Spiraling Behavior of Photorefractive Screening Solitons

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The question of spiraling or no spiraling of incoherent screening spatial solitons in photorefractive crystals is addressed by a careful numerical analysis. We find that the interaction of solitons typically results in an initial mutual rotation of their trajectories, followed by damped oscillations and the fusion of solitons. The rotation can be propelled to prolonged spiraling by the skewed launching of beams. This behavior is caused by the anisotropy of the refractive index change in the crystal. [S0031-9007(98)08275-1]

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Recently, a controversy erupted in the literature [1,2] and at conferences [3] regarding the possibility of spiraling of incoherent spatial screening solitons in photorefractive (PR) crystals [4]. While the question is of little importance on its own, it gains in weight when one realizes that the departure from spiraling hints at nonisotropic interaction of screening solitons.

Segev and co-workers claim that the interaction between PR screening solitons is primarily isotropic in nature, regardless of the inherent anisotropy of PR media [1]. Isotropic interaction produces indefinite spiraling. Królikowski and co-workers claim that the PR anisotropy leads to the repulsion of solitons in the direction of the biasing electric field [2], which prevents the indefinite spiraling and induces oscillation in the direction perpendicular to the biasing field.

The claims of both groups are corroborated by experimental findings; however, the whole argument is rather academic if one stays at the level of experimental results. The devil is in the details. The resolution asks for a detailed consideration of beam propagation within the crystal over large distances, which may not be accessible to experimentation.

We address the question by a careful numerical analysis of the model which adequately describes the generation and propagation of screening solitons in PR media. We reproduce the findings of both groups by choosing the material parameters and initial conditions according to the experimental situation, then show how one picture blends with the other by an appropriate change in initial conditions. The crucial ingredient is the existence of "angular momentum" of two interacting solitons relative to the common "center of mass." In experiment this is accomplished by a nonparallel launching of initial beams. It is worth mentioning that early numerical simulations [2] could not capture the tilted launching adequately, and this may have added to the confusion.

Spatial optical solitons are important on their own for considerable applicative potential [5–8]. They are also useful for experimental verification of theoretical predictions regarding properties of individual solitons, as well as soliton pairs. Recent experiments with the PR strontium barium niobate (SBN) crystal have revealed curious effects, such as the creation [9] and annihilation [10] of spatial solitons.

Let us state the facts of the case. A while ago it was predicted [11] that the incoherent spatial solitons in a self-focusing medium should spiral [12] around each other if their mutual attraction could counterbalance the divergence of trajectories. A report on the experimental observation of this effect in a PR crystal has recently been published [1]. Spiraling for up to 3π over a propagation distance of 13 mm has been seen. For the spiraling to occur it is necessary to have attractive interaction between solitons. While this certainly is the case in isotropic self-focusing Kerr-type materials, the situation with PR media is more complex. The nonlinear response of a PR crystal can be anisotropic [13]. Two incoherent solitons may experience both attractive and repulsive forces, depending on relative separation and location in the crystal [2,14]. After an initial mutual rotation, the beams are slowed down by the repulsive shoulders along the direction of the external field, and arrested by the attractive well perpendicular to it.

We find that the generic behavior of soliton interaction includes the elements of both pictures. The beams always initially rotate. For low intensities above saturation and parallel launching (or nonparallel, but with an insufficient tilt), the initial rotation is followed by the oscillation perpendicular to the direction of the applied field, and eventually the beams fuse. On the other hand, the initial rotation can be propelled to long-lasting spiraling by increasing the initial intensity and the launching tilt. The distance where the beams stop spiraling and start oscillating might be much larger than the typical thickness of the crystal. In this manner the influence of anisotropy is decreased, and for all practical purposes the beams spiral indefinitely.

