}(\Psi) = (\Psi - \epsilon_1)^2(\Psi - \epsilon_2)^2$,

$$\begin{aligned} q(x, y, t) &= \left\{ \tau_0 + \tau_1\epsilon_2 + \frac{\tau_1(\epsilon_2 - \epsilon_1)}{\exp \left[\frac{\epsilon_1 - \epsilon_2}{\Omega}(B_1x + B_2y + a(\kappa_1B_2 + \kappa_2B_1)t - \xi_0) \right] - 1} \right\} \\ &\times \exp \left[i \left\{ -\kappa_1x - \kappa_2y - \left(\frac{\kappa_1[2b\tau_0^2(2\mu_2\tau_0 - 3\mu_1\tau_1) + a\kappa_2(2\mu_2\tau_0 - \mu_1\tau_1)]}{2\mu_2\tau_0 - \mu_1\tau_1} \right) t + \theta \right\} \right], \end{aligned} \tag{18}$$

and

$$\begin{aligned} q(x, y, t) &= \left\{ \tau_0 + \tau_1\epsilon_1 + \frac{\tau_1(\epsilon_1 - \epsilon_2)}{\exp \left[\frac{\epsilon_1 - \epsilon_2}{\Omega}(B_1x + B_2y + a(\kappa_1B_2 + \kappa_2B_1)t - \xi_0) \right] - 1} \right\} \\ &\times \exp \left[i \left\{ -\kappa_1x - \kappa_2y - \left(\frac{\kappa_1[2b\tau_0^2(2\mu_2\tau_0 - 3\mu_1\tau_1) + a\kappa_2(2\mu_2\tau_0 - \mu_1\tau_1)]}{2\mu_2\tau_0 - \mu_1\tau_1} \right) t + \theta \right\} \right]. \end{aligned} \tag{19}$$

Whenever $\mathfrak{U}(\Psi) = (\Psi - \epsilon_1)^2(\Psi - \epsilon_2)(\Psi - \epsilon_3)$ and $\epsilon_1 > \epsilon_2 > \epsilon_3$,

$$q(x, y, t) = \left\{ \tau_0 + \tau_1 \epsilon_1 - \frac{2\tau_1(\epsilon_1 - \epsilon_2)(\epsilon_1 - \epsilon_3)}{2\epsilon_1 - \epsilon_2 - \epsilon_3 + (\epsilon_3 - \epsilon_2) \cosh \left[\frac{\sqrt{(\epsilon_1 - \epsilon_2)(\epsilon_1 - \epsilon_3)}}{\Omega} (B_1 x + B_2 y + a(\kappa_1 B_2 + \kappa_2 B_1)t) \right]} \right\} \times \exp \left[i \left\{ -\kappa_1 x - \kappa_2 y - \left(\frac{\kappa_1 [2b\tau_0^2 (2\mu_2 \tau_0 - 3\mu_1 \bar{\tau}_1)] + a\kappa_2 (2\mu_2 \tau_0 - \mu_1 \bar{\tau}_1)}{2\mu_2 \tau_0 - \mu_1 \bar{\tau}_1} \right) t + \theta \right\} \right]. \tag{20}$$

On the other hand, if $\mathcal{U}(\Psi) = (\Psi - \epsilon_1)(\Psi - \epsilon_2)(\Psi - \epsilon_3)(\Psi - \epsilon_4)$ and $\epsilon_1 > \epsilon_2 > \epsilon_3 > \epsilon_4$,

$$q(x, y, t) = \left\{ \tau_0 + \tau_1 \epsilon_2 + \frac{\tau_1(\epsilon_1 - \epsilon_2)(\epsilon_4 - \epsilon_2)}{\epsilon_4 - \epsilon_2 + (\epsilon_1 - \epsilon_4) \operatorname{sn}^2 \left[\pm \frac{\sqrt{(\epsilon_1 - \epsilon_3)(\epsilon_2 - \epsilon_4)}}{2\Omega} (B_1 x + B_2 y + a(\kappa_1 B_2 + \kappa_2 B_1)t - \xi_0), m \right]} \right\} \times \exp \left[i \left\{ -\kappa_1 x - \kappa_2 y - \left(\frac{\kappa_1 [2b\tau_0^2 (2\mu_2 \tau_0 - 3\mu_1 \bar{\tau}_1)] + a\kappa_2 (2\mu_2 \tau_0 - \mu_1 \bar{\tau}_1)}{2\mu_2 \tau_0 - \mu_1 \bar{\tau}_1} \right) t + \theta \right\} \right], \tag{21}$$

where modulus m is given as

$$m^2 = \frac{(\epsilon_2 - \epsilon_3)(\epsilon_1 - \epsilon_4)}{(\epsilon_1 - \epsilon_3)(\epsilon_2 - \epsilon_4)}. \tag{22}$$

