Optik - International Journal for Light and Electron Optics 182 (2019) 930–943

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.com/locate/ijleo

Original research article

Highly dispersive optical solitons with quadratic-cubic law by *F*-expansion

Anjan Biswas[a,b,c], Mehmet Ekici[d,*], Abdullah Sonmezoglu[d], Milivoj R. Belic[e]

[a] Department of Physics, Chemistry and Mathematics, Alabama A&M University, Normal, AL 35762-7500, USA
[b] Department of Mathematics, King Abdulaziz University, Jeddah 21589, Saudi Arabia
[c] Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria 0008, South Africa
[d] Department of Mathematics, Faculty of Science and Arts, Yozgat Bozok University, 66100 Yozgat, Turkey
[e] Institute of Physics Belgrade, Pregrevica 118, 11080 Zemun, Serbia

ARTICLE INFO

OCIS:
060.2310
060.4510
060.5530
190.3270
190.4370

Keywords:
Solitons
Complexitons
Quadratic-cubic law

ABSTRACT

This paper retrieves, bright, dark and singular highly dispersive optical solitons with quadratic-cubic law of nonlinearity. The *F*-expansion scheme is the integration algorithm adopted in the paper. There are other solutions to the model that are also recovered such as doubly periodic functions, Weierstrass elliptic functions and periodic solutions.

1. Introduction

The fundamental principle which permits optical solitons to sustain during long haul communications is the existence of a delicate balance that persists between dispersion and nonlinearity which is also known as self-phase modulation (SPM) [1–15]. However, when SPM is minimal or totally absent, solitons still do persist as described by Biswas-Arshed model [4,11]. This paper studies optical solitons when, on the other hand, dispersive effect is a dominating factor. In addition to group velocity dispersion (GVD), when higher order dispersion terms are also present, one encounters, highly dispersive optical solitons. This paper addresses the governing nonlinear Schrödinger's equation (NLSE) in presence of inter-modal dispersion (IMD), third-order dispersion (3OD), fourth-order dispersion (4OD), fifth-order dispersion (5OD) and sixth-order dispersion (6OD) terms. The model is considered with quadratic-cubic (QC) nonlinearity. While such solitons with Kerr law nonlinearity has already been addressed in the past using the method of undetermined coefficients as well as *F*–expansion scheme, this paper utilizes the second methodology. After a quick intro to the model, the results are derived and sequentially enumerated.

1.1. Governing model

The dimensionless form of NLSE with QC nonlinearity in presence of dispersion terms of all orders is [6,7]:

$$iq_t + ia_1 q_x + a_2 q_{xx} + ia_3 q_{xxx} + a_4 q_{xxxx} + ia_5 q_{xxxxx} + a_6 q_{xxxxxx} + (b_1 |q| + b_2 |q|^2)q = 0. \tag{1}$$

* Corresponding author.
E-mail address: mehmet.ekici@bozok.edu.tr (M. Ekici).

https://doi.org/10.1016/j.ijleo.2019.01.041
Received 23 December 2018; Accepted 20 January 2019

Here, in (1), $q(x, t)$ represents soliton molecules and other forms of nonlinear waves where x and t are independent spatial and temporal variables respectively. The first term represents linear temporal evolution, while a_1, a_2, a_3, a_4, a_5 and a_6 are coefficients of IMD, GVD, 3OD, 4OD, 5OD and 6OD respectively. Finally, while b_1 and b_2 together comprise the QC nonlinearity. The complex-valued function $q(x, t)$ is the wave profile that is the dependent variable. The coefficients are all real–valued constants while $i = \sqrt{-1}$.

