Transverse localization of light in nonlinear photonic lattices with dimensionality crossover

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In a numerical study, we demonstrate the dimensionality crossover in Anderson localization of light. We consider crossover from the two-dimensional (2D) to the one-dimensional (1D) lattice, optically induced in both linear and nonlinear dielectric media. The joint influence of nonlinearity and disorder on Anderson localization in such systems is discussed in some detail. We find that, in the linear regime, the localization is more pronounced in two dimensions than in one dimension. We also find that the localization in the intermediate cases of crossover is less pronounced than in both the pure 1D and 2D cases in the linear regime, whereas in the nonlinear regime this depends on the strength of the nonlinearity. There exist strongly nonlinear regimes in which 1D localization is more pronounced than the 2D localization, opposite to the case of the linear regime. We find that the dimensionality crossover is characterized by two different localization lengths, whose behavior is different along different transverse directions.

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I. INTRODUCTION

In recent years there has been increased interest in the study of Anderson localization in disordered systems [1,2]. Both physicists and mathematicians have contributed greatly to the understanding of various features of Anderson localization theoretically and experimentally and continue to contribute even more than 50 years since its discovery [3].

Originally proposed for electrons and one-particle excitations in solids [1], Anderson localization was soon observed in many other fields of physics, such as acoustics [4], Bose-Einstein condensates [5], and optics [6,7]. It is a universal wave phenomenon that is at the center of recent investigations in discrete photonic lattices with random structure. Anderson localization is realized experimentally as transverse localization in two-dimensional (2D) [8] and in one-dimensional (1D) [9] random lattice potentials.

A renewed interest is devoted to physical systems with dimensionality crossover, for example the continuous transformation of the lattice structure from one dimension to two dimensions [10]. A similar problem is investigated within the context of electric properties of disordered media [11,12]. A natural question arises in such systems: When and how does a system cross over from one to two dimensions? Owing to the mathematical analogy in describing the evolution of electronic and photonic wave packets, many related phenomena can be explored better in the optical domain.

In this paper we analyze the effect of lattice dimensionality crossover on the Anderson localization of light, specifically localization in 1D and 2D photonic lattices, including also localization upon continuously transforming the lattice structure from two dimensions to one dimension. A systematic quantitative study of the dependence on both the strength of disorder and the strength of nonlinearity on the Anderson localization in such a system is presented. Here, we consider the Kerr-type cubic nonlinearity. While in the linear regime Anderson localization is more pronounced in two dimensions than in one dimension, the situation is the inverse in the nonlinear regime. We study localization in the nonlinear regime with both focusing and defocusing nonlinearity and compare them with the localization in the linear regime. There exist strongly focusing nonlinear regimes, where the localization is more pronounced in 1D than in 2D lattices. However, in the defocusing regimes, localization is always less pronounced than in the linear regime.

We investigate a gradual transition of Anderson localization behavior from the 2D to the 1D case by measuring the quantities of interest: the inverse participation ratio, the effective width, and the localization length. We observe two different localization lengths, along the two transverse directions in the system with dimensionality crossover. The localization of intermediate states is less pronounced than both the pure 1D and 2D cases in the linear regime. However, in the nonlinear regime localization depends on the strength of the nonlinearity.

The paper is organized as follows. In Sec. II we introduce the model, which describes the propagation of light in a nonlinear medium with an induced lattice potential. Section III summarizes our results in the linear regime, while Sec. IV discusses the nonlinear regime vs the linear regime, considering as well the defocusing localization. In Sec. V we study dimensionality crossover in the system. Finally, Sec. VI concludes the paper.

II. THEORETICAL MODEL AND SYSTEM GEOMETRY

We study localization of light in optically induced photonic lattices and describe the propagation of a beam along the z axis using the effective nonlinear Schrödinger equation for the complex electric field amplitude E:

$$i\frac{\partial E}{\partial z} = -\Delta E - \gamma |E|^2 E - VE, \tag{1}$$

where Δ is the transverse Laplacian, γ is the dimensionless nonlinearity strength, and V is the transverse lattice potential. Here, it is defined as a sum of Gaussian beams, with peak intensity V_0. A scaling $x/x_0 \to x$, $y/x_0 \to y$, $z/L_D \to z$ is