It is important to highlight that ϵ_j for $j = 1, \dots, 4$ are the roots of

$$\mathcal{U}(\Psi) = 0. \tag{23}$$

Let $\tau_0 = -\tau_1 \epsilon_1$ and $\xi_0 = 0$. In this case, the solutions in Eqs. (16)–(20) can be converted to plane wave solutions

$$q(x, y, t) = \left\{ \pm \frac{\tau_1 \Omega}{B_1 x + B_2 y + a(\kappa_1 B_2 + \kappa_2 B_1)t} \right\} \times \exp \left[i \left\{ -\kappa_1 x - \kappa_2 y - \left(\frac{\kappa_1 [2b\tau_0^2 (2\mu_2 \tau_0 - 3\mu_1 \bar{\tau}_1)] + a\kappa_2 (2\mu_2 \tau_0 - \mu_1 \bar{\tau}_1)}{2\mu_2 \tau_0 - \mu_1 \bar{\tau}_1} \right) t + \theta \right\} \right], \tag{24}$$

$$q(x, y, t) = \left\{ \frac{4\tau_1 \Omega^2 (\epsilon_2 - \epsilon_1)}{4\Omega^2 - [(\epsilon_1 - \epsilon_2)(B_1 x + B_2 y + a(\kappa_1 B_2 + \kappa_2 B_1)t)]^2} \right\} \times \exp \left[i \left\{ -\kappa_1 x - \kappa_2 y - \left(\frac{\kappa_1 [2b\tau_0^2 (2\mu_2 \tau_0 - 3\mu_1 \bar{\tau}_1)] + a\kappa_2 (2\mu_2 \tau_0 - \mu_1 \bar{\tau}_1)}{2\mu_2 \tau_0 - \mu_1 \bar{\tau}_1} \right) t + \theta \right\} \right], \tag{25}$$

singular soliton solutions

$$q(x, y, t) = \left\{ \frac{\tau_1(\epsilon_2 - \epsilon_1)}{2} \left(1 \mp \coth \left[\frac{\epsilon_1 - \epsilon_2}{2\Omega} (B_1 x + B_2 y + a(\kappa_1 B_2 + \kappa_2 B_1)t) \right] \right) \right\} \times \exp \left[i \left\{ -\kappa_1 x - \kappa_2 y - \left(\frac{\kappa_1 [2b\tau_0^2 (2\mu_2 \tau_0 - 3\mu_1 \bar{\tau}_1)] + a\kappa_2 (2\mu_2 \tau_0 - \mu_1 \bar{\tau}_1)}{2\mu_2 \tau_0 - \mu_1 \bar{\tau}_1} \right) t + \theta \right\} \right], \tag{26}$$

and bright soliton solution

$$q(x, y, t) = \left\{ \frac{\mathcal{P}}{\mathcal{R} + \cosh[\mathcal{S}(B_1 x + B_2 y + a(\kappa_1 B_2 + \kappa_2 B_1)t)]} \right\} \times \exp \left[i \left\{ -\kappa_1 x - \kappa_2 y - \left(\frac{\kappa_1 [2b\tau_0^2 (2\mu_2 \tau_0 - 3\mu_1 \bar{\tau}_1)] + a\kappa_2 (2\mu_2 \tau_0 - \mu_1 \bar{\tau}_1)}{2\mu_2 \tau_0 - \mu_1 \bar{\tau}_1} \right) t + \theta \right\} \right], \tag{27}$$

where

$$\mathcal{P} = \frac{2\tau_1(\epsilon_1 - \epsilon_2)(\epsilon_1 - \epsilon_3)}{\epsilon_3 - \epsilon_2}, \tag{28}$$

$$\mathcal{S} = \frac{\sqrt{(\epsilon_1 - \epsilon_2)(\epsilon_1 - \epsilon_3)}}{\Omega}, \tag{29}$$

$$\mathcal{R} = \frac{2\epsilon_1 - \epsilon_2 - \epsilon_3}{\epsilon_3 - \epsilon_2}. \tag{30}$$

