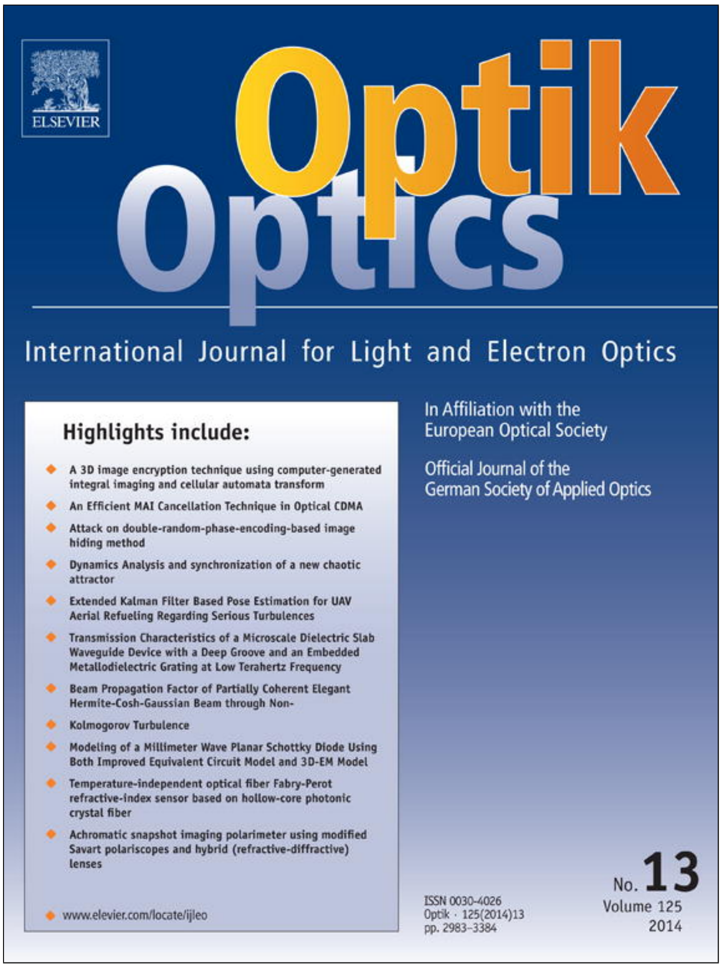
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Bright and dark solitons in optical metamaterials

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ABSTRACT

This paper addresses the dynamics of solitons in optical metamaterials. Both bright and dark soliton solutions are obtained. The ansatz method of integration is employed to extract the 1-soliton solutions to the governing equations. A couple of constraint relations are obtained in order for these solitons to exist. A few numerical simulations are also given to expose the dissipative effects.

1. Introduction

Electromagnetic properties of complex materials with simultaneous negative and real dielectric permittivity (ϵ) and magnetic permeability (μ) have attracted a lot of attention in research [1–15]. Russian physicist Vaselago predicted that electromagnetic wave propagation in these media should give rise to several peculiar characteristics [14]. These media typically referred to as left-handed (L-H) media possess interesting features that may lead to unconventional phenomena in guidance, radiation and scattering of electromagnetic waves. Even though not found in nature, the novel and interesting features of these engineered materials and their possible applications to support short duration soliton and non-soliton pulses are the primary motivation of this research work. A clear distinction is present in terms of single negative such as negative refraction and with double negative (DNG) material [4,11,13,14]. Regular photonic crystal often time shows the negative refraction for the optical wave. For both cases, the optical wave encounter higher degree of losses. Even the soliton pulses which are evolved due to delicate balance between dispersion and nonlinearity will be dissipative in nature. Loss compensation is a challenge to engineer these types of materials.

Recently reported DNG materials in visible infrared region by Shalaev and others have shown promise to make optical waveguide with these materials. We use the dispersion profile of the reported metamaterial to determine the nature of the soliton pulse. The effect of loss has also been considered. Comprehensive analytical and numerical studies using split step Fourier method (SSFM) have been conducted to treat soliton wave propagation in regular positive index materials. Negative index materials attract interest in the nonlinear domain as it enhances the nonlinearity due the confinement of electric field in a small region and other multiple frequencies can be generated due to efficient phase matching process. We, then, extend the study for negative-indexed materials. Fig. 1 shows dissipative soliton wave propagating through the DNG material. High losses in DNG material cause the dissipation of the soliton.