2. Mathematical analysis

In order to tackle (1), the starting hypothesis is selected as follows:

$$q(x, t) = g(s)e^{i\phi(x,t)}, \tag{2}$$

where $g(s)$ represents the shape of the pulse and

$$s = x - \mathrm{v}\,\mathrm{t}, \tag{3}$$

where v is the velocity of the soliton and

$$\phi(x, t) = -\kappa x + \omega t + \theta. \tag{4}$$

From the phase component $\phi(x, t)$, κ is the soliton frequency, while ω is the soliton wave number and θ is the phase constant. Insert (2) into (1) and then split into real and imaginary parts respectively. Thus real part give rise to

$$\begin{aligned} &-(\omega + \kappa(-a_1 + \kappa(a_2 + \kappa(a_3 - \kappa(a_4 + a_5\kappa) + a_6\kappa^3))))g + b_1g^2 + b_2g^3 \\ &+ (a_2 + 3a_3\kappa - 6a_4\kappa^2 - 10a_5\kappa^3 + 15a_6\kappa^4)g'' + (a_4 + 5a_5\kappa - 15a_6\kappa^2)g^{(4)} + a_6g^{(6)} = 0, \end{aligned} \tag{5}$$

while imaginary part implies

$$(v - a_1 + 2a_2\kappa + 3a_3\kappa^2 - 4a_4\kappa^3 - 5a_5\kappa^4 + 6a_6\kappa^5)g + (-a_3 + 2\kappa(2a_4 + 5\kappa(a_5 - 2a_6\kappa)))g'' - (a_5 - 6a_6\kappa)g^{(4)} = 0. \tag{6}$$

From (6), one has the constraint as

$$a_5 = 6a_6\kappa. \tag{7}$$

Therefore, the coefficients of the remaining linearly independent functions lead to the other constraint condition

$$a_3 = 4\kappa(a_4 + 10a_6\kappa^2), \tag{8}$$

and then the velocity of the soliton emerges as

$$v = a_1 - 2a_2\kappa - 8a_4\kappa^3 - 96a_6\kappa^5. \tag{9}$$

Eq. (5) will now be examined by F–expansion scheme [9,10,13] in the next section.

3. *F*-expansion scheme

In order to kick off the integration process, the solution to (5) is taken to be

$$g(s) = \sum_{j=0}^{N} \vartheta_j F^j(s), \tag{10}$$

where ϑ_j are constants to be determined and also $F = F(s)$ is a solution of

$$(F')^2 = \mathrm{P}\mathrm{F}^4 + \mathrm{Q}\mathrm{F}^2 + R, \tag{11}$$

where P, Q and R are constants. Balancing g^3 with $g^{(6)}$ in (5) gives $N = 3$. Thus one reaches

$$g(s) = \vartheta_0 + \vartheta_1F(s) + \vartheta_2F^2(s) + \vartheta_3F^3(s). \tag{12}$$

Putting (12) into (5), collecting the coefficients of F, and solving the resulting system one has

$$\begin{aligned} \vartheta_0 &= \frac{\sqrt{\aleph_0 + \aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}, \quad \vartheta_1 = \frac{\aleph_2}{2}, \quad \vartheta_2 = 0, \quad \vartheta_3 = \vartheta_3, \\ \omega &= \frac{\mathcal{H}_8 + a_6(15\aleph_0 + 15\aleph_1 - P\aleph_2(\vartheta_3^2\mathcal{H}_9 + 5P\aleph_2(\vartheta_3(1177\kappa^2 - 8649Q) + 1343P\aleph_2)))}{5\vartheta_3^3}, \\ a_2 &= 4a_4\kappa^2 - 3a_3\kappa + \frac{a_6(\mathcal{H}_{10} + \mathcal{L}_5 + \aleph_3)}{2422300607}, \\ a_5 &= \frac{a_6\left(83P(\sqrt{\mathcal{H}_0} + \sqrt{\mathcal{L}_0}) + \vartheta_3\left(15\kappa^2 + \frac{43575Q}{1343}\right)\right) - a_4\vartheta_3}{5\vartheta_3\kappa}, \\ b_1 &= \frac{72\sqrt{35}a_6P^{3/2}\sqrt{\aleph_0 + \aleph_1}}{\vartheta_3^{5/2}}, \quad b_2 = -\frac{20160a_6P^3}{\vartheta_3^2}, \end{aligned} \tag{13}$$