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The model describing the interaction of screening solitons in PR media is based on the paraxial approximation to the propagation of optical beams and the Kukhtarev material equations [13,14]. The propagation of beams along the z axis is described by the following equations:

$$\left[\frac{\partial}{\partial z} + \vec{\theta}_1 \cdot \nabla - \frac{i}{2}\nabla^2\right]A_1 = \frac{i\gamma}{2}\left(\frac{\partial \varphi}{\partial x} - E_0\right)A_1, \quad (1a)$$

$$\left[\frac{\partial}{\partial z} + \vec{\theta}_2 \cdot \nabla - \frac{i}{2}\nabla^2\right]A_2 = \frac{i\gamma}{2}\left(\frac{\partial \varphi}{\partial x} - E_0\right)A_2, \quad (1b)$$

for the slowly varying envelopes of the two beams $A_1(\vec{r})$ and $A_2(\vec{r})$. The vectors $\vec{\theta}_1$ and $\vec{\theta}_2$ specify the directions of beam launching, ∇ is the transverse gradient, and $\gamma = k^2 n^4 x_0^2 r_{\rm eff}$ is the medium-light coupling constant. Here k is the wave number of light, n is the index of refraction, x_0 is the input-beam spot size, and $r_{\rm eff}$ is the effective element of the electro-optic tensor. The transverse coordinates x and y are scaled by x_0 and the propagation coordinate z is scaled by the diffraction length $L_D = knx_0^2$. φ is the electrostatic potential induced by the light, whose evolution is described by the following relaxation equation:

$$\tau \frac{\partial}{\partial t}(\nabla^2 \varphi) + \nabla^2 \varphi + \nabla\varphi \cdot \nabla \ln I$$
$$= E_0 \frac{\partial}{\partial x}\ln I + \frac{k_B T}{e}\left[\nabla^2 \ln I + (\nabla \ln I)^2\right], \quad (2)$$

where τ is the relaxation time of the crystal, and E_0 is the external biasing field directed along the crystalline c axis, which is also the x axis of our coordinate system. The total intensity $I = 1 + |A_1|^2 + |A_2|^2$ is measured in units of the saturation intensity. The terms on the right side of Eq. (2) describe the drift and diffusion of charges in the crystal.

The set of equations (1) and (2) is integrated numerically to steady state, for a range of initial conditions. A beam propagation method is used, with absorptive boundary conditions. All material parameters correspond to the values found for SBN crystals in actual experiments [1,2]. In all simulations the input beams are assumed to be Gaussian of sufficient intensity ($I_{1,2} = \sim 2 - 5$) and shape to yield solitons.

The trajectory of a soliton is defined as the spatial expectation value of its transverse coordinates,

$$\langle x \rangle_i(z) = I_{\rm tot}^{-1}\int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dx\ x|A_i(x,y,z)|^2, \quad (3a)$$

$$\langle y \rangle_i(z) = I_{\rm tot}^{-1}\int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dx\ y|A_i(x,y,z)|^2, \quad (3b)$$

where $i = 1, 2$. These quantities are normalized by the total power of the beam $I_{\rm tot} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \times |A_i(x,y,z)|^2 dxdy$. This center-of-beam representation is more appropriate than just determining the point of maximum intensity of the beam. It is a good approximation to the beam position even when the deformation of beams is large and the beams split.

The launching of incoherent solitons along the direction of an applied field leads to their anomalous interaction [2]. Owing to the anisotropy of screening, the soliton interaction is repulsive for well-separated beams and attractive for the overlapping beams. As a consequence, there exist domains of attraction and repulsion in the transverse plane which lead to the nontrivial topology of soliton trajectories when the beams are launched slanted to the direction of external field.

In Fig. 1, ten separate soliton pairs are launched perpendicularly to the transverse plane, oriented at some angle to the direction of the biasing field, corresponding to the Królikowski conditions (no tilt, low intensity). The intensity of all beams equals 1.2. The origin is chosen to be the center of mass of all pairs. 1–3 are shot outside the domain of attraction; thus the beams fly apart. Other pairs initially rotate counterclockwise trying to align along the y axis. As the beams cross the y axis the anisotropy of screening slows down the rate of rotation and reverses its direction. The distance between solitons decreases and the pairs twist and turn about the z axis in a damped motion. When viewed along the z axis the initial motion is rotation, followed by oscillation predominantly in the y direction.