Fig. 2 shows the 3D view of bright and dark soliton pulses propagating through 0.125 m optical waveguide with DNG metamaterial. The pulse was launched with 1.55 μm telecommunications wavelength. Because of high loss in the DNG material, the soliton pulse

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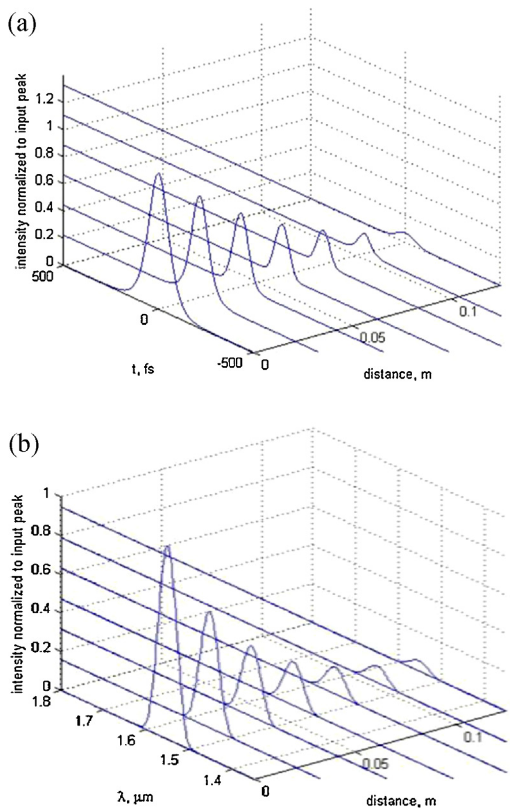

Fig. 1. Dissipative femto-second soliton pulse propagated through bulk DNG material (a) temporal view and (b) spectral view.

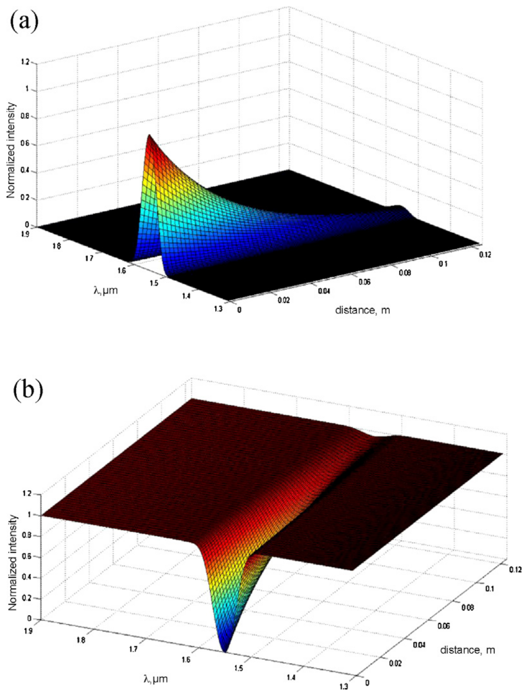

Fig. 2. 3D view of guided optical soliton pulses after propagating through a 0.125 m optical waveguide with meta materials: (a) bright soliton and (b) dark soliton.

gets attenuated over the distance. The shape of the soliton is conserved as long as the balance between power and dispersion is maintained.

2. Governing equation and soliton solutions

The dynamics of solitons in optical metamaterials is governed by the nonlinear Schrödinger's equation (NLSE) which in the dimensionless form is given by [15]

$$iq_t + aq_{xx} + b|q|^2q = i\alpha q_x + i\lambda(|q|^2q)_x + i\nu(|q|^2)_x q + \theta_1(|q|^2q)_{xx} + \theta_2|q|^2q_{xx} + \theta_3 q^2 q^*_{xx} \tag{1}$$

Eq. (1) is the NLSE that is studied in the context of metamaterials. Here in (1), a and b are the group velocity dispersion and the self-phase modulation terms respectively. This pair produces the delicate balance between dispersion and nonlinearity that accounts for the formation of the stable solitons. On the right hand side λ represents the self-steepening term in order to avoid the formation of shocks and ν is the nonlinear dispersion, while α represents the intermodal dispersion. Then finally, θ_j for $j = 1, 2, 3$ are the perturbation terms that appears in the context of metamaterials [1].