where

$$\begin{aligned}
\aleph_0 &= 1343P^3\mathcal{L}_0^{3/2} + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 \\
&\quad + \sqrt{\mathcal{L}_0}\left(-6090\vartheta_3 P^2 Q\sqrt{\mathcal{H}_0} + \frac{21\vartheta_3^2 P(90833Q^2 - 1112004\mathrm{P\,R})}{1343} + \frac{\sqrt[3]{\mathcal{H}_2 - \sqrt{\mathcal{H}_1}}}{\sqrt[3]{2}}\right) \\
\aleph_1 &= -\frac{3024\sqrt[3]{2}\,\vartheta_3^4 P^2\sqrt{\mathcal{L}_0}\,(4611P^2R^2 - 1104PQ^2R - 16Q^4)}{\sqrt[3]{\mathcal{H}_2 - \sqrt{\mathcal{H}_1}}} \\
\aleph_2 &= \sqrt{\mathcal{H}_0} + 2\mathcal{H}_7 + \sqrt{\mathcal{L}_0} \\
\aleph_3 &= \frac{3607298P(1580711P\sqrt{\mathcal{H}_0}\sqrt{\mathcal{L}_0} + \vartheta_3(111469\kappa^2 + 335895Q)(\sqrt{\mathcal{H}_0} + \sqrt{\mathcal{L}_0}))}{\vartheta_3^2}
\end{aligned} \tag{14}$$

and

$$\begin{aligned}
\mathcal{L}_0 &= \mathcal{H}_3 + \frac{\mathcal{H}_4}{4\sqrt{\mathcal{H}_0}} - \frac{1008\mathcal{H}_5}{1343P^3\sqrt[3]{\mathcal{H}_2 - \sqrt{\mathcal{H}_1}}} - \frac{\sqrt[3]{\mathcal{H}_2 - \sqrt{\mathcal{H}_1}}}{4029\sqrt[3]{2}\,P^3} \\
\mathcal{L}_1 &= 1343P^3\mathcal{H}_0^{3/2} + \mathcal{H}_6 - \frac{\sqrt{\mathcal{H}_0}\sqrt[3]{\mathcal{H}_2 - \sqrt{\mathcal{H}_1}}}{\sqrt[3]{2}} \\
\mathcal{L}_2 &= \frac{315\vartheta_3^2 P\sqrt{\mathcal{H}_0}\,(8591Q^2 - 64464\mathrm{P\,R})}{1343} \\
\mathcal{L}_3 &= \frac{487200\vartheta_3^4(5441984Q^4 - 40431015PQ^2R)}{2422300607P\sqrt{\mathcal{H}_0}} \\
\mathcal{L}_4 &= \frac{3024\sqrt[3]{2}\,\vartheta_3^4 P^2\sqrt{\mathcal{H}_0}\,(4611P^2R^2 - 1104PQ^2R - 16Q^4)}{\sqrt[3]{\mathcal{H}_2 - \sqrt{\mathcal{H}_1}}} \\
\mathcal{L}_5 &= \frac{188320\vartheta_3 Q(40431015\mathrm{P\,R} - 5441984Q^2)}{P\sqrt{\mathcal{H}_0}}
\end{aligned} \tag{15}$$

in which

$$\begin{aligned}
\mathcal{H}_0 &= \ell_3 + \frac{1008\,\ell_4}{1343P^3\sqrt[3]{\ell_2 - \sqrt{\ell_0 + \ell_1}}} + \frac{\sqrt[3]{\ell_2 - \sqrt{\ell_0 + \ell_1}}}{4029\sqrt[3]{2}\,P^3} \\
\mathcal{H}_1 &= \ell_0 + \ell_1 \\
\mathcal{H}_2 &= \ell_2 \\
\mathcal{H}_3 &= \frac{56\vartheta_3^2(36261\mathrm{P\,R} + 9508Q^2)}{1803649P^2} \\
\mathcal{H}_4 &= \frac{640\vartheta_3^3 Q(40431015\mathrm{P\,R} - 5441984Q^2)}{2422300607P^3} \\
\mathcal{H}_5 &= \ell_4 \\
\mathcal{H}_6 &= \frac{5\vartheta_3^3 Q(812971620\mathrm{P\,R} - 1142904209Q^2)}{1803649} \\
\mathcal{H}_7 &= \frac{934\vartheta_3 Q}{1343P} \\
\mathcal{H}_8 &= \vartheta_3^3(5a_1\kappa + 2\kappa^3(5a_3 - 8a_4\kappa) + a_6(-65\kappa^6 + 9Q(83\kappa^4 - 13320\mathrm{P\,R}) \\
&\quad + 7560\kappa^2\mathrm{P\,R} + 55125Q^3 - 9455\kappa^2Q^2)) \\
\mathcal{H}_9 &= 747\kappa^4 - 98280\mathrm{P\,R} + 85455Q^2 - 13870\kappa^2Q \\
\mathcal{H}_{10} &= 20145(1343(1343\kappa^4 + 42588\mathrm{P\,R}) + 99905925Q^2 + 7802830\kappa^2Q).
\end{aligned} \tag{16}$$