Experimental results of both Króliklowski and Segev are reproduced numerically in Fig. 2.

The figure presents generic examples of the motion of a soliton pair launched obliquely in the direction of the external field. All pairs are launched from the same position, which corresponds to the pair 8 in Fig. 1. The pair shown in Fig. 2(a) is launched under the Królikowski conditions, i.e., no tilt, low intensity. It starts to rotate;

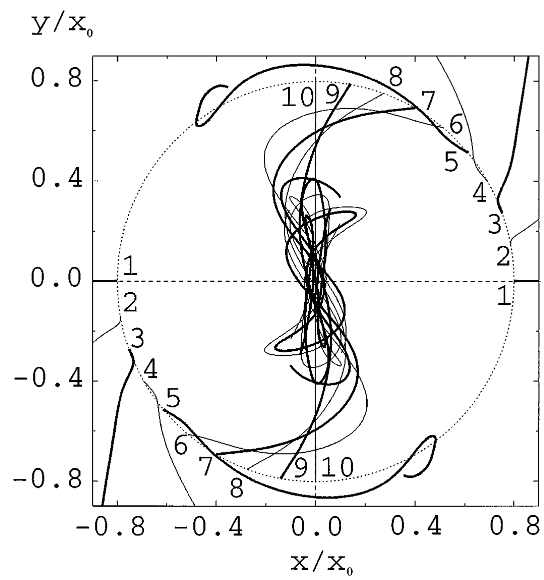

FIG. 1. Projection in the transverse plane of the trajectories of ten soliton pairs launched separately at different initial positions along a circle. Pairs 1–3 are launched outside the domain of attraction. All pairs propagated for 13 mm.

however, after one twist the beams unwind and remain on the same side of each other, producing damped oscillations. The same pair is launched in Fig. 2(b) with a tilt $\theta_x = \mp 0.8$, $\theta_y = 0$ for the beams, respectively, and it spirals for more than 2π before it is captured by the y axis. This example corresponds to an experiment in Ref. [15], where the spiraling for more than π is seen in a crystal of 10 mm thickness. Both solitons are shot with the initial intensity $I_1 = I_2 = 2$.

When the initial conditions are changed to correspond to the Segev experiment, i.e., high intensity and tilt, the pair spirals for 3π [Fig. 2(c)] as in Ref. [1]. The pair is launched with the same tilt as in Fig. 2(b), but with the intensity $I_1 = I_2 = 5.5$. It spirals for the whole length, with a swing about the y axis each time it is crossed. In Fig. 2(d) the pair is launched with the same conditions, but with a smaller tilt $\theta_x = \mp 0.3$, $\theta_y = 0$. Now it oscillates. In all of the spiraling cases, if the propagation is extended, the beams eventually stop spiraling and start oscillating. For the experimental case shown in Fig. 2(c) the spiraling stops at 14 mm, just beyond the actual thickness of the crystal. However, for similar conditions we were able to prolong the spiraling for more than 4π over the distance of 29 mm.

To corroborate the designation of soliton behavior in Fig. 2 quantitatively, we introduce the winding angle, defined as the angle between the x axis and the orientation vector, pointing transversely from one beam to the other. Figure 3 depicts the winding angle for the solitons presented in Fig. 2. Clearly, the launching of skewed beams at a higher intensity changes the behavior from rotation/oscillation to prolonged spiraling. Tilted beams carry initial angular momentum relative to the origin. However, the trajectories observed are not simple, smooth spirals. The beams wobble while spiraling. The "potential" in which the solitons rotate is not central. The long attractive well along the y axis and the repulsive shoulders along the x axis break the symmetry and prevent indefinite spiraling. It should be mentioned that, when the solitons are close to each other and interact strongly, they entangle and their individual identities are rather dubious. The light intensity distributions do not show two distinct beams anymore. However, as soon as the beams disentangle, two bright spots reappear.