This governing NLSE given by (1) will be solved by the aid of ansatz method. The bright and dark soliton solutions will be derived and discussed in the following two subsections. In order to proceed with the soliton solution, the following ansatz is adopted [1]

$$q(x, t) = P(x, t)e^{i\phi}, \tag{2}$$

In (2), $P(x, t)$ represents the bright or dark solitary wave profile and $\phi(x, t)$ is the phase component of the soliton that is defined as

$$\phi = -\kappa x + \omega t + \theta \tag{3}$$

where κ gives the soliton frequency and ω being the soliton wave number while θ represent the phase constant. Substituting (2) into (1) and then decomposing into real and imaginary parts leads to

$$(\omega + \alpha\kappa + a\kappa^2) + \{\kappa(\lambda - \theta_1\kappa^2 - \theta_2\kappa^2 - \theta_3\kappa) - b\}P^3 - a\frac{\partial^2 P}{\partial x^2} + 6\theta_1\left(\frac{\partial P}{\partial x}\right)^2 + (3\theta_1 + \theta_2 + \theta_3)P^2\frac{\partial^2 P}{\partial x^2} = 0 \tag{4}$$

and

$$\frac{\partial P}{\partial t} - (\alpha + 2a\kappa)\frac{\partial P}{\partial x} = (3\lambda + 2\nu - 6\theta_1\kappa - 2\theta_2\kappa + 2\theta_3\kappa)P^2\frac{\partial P}{\partial x} \tag{5}$$

The imaginary part leads to the relations

$$\nu = -\alpha - 2a\kappa \tag{6}$$

and

$$3\lambda + 2\nu - 2\kappa(3\theta_1 + \theta_2 - \theta_3) = 0 \tag{7}$$

Eq. (6) is the velocity of the soliton while relation (7) is the constraint condition that must be valid in order for the solitons to exist. These relations (6) and (7) remain valid for both bright and dark solitons. The real part equation, given by (4) will now be analyzed individually for bright and dark solitons in the following two subsections

2.1. Bright solitons

For bright solitons, the choice for the wave profile given by [1]

$$P(x, t) = A\,\text{sech}^p\,\tau \tag{8}$$

where

$$\tau = B(x - vt) \tag{9}$$

is picked. In (9), the soliton velocity is given by v while the inverse width of the soliton is B. The value of the unknown exponent p will fall out during the process of deriving the exact soliton solution. Substituting this ansatz into (4) leads to the relation

$$\begin{aligned}&(\omega + \alpha\kappa + a\kappa^2)\,\mathrm{sech}^p\,\tau + \{\kappa(\lambda - \theta_1\kappa^2 - \theta_2\kappa^2 - \theta_3\kappa) - b\}A^2\,\mathrm{sech}^{3p}\,\tau\\ &\quad - ap^2B^2\,\mathrm{sech}^p\,\tau + ap(p+1)B^2\,\mathrm{sech}^{p+2}\,\tau + 6\theta_1 p^2A^2B^2\,\mathrm{sech}^{3p}\,\tau\\ &\quad + (3\theta_1 + \theta_2 + \theta_3)pA^2B^2\{p\,\mathrm{sech}^{3p}\,\tau - (p+1)\,\mathrm{sech}^{3p+2}\,\tau\} = 0\end{aligned} \tag{10}$$