Also here

$$\begin{aligned}
\ell_0 &= -\,206391214080\vartheta_3^{12}P^6Q^6(30579312213P^3R^3 - 5666847084P^2Q^2R^2 + 530650080PQ^4R - 19159424Q^6) \\
\ell_1 &= 174142586880\vartheta_3^{12}P^{10}R^4(127450522724P^2R^2 - 173539586829PQ^2R + 107366239041Q^4) \\
\ell_2 &= -\,93312\vartheta_3^6P^3(1141749P^3R^3 - 1128194P^2Q^2R^2 + 294848PQ^4R - 21312Q^6) \\
\ell_3 &= \frac{28\vartheta_3^2(36261\mathrm{P\,R} + 9508Q^2)}{1803649P^2} \\
\ell_4 &= \sqrt[3]{2}\,\vartheta_3^4P^2(-4611P^2R^2 + 1104PQ^2R + 16Q^4).
\end{aligned} \tag{17}$$

As a result, the formal solution of Eq. (1) can be written as:

$$q(x, t) = \left\{\frac{\sqrt{\aleph_0 + \aleph_1}}{24\sqrt{35\vartheta_3}\,P^{3/2}} + \frac{\aleph_2}{2}F(s) + \vartheta_3F^3(s)\right\}\exp[i(-\kappa x + \omega t + \theta)]. \tag{18}$$

3.1. Jacobi elliptic function solutions

By utilizing the solutions of (11), one can reveal Jacobi elliptic function solutions to the model as below:

Case 1: $P = m^2,\quad Q = -(1 + m^2),\quad R = 1,\quad F(s) = \mathrm{sn}\, s,$

$$\begin{aligned} q(x,t) = & \left\{ \frac{\sqrt{\aleph_0 + \aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}} + \frac{\aleph_2}{2}\operatorname{sn}[x - (a_1 - 2a_2\kappa - 8a_4\kappa^3 - 96a_6\kappa^5)t] \right. \\ & \left. + \vartheta_3 \operatorname{sn}^3[x - (a_1 - 2a_2\kappa - 8a_4\kappa^3 - 96a_6\kappa^5)t] \right\} \\ & \times \exp\left[i\left\{ -\kappa x + \left(\frac{\mathcal{H}_8 + a_6(15\aleph_0 + 15\aleph_1 - P\aleph_2(\vartheta_3^2\mathcal{H}_9 + 5P\aleph_2(\vartheta_3(1177\kappa^2 - 8649Q) + 1343P\aleph_2)))}{5\vartheta_3^3} \right) t + \theta \right\} \right]. \end{aligned} \tag{19}$$

Case 2: $P = -m^2, \quad Q = 2m^2 - 1, \quad R = 1 - m^2, \quad F(s) = \operatorname{cn} s,$

$$\begin{aligned} q(x,t) = & \left\{ \frac{\sqrt{\aleph_0 + \aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}} + \frac{\aleph_2}{2}\operatorname{cn}[x - (a_1 - 2a_2\kappa - 8a_4\kappa^3 - 96a_6\kappa^5)t] \right. \\ & \left. + \vartheta_3 \operatorname{cn}^3[x - (a_1 - 2a_2\kappa - 8a_4\kappa^3 - 96a_6\kappa^5)t] \right\} \\ & \times \exp\left[i\left\{ -\kappa x + \left(\frac{\mathcal{H}_8 + a_6(15\aleph_0 + 15\aleph_1 - P\aleph_2(\vartheta_3^2\mathcal{H}_9 + 5P\aleph_2(\vartheta_3(1177\kappa^2 - 8649Q) + 1343P\aleph_2)))}{5\vartheta_3^3} \right) t + \theta \right\} \right]. \end{aligned} \tag{20}$$