Here the amplitude of the soliton is given by (28), while the inverse width of the soliton is given by (29). These solitons will exist for $\tau_1 < 0$. Moreover, when $\tau_0 = -\tau_1\epsilon_2$ and $\xi_0 = 0$, Jacobi elliptic function solutions given by (21) are degraded to

$$q(x, y, t) = \left\{ \frac{\mathcal{P}_1}{\mathcal{R}_1 + \text{sn}^2 \left[\mathcal{S}_j(B_1x + B_2y + a(\kappa_1B_2 + \kappa_2B_1)t), \frac{(\epsilon_2 - \epsilon_3)(\epsilon_1 - \epsilon_4)}{(\epsilon_1 - \epsilon_3)(\epsilon_2 - \epsilon_4)} \right]} \right\} \times \exp \left[i \left\{ -\kappa_1x - \kappa_2y - \left(\frac{\kappa_1 [2b\tau_0^2(2\mu_2\tau_0 - 3\mu_1\bar{\tau}_1)] + a\kappa_2(2\mu_2\tau_0 - \mu_1\bar{\tau}_1)}{2\mu_2\tau_0 - \mu_1\bar{\tau}_1} \right) t + \theta \right\} \right], \tag{31}$$

where

$$\mathcal{P}_1 = \frac{\bar{\tau}_1(\epsilon_1 - \epsilon_2)(\epsilon_4 - \epsilon_2)}{\epsilon_1 - \epsilon_4}, \tag{32}$$

$$\mathcal{R}_1 = \frac{\epsilon_4 - \epsilon_2}{\epsilon_1 - \epsilon_4}, \tag{33}$$

$$\mathcal{S}_j = \frac{(-1)^j \sqrt{(\epsilon_1 - \epsilon_3)(\epsilon_2 - \epsilon_4)}}{2\Omega} \text{ for } j = 1, 2. \tag{34}$$

Remark-1: When the modulus $m \rightarrow 1$, singular optical soliton solutions take place

$$q(x, y, t) = \left\{ \frac{\mathcal{P}_1}{\mathcal{R}_1 + \tanh^2[\mathcal{S}_j(B_1x + B_2y + a(\kappa_1B_2 + \kappa_2B_1)t)]} \right\} \times \exp \left[i \left\{ -\kappa_1x - \kappa_2y - \left(\frac{\kappa_1 [2b\tau_0^2(2\mu_2\tau_0 - 3\mu_1\bar{\tau}_1)] + a\kappa_2(2\mu_2\tau_0 - \mu_1\bar{\tau}_1)}{2\mu_2\tau_0 - \mu_1\bar{\tau}_1} \right) t + \theta \right\} \right], \tag{35}$$

for $\epsilon_3 = \epsilon_4$.

Remark-2: However, if $m \rightarrow 0$, Eq. (31) degenerates to periodic singular solutions

$$q(x, y, t) = \left\{ \frac{\mathcal{P}_1}{\mathcal{R}_1 + \text{sin}^2[\mathcal{S}_j(B_1x + B_2y + a(\kappa_1B_2 + \kappa_2B_1)t)]} \right\} \times \exp \left[i \left\{ -\kappa_1x - \kappa_2y - \left(\frac{\kappa_1 [2b\tau_0^2(2\mu_2\tau_0 - 3\mu_1\bar{\tau}_1)] + a\kappa_2(2\mu_2\tau_0 - \mu_1\bar{\tau}_1)}{2\mu_2\tau_0 - \mu_1\bar{\tau}_1} \right) t + \theta \right\} \right], \tag{36}$$

for $\epsilon_2 = \epsilon_3$.

3. Conclusion

This paper secured bright and singular optical soliton solutions to KMN equation that is proposed to govern soliton dynamics in (2+1)-dimensions along excited resonant waveguides that is doped with Erbium atoms. The extended trial function method was adopted to obtain these soliton solutions as well as other solutions that are written in terms of Jacobi’s elliptic function method. The listed solutions are being reported for the first time. It must be noted that the stability of the solutions have not been addressed in the paper. One of the future activities will involve studying the model using numerical algorithms and analyzing the stability of the solutions.

Being a newly proposed model, there is a lot of scope to broaden the horizon in this context. The extended trial function scheme being a nontrivial approach, additional integration methodologies are to be applied, in future, to secure further new solutions to KMN equation. These include Lie symmetry analysis, mapping methods, trial solution approach, Kudryashov’s scheme and several others. These results will be surely and sequentially available but is currently awaited.

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Conflicts of interest

The authors declare no conflict of interest.

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