In summary, we have clarified the controversy surrounding the spiraling of incoherent screening solitons in PR media. The behavior is the same generically, but appears different when observed under different experimental conditions. The beams perform complicated motion in the transverse plane. Initially, the beams always rotate. For low intensities above saturation and parallel launching, the initial rotation is followed by the oscillation perpendicular to the direction of applied field, and eventually the beams fuse. The off-axes launching at higher intensities, in combination with the initial angular tilt, produces prolonged spiraling. The interaction between incoherent screening solitons is basically anisotropic, which is less apparent

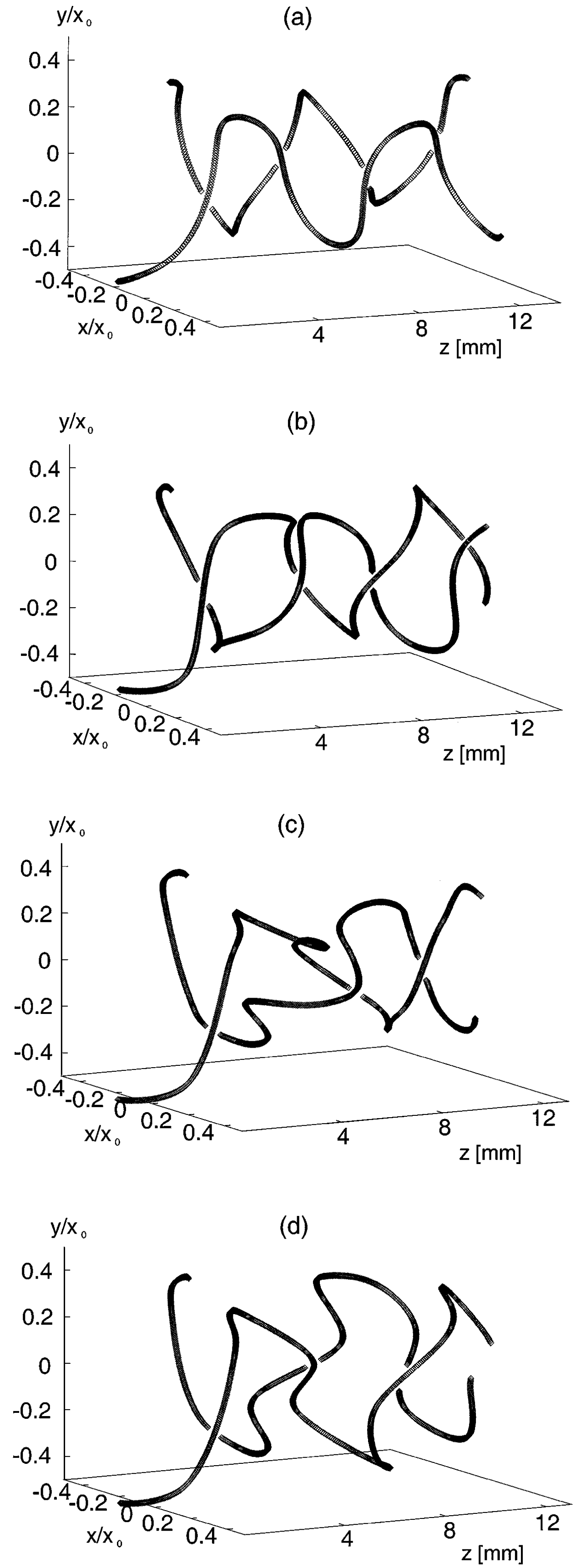

FIG. 2. Three-dimensional trajectories for different experimental conditions. (a) Królikowski oscillation, (b) Królikowski spiraling, (c) Segev spiraling, and (d) Segev oscillation.