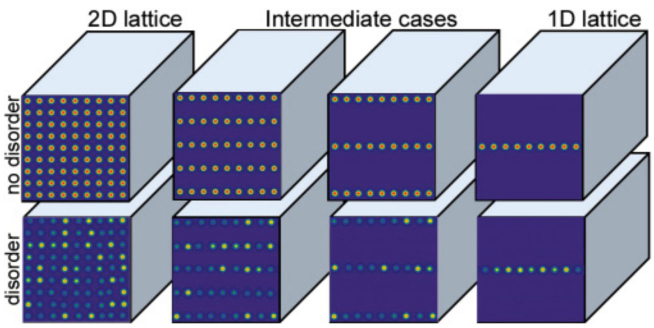

FIG. 1. (Color online) Dimensionality crossover from 2D to 1D photonic lattice, showing two intermediate cases.

used for the dimensionless equation, where x_0 is the typical full width at half maximum (FWHM) beam waist and L_D is the diffraction length. The propagation equation is solved numerically by employing a numerical approach developed earlier [13]. To study Anderson localization effects, we realize disorder using random lattice intensity and also random lattice period. The random lattice intensity V_{0r} takes the values in the range $V_0(1 - Nr) < V_{0r} < V_0(1 + Nr)$, where r is the random number generator from the interval [0,1], and N determines the degree of disorder. The random lattice period takes the values from the range $[d \pm 0.5d]$, where d is the lattice period.

First, we investigate localization effects in the 1D disordered photonic lattice and compare them with the localization in the 2D photonic lattice. To observe dimensionality crossover effects, we start from the 2D square photonic lattice and increase the lattice period along one transverse direction, keeping the period along the other transverse direction fixed (Fig. 1). In such a procedure, after a while one effectively reaches the 1D lattice, when the 1D arrays are too far away from each other to feel any interaction. We can then analyze the crossover from 2D to 1D systems and localization effects in the intermediate cases, when the distance between 1D arrays is not that great. With such a gradual transition from a 2D to a 1D lattice, one may study the dimensionality crossover in Anderson localization of light.

III. LOCALIZATION IN THE LINEAR REGIME

We start with the investigation of the localization effects in the linear regime in 1D and 2D lattices. The linear regime means that the nonlinearity is turned off ($\gamma = 0$). To observe the effect of Anderson localization we increase the level of disorder. Typical results are summarized in Fig. 2. For quantitative analysis we use the standard quantities for the description of Anderson localization: the inverse participation ratio, $P = \int I^2(x,y,L)dxdy/[\int I(x,y,L)dxdy]^{-2}$, and the effective beam width $\omega_{\mathrm{eff}} = P^{-1/2}$. To compare 1D and 2D Anderson localization, we measure the effective beam width at the lattice output for different disorder levels. Many realizations of disorder are needed to measure such quantities. In our numerics, different disorder realizations are realized by starting each simulation with different seeds for the random-number generator. We take 100 realizations of disorder for each disorder level. Error bars in Figs. 3 and 4 depict the spread in values coming from different runs.

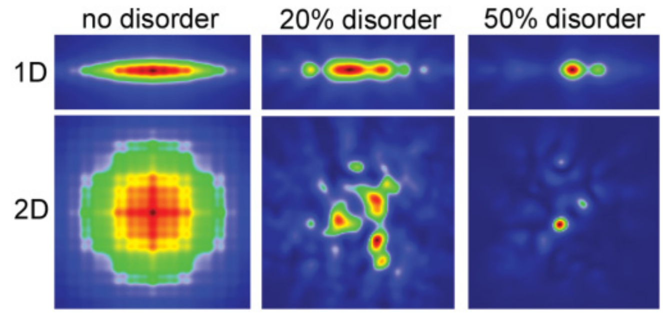

FIG. 2. (Color online) Examples of Anderson localization in 1D and 2D photonic lattices. Localized modes are shown in the linear regime. Physical parameters are crystal length $L = 20$ mm, input lattice intensity $V_0 = 1$, lattice period $d = 15$ μm, input beam intensity $|E_0|^2 = 0.5$, input beam FWHM $= 13$ μm.

Figure 3 presents a comparison between 1D and 2D localization in the linear regime. Averaged effective widths normalized with the corresponding input values are presented as functions of the disorder level. The effective beam width decreases as the level of disorder is increased in both 1D and 2D cases, but the decrease is more pronounced in the 2D than in the 1D lattice. We interpret this steeper decrease as the more pronounced localization. The effective beam width decreases faster in the 2D lattice as compared to the 1D lattice, as the level of disorder is increased.