From (10), by the balancing principle, equating the exponents $3p$ and $p+2$ leads to

$$3p = p + 2 \tag{11}$$

which gives

$$p = 1 \tag{12}$$

Again, from (10) setting the coefficients of the linearly independent functions $\mathrm{sech}^{p+j}\,\tau$ for $j = 0, 2$ and $\mathrm{sech}^{3p+2}\,\tau$ to zero gives

$$\omega = a(B^2 - \kappa^2) - \alpha\kappa \tag{13}$$

$$\{\kappa(\lambda - \theta_1\kappa^2 - \theta_2\kappa^2 - \theta_3\kappa) - b\}A^2 + 3\theta_1A^2B^2 + 2aB^2 = 0 \tag{14}$$

and

$$6\theta_1 + \theta_2 + \theta_3 = 0 \tag{15}$$

This finally shows that the bright 1-soliton solution in optical metamaterials for the governing Eq. (1) is

$$q(x,t) = A\,\mathrm{sech}[B(x - vt)]e^{i(-\kappa x + \omega t + \theta)} \tag{16}$$

where the amplitude A and the inverse width B of the soliton are connected as given by (14), the velocity of the soliton is given by (6) while the wave number is given by (13). Additionally, the constraint conditions given by (7) and (15) must hold in order for the solitons to exist.

2.2. Dark solitons

For dark solitons, the hypothesis is [1]

$$P(x,t) = A\,tanh^p\tau \tag{17}$$

where the same value of τ as given by (9) is valid. In this case, however, the parameters A and B are referred to as free parameters. Therefore substituting (17) into the real part equation given by (4) leads to

$$\begin{aligned}&(\omega + \alpha\kappa + a\kappa^2)\tanh^p\tau + \{\kappa(\lambda - \theta_1\kappa^2 - \theta_2\kappa^2 - \theta_3\kappa) - b\}A^2\tanh^{3p}\tau\\ &\quad - apB^3\{(p-1)\tanh^{p-2}\tau - 2p\,tanh^p\tau + (p+1)\tanh^{p+2}\tau\}\\ &\quad + 6\theta_1p^2A^2B^2(\tanh^{3p-2}\tau - 2\,tanh^{3p}\tau + \tanh^{3p+2}\tau) + (3\theta_1 + \theta_2\\ &\quad + \theta_3)pA^2B^2\{(p-1)\tanh^{3p-2}\tau - 2p\,tanh^{3p}\tau\\ &\quad + (p+1)\tanh^{3p+2}\tau\} = 0\end{aligned} \tag{18}$$

Similarly as in the case of bright solitons, the balancing principle yields the same value of p as in (12). An additional observation in this case of dark solitons is that the coefficient of the stand-alone perturbation term given by $\tanh^{p-2}\tau$ must be zero. This also leads to the same value of p as in (12). Now, setting the coefficients of the other linearly independent functions $\tanh^{p+j}\tau$ for $j = 0, 2$ and $\tanh^{3p+l}\tau$ for $l = -2, 2$ gives the following

$$\omega = -a(2B^2 + \kappa^2) - \alpha\kappa \tag{19}$$

$$\{\kappa(\lambda - \theta_1\kappa^2 - \theta_2\kappa^2 - \theta_3\kappa) - b\}A^2 - 2(9\theta_1 + \theta_2 + \theta_3) - 2aB^2 = 0 \tag{20}$$

$$\theta_1 = 0 \tag{21}$$

and

$$\theta_2 + \theta_3 = 0 \tag{22}$$

Implementing conditions (21) and (22) into (20) leads to the relation between the free parameters as

$$B = \left[\frac{\kappa(\lambda - \theta_1\kappa^2 - \theta_2\kappa^2 - \theta_3\kappa) - b}{2a}\right]^{1/2}A \tag{23}$$

while the constraint condition given by (7) simplifies to

$$3\lambda + 2\nu - 2\kappa(\theta_2 - \theta_3) = 0 \tag{24}$$

Again the relation (23) induces the restriction

$$a\{\kappa(\lambda - \theta_2\kappa^2 - \theta_3\kappa) - b\} > 0 \tag{25}$$

Hence finally, with the constraint condition (21), the model Eq. (1), for dark solitons in optical metamaterials, reduces to

$$\begin{aligned}&iq_t + aq_{xx} + b|q|^2q = i\alpha q_x + i\lambda(|q|^2q)_x + i\nu(|q|^2)_xq + \theta_2|q|^2q_{xx}\\ &\quad + \theta_3q^2q^*_{xx}\end{aligned} \tag{26}$$

whose dark 1-soliton solution is given by

$$q(x,t) = A\,tanh[B(x - vt)]e^{i(-\kappa x + \omega t + \theta)} \tag{27}$$

where the velocity and the wave numbers are (6) and (19) respectively. The relation between the free parameters is then given by (23). The constraints and restriction given by (22), (24) and (25) must hold in order for the dark soliton solution to exist in metamaterials.