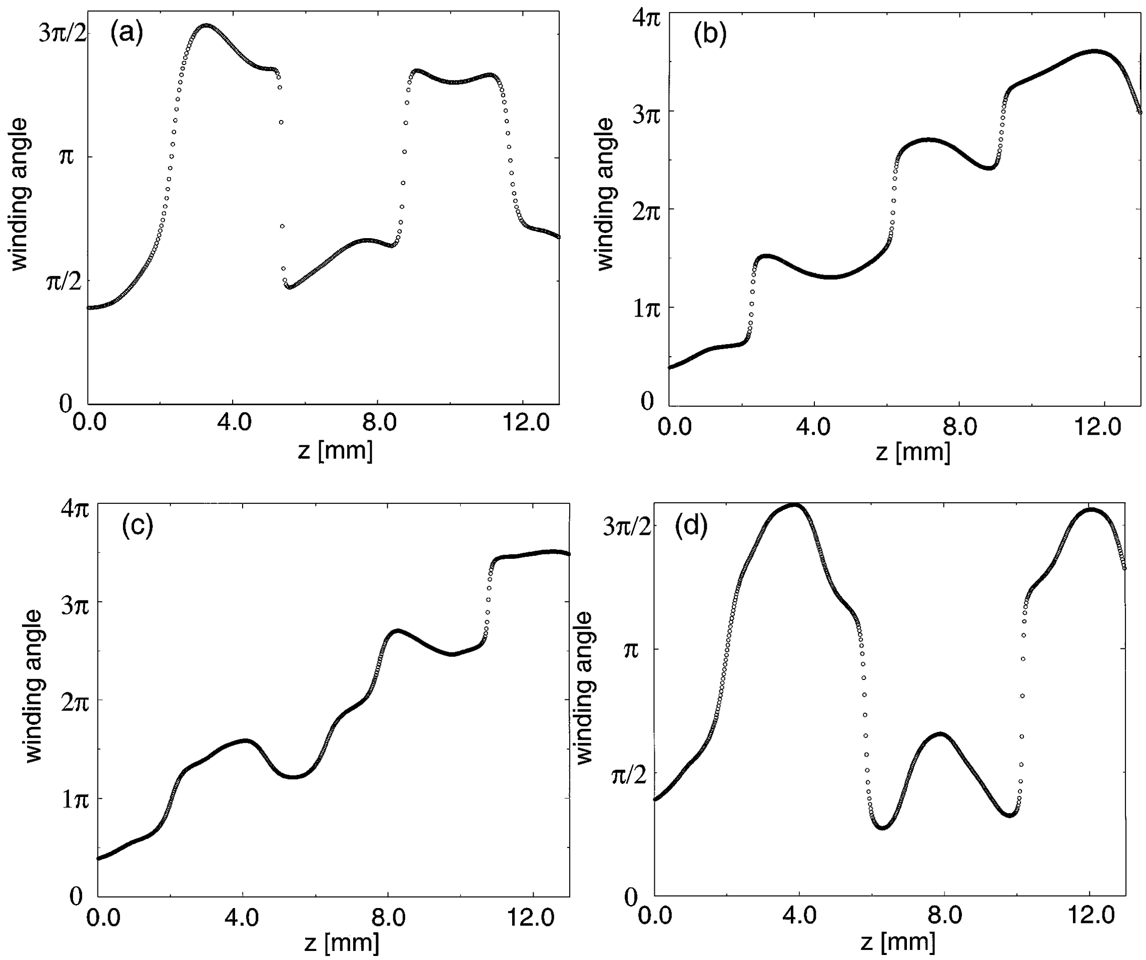

FIG. 3. Winding angle for the beams presented in Fig. 2. (a) Oscillation about the y axis, (b) spiraling for 2π, (c) spiraling for 3π, and (d) oscillation about the y axis.

for skewed launching at high intensities, and for an arbitrary angle between the c axis and the external field.

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