IV. NONLINEAR VERSUS LINEAR REGIME

Next, it is of interest to consider localization effects in the nonlinear regime and to investigate the influence of nonlinearity on the Anderson localization. Again, we study both 1D and 2D cases, in different nonlinear regimes. We investigate localization for focusing ($\gamma > 0$) as well as defocusing ($\gamma < 0$) nonlinearity and compare them with the linear regime localization. In the case of defocusing nonlinearity,

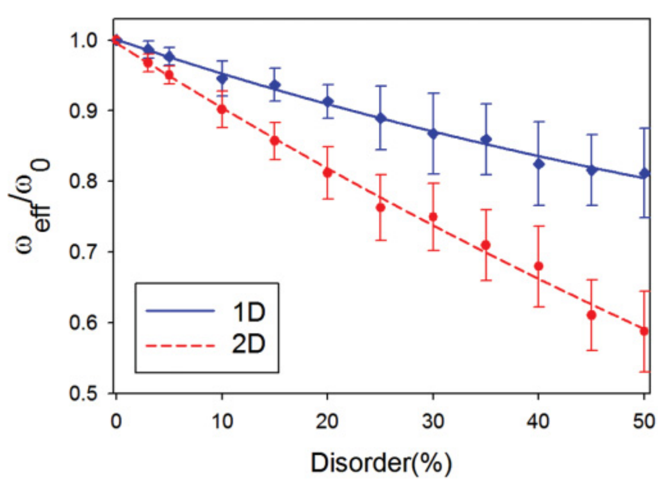

FIG. 3. (Color online) Comparison between 1D and 2D Anderson localization in the linear regime. Effective beam width at the lattice output vs disorder level. The widths are normalized to their values without disorder. Points are ensemble averages and lines are least-square fits through the points. Error bars depict the spread in values coming from statistics. Parameters are as in Fig. 2.

localization is less pronounced than in the linear regime, for both 1D and 2D cases. In the focusing case, the situation is more complex.

As one starts increasing the strength of the focusing nonlinearity in both 1D and 2D lattices, the effective beam width gets smaller as the level of disorder is increased and localization gets more pronounced than in the linear regime. However, as the focusing nonlinearity is further increased, the coexistence of nonlinearity and disorder leads to different conclusions. At a certain value of focusing nonlinearity, a threshold is reached after which disorder produces little influence on the localization. This strong nonlinearity threshold, when disorder ceases to produce a significant effect on the localization process, is different for 1D and 2D cases. For the parameters we used, these threshold values are $\gamma \approx 5$ for the 2D case, and $\gamma \approx 9$ for the 1D case.

Figure 4 presents a comparison of localization in the linear and nonlinear regimes for 1D and 2D cases. The value of $\gamma = 0$, naturally, is representative of the linear regime. In both 1D and 2D cases we present the strongly focusing nonlinear regime with $\gamma = 5$. In such a focusing regime, the 1D localization is more pronounced than the 2D localization, and even the defocusing localization is more pronounced than the focusing for the 2D case [Fig. 4(b)]. In one dimension, localization is more pronounced in the nonlinear regime than in the linear. In two dimensions, it is the opposite: localization is more pronounced in the linear regime. These sharp differences between the linear and nonlinear regimes are signs of competition between disorder and nonlinearity influences on the localization process.