3. Conclusions

This paper addresses the dynamics of the solitons in optical metamaterials with Kerr law nonlinearity. Both bright and dark soliton solutions are obtained. There a few constraint conditions that naturally fell off during the course of derivation of the soliton solution. These conditions must remain valid in order for the solitons to exist. There are a couple of numerical simulations that are included for both bright and dark solitons in order to expose the fact that these solitons undergo dissipative effect.

These results will be extended in future. There are other laws of nonlinearity that will be taken into consideration. These are the power law, parabolic law, dual-power law and logarithmic law. Several other integration techniques will be applied and the corresponding bright, dark and singular solitons will retrieved. These results will be subsequently extended to birefringent optical metamaterials, DWDM systems as well as nonlinear directional couplers. They will be reported in future. This is just the tip of the iceberg.

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References

[1] A. Biswas, K. Khan, A. Rahman, A. Yildirim, T. Hayat, O.M. Aldossary, Bright and dark optical solitons in birefringent fibers with Hamiltonian perturbations and Kerr law nonlinearity, J. Optoelectron. Adv. Mater. 14 (7/8) (2012) 571–576.

[2] A.D. Boardman, R.C. Mitchell-Thomas, N.G. King, Y.G. Rapport, Bright spatial solitons in controlled negative phase metamaterials, Opt. Commun. 283 (8) (2010) 1585–1587.

[3] S. Crutcher, A. Oseo, A. Yildirim, A. Biswas, Oscillatory parabolic law optical solitons, J. Optoelectron. Adv. Mater. 14 (1/2) (2012) 29–40.

[4] K.R. Khan, T.X. Wu, Short pulse propagation in wavelength select index guided photonics crystal fiber coupler, IEEE J. Sel. Topics Quantum Electron. 14 (2008).

[5] K.R. Khan, T.X. Wu, D.N. Christodoulides, G.I. Stegeman, Soliton switching and multi-frequency generation in nonlinear photonis cryatal fiber, Opt. Express 16 (13) (2008) 9417–9428.

[6] H. Leblond, D. Mihalache, Models for few-cycle optical solitons, J. Optoelectron. Adv. Mater. 12 (1) (2010) 1–5.

[7] H. Leblond, B.A. Malomed, D. Mihalache, Spatio-temporal solitons in waveguide arrays, J. Optoelectron. Adv. Mater. 12 (1) (2010) 6–11.

[8] D. Mihalache, Formation and stability of light bullets: recent theoretical results, J. Optoelectron. Adv. Mater. 12 (1) (2010) 12–18.

[9] D. Mihalache, Recent trends in micro- and nanophotonics: a personal selection, J. Optoelectron. Adv. Mater. 13 (9) (2011) 1055–1066.

[10] H. Leblond, D. Mihalache, Models of few cycles optical solitons beyond the slowly varying envelope approximations, Phys. Rep. 523 (2013) 61–126.

[11] J.B. Pendry, Negative refraction makes a perfect lens, Phys. Rev. Lett. 85 (18) (2000) 3966–3969.

[12] M. Saha, A.K. Sarma, Modulational instability in nonlinear metamaterials induced by cubic–quintic nonlinearities and higher order dispersive effects, Opt. Commun. 291 (2013) 321–325.

[13] V.M. Shalaev, Optical negative-index metamaterials, Nat. Photon. 1 (2007) 41–48.

[14] V.G. Vaselago, Electrodynamics of media with simultaneous negative electric permittivity and magnetic permeability, Soviet Phys. Uspehki 10 (1967) 509–514.

[15] Y. Xiang, X. Dai, S. Wen, J. Guo, D. Fan, Controllable Raman soliton self-frequency shift in nonlinear metamaterials, Phys. Rev. A 84 (2011) 033815.