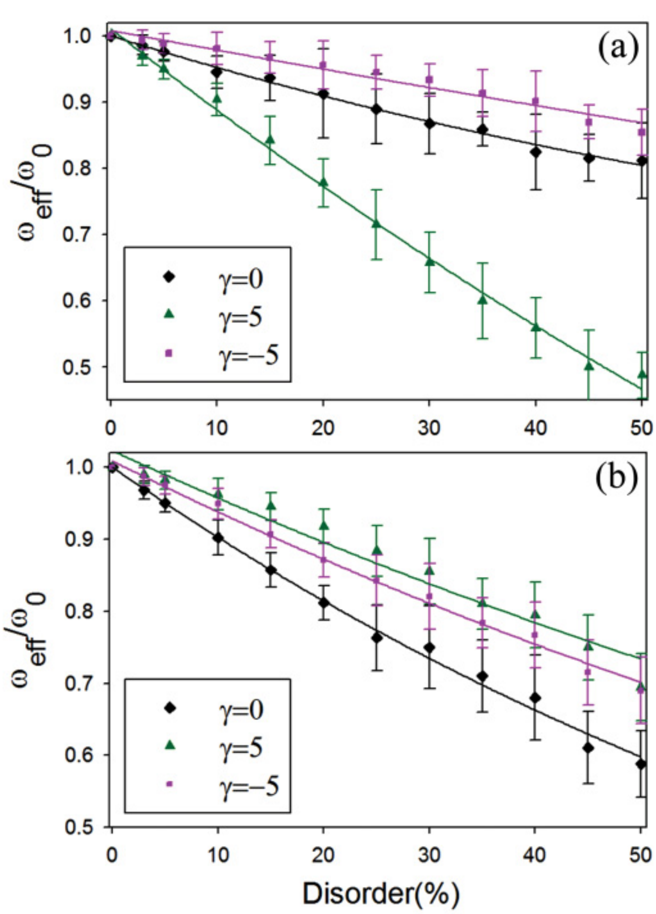

FIG. 4. (Color online) Linear vs nonlinear localization. Normalized effective beam width at the lattice output vs disorder level for (a) 1D lattice and (b) 2D lattice. Parameters are as in Fig. 2.

V. DIMENSIONALITY CROSSOVER

Finally, we investigate the dimensionality crossover from 1D to 2D systems and localization effects in the intermediate regime. We start from a 2D photonic lattice and increase the lattice period along the y transverse direction, to reach separated horizontal 1D lattices. For quantitative description of the dimensionality crossover on the beam localization, we use localization length ξ. The localization length is found by fitting the averaged intensity profile to an exponentially decaying profile $I \sim \exp(-2|r|/\xi)$. As the level of disorder is increased, the output intensity beam profile narrows down, with exponentially decaying tails, as a direct indication of strong localization. To study the crossover regime, we calculate localization lengths from averaged intensity profiles obtained at the 50% disorder level. We find different behavior along the stretched and nonstretched transverse directions.

We find two different localization lengths along two transverse directions in the system with dimensionality crossover.

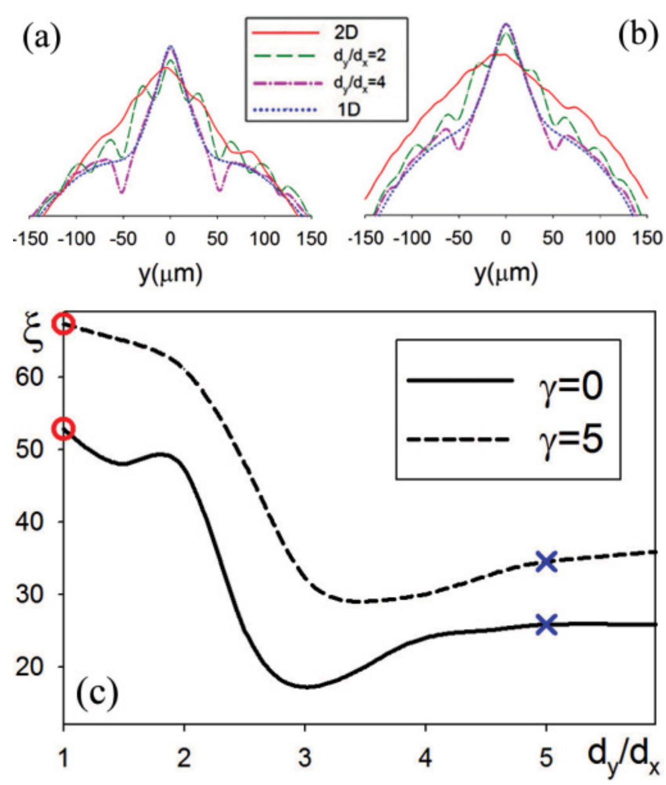

FIG. 5. (Color online) Crossover regime along the stretched (y) transverse direction. Averaged intensity profiles are shown for 50% disorder level in (a) linear and (b) nonlinear regimes. (c) Localization length measured along the stretched transverse direction vs the lattice period ratio, for the linear and nonlinear regimes. Red circles represent localization lengths for the 2D lattice, and blue crosses represent the corresponding localization lengths for the 1D lattice. Other parameters are as in Fig. 2.

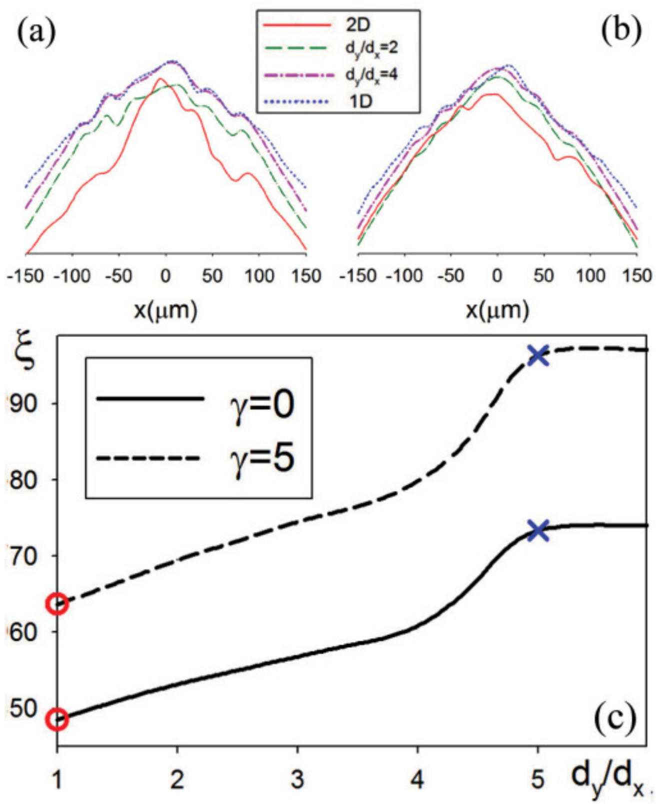

FIG. 6. (Color online) Crossover regime along the nonstretched (x) transverse direction. Averaged intensity profiles are shown for 50% disorder level in (a) linear and (b) nonlinear regime. (c) Localization length measured along the nonstretched transverse direction vs the lattice period ratio, for the linear and nonlinear regimes. Red circles represent localization lengths for the 2D lattice, and blue crosses represent the corresponding localization lengths for the 1D lattice. Other parameters are as in Fig. 2.

They start having the same values for the ratio of lattice periods equal to 1, but then start to differ. Thus, localization lengths measured from the localized profiles in the 2D case have very similar values along both transverse directions. But, in the crossover regime, as well as in the 1D photonic lattice, there are different values of the localization lengths along different transverse directions. Figure 5 shows the localization effects in the intermediate regime along the stretched transverse direction. Localization lengths are calculated from the intensity beam profiles; some of those profiles are presented in Fig. 5(a) for the linear and in Fig. 5(b) for the nonlinear regime. The Gaussian signature of profiles is still visible in the wings. The localization lengths along the stretched transverse direction are presented vs the lattice period ratio, for both linear and nonlinear regimes [Fig. 5(c)]. Along the stretched transverse direction, the localization length gets smaller as the lattice period ratio is increased, although there are some oscillations visible in the linear regime. At a certain value of the period ratio (different for different regimes), a minimum of ξ is reached in both the linear and nonlinear regimes. For the lattice period ratio of 5, we already observe the 1D localization (blue crosses). The localization lengths there are smaller than the values for the 2D lattice.

The dimensionality crossover of Anderson localization along the nonstretched transverse direction is presented in Fig. 6. The corresponding output intensity beam profiles are presented in Fig. 6(a) for the linear and in Fig. 6(b) for the nonlinear regime. While in the 2D case the localization lengths along different transverse directions are very close to each other (red circles), in the intermediate regime and in the 1D case this is not the case. Along the nonstretched transverse direction, the values of the localization length get larger in the intermediate regime and reach maximum values for the 1D case [Fig. 6(c)]. The localization lengths for the lattice period ratio of 5 and greater are larger than the 2D lattice lengths and correspond to the 1D case (blue crosses). The existence of two different localization lengths is closely connected with the anisotropy of the system, induced by the lattice stretching in one transverse direction.

VI. CONCLUSIONS

We have analyzed numerically the influence of dimensionality crossover on Anderson localization of light. We have investigated the transition from 1D to 2D Anderson localization behavior. We have considered the coexistence of nonlinearity and disorder in such a system. In the linear regime, localization is more pronounced in the 2D than in the 1D photonic lattice. But in the nonlinear regime, localization effects depend on the strength of the nonlinearity. We have investigated also the localization in the defocusing regime and compared it with the focusing regime. We have found a gradual transition in the system with dimensionality crossover, where there exist two different localization lengths. In the intermediate cases between one and two dimensions, Anderson localization is less pronounced than in both the pure 1D and 2D cases in the linear regime. In the nonlinear regime, the localization in the intermediate cases depends on the strength of the nonlinearity.

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