Case 3: $P = 1, \quad Q = -(1 + m^2), \quad R = m^2, \quad F(s) = \operatorname{ns} s,$

$$\begin{aligned} q(x,t) = & \left\{ \frac{\sqrt{\aleph_0 + \aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}} + \frac{\aleph_2}{2}\operatorname{ns}[x - (a_1 - 2a_2\kappa - 8a_4\kappa^3 - 96a_6\kappa^5)t] \right. \\ & \left. + \vartheta_3 \operatorname{ns}^3[x - (a_1 - 2a_2\kappa - 8a_4\kappa^3 - 96a_6\kappa^5)t] \right\} \\ & \times \exp\left[i\left\{ -\kappa x + \left(\frac{\mathcal{H}_8 + a_6(15\aleph_0 + 15\aleph_1 - P\aleph_2(\vartheta_3^2\mathcal{H}_9 + 5P\aleph_2(\vartheta_3(1177\kappa^2 - 8649Q) + 1343P\aleph_2)))}{5\vartheta_3^3} \right) t + \theta \right\} \right]. \end{aligned} \tag{21}$$

Case 4: $P = 1, \quad Q = -(1 + m^2), \quad R = m^2, \quad F(s) = \operatorname{dc} s,$

$$\begin{aligned} q(x,t) = & \left\{ \frac{\sqrt{\aleph_0 + \aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}} + \frac{\aleph_2}{2}\operatorname{dc}[x - (a_1 - 2a_2\kappa - 8a_4\kappa^3 - 96a_6\kappa^5)t] \right. \\ & \left. + \vartheta_3 \operatorname{dc}^3[x - (a_1 - 2a_2\kappa - 8a_4\kappa^3 - 96a_6\kappa^5)t] \right\} \\ & \times \exp\left[i\left\{ -\kappa x + \left(\frac{\mathcal{H}_8 + a_6(15\aleph_0 + 15\aleph_1 - P\aleph_2(\vartheta_3^2\mathcal{H}_9 + 5P\aleph_2(\vartheta_3(1177\kappa^2 - 8649Q) + 1343P\aleph_2)))}{5\vartheta_3^3} \right) t + \theta \right\} \right]. \end{aligned} \tag{22}$$

Case 5: $P = 1 - m^2, \quad Q = 2 - m^2, \quad R = 1, \quad F(s) = \operatorname{sc} s,$

$$\begin{aligned} q(x,t) = & \left\{ \frac{\sqrt{\aleph_0 + \aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}} + \frac{\aleph_2}{2}\operatorname{sc}[x - (a_1 - 2a_2\kappa - 8a_4\kappa^3 - 96a_6\kappa^5)t] \right. \\ & \left. + \vartheta_3 \operatorname{sc}^3[x - (a_1 - 2a_2\kappa - 8a_4\kappa^3 - 96a_6\kappa^5)t] \right\} \\ & \times \exp\left[i\left\{ -\kappa x + \left(\frac{\mathcal{H}_8 + a_6(15\aleph_0 + 15\aleph_1 - P\aleph_2(\vartheta_3^2\mathcal{H}_9 + 5P\aleph_2(\vartheta_3(1177\kappa^2 - 8649Q) + 1343P\aleph_2)))}{5\vartheta_3^3} \right) t + \theta \right\} \right]. \end{aligned} \tag{23}$$

Case 6: $P = 1, \quad Q = 2 - m^2, \quad R = 1 - m^2, \quad F(s) = \operatorname{cs} s,$

$$\begin{aligned} q(x,t) = & \left\{ \frac{\sqrt{\aleph_0 + \aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}} + \frac{\aleph_2}{2}\operatorname{cs}[x - (a_1 - 2a_2\kappa - 8a_4\kappa^3 - 96a_6\kappa^5)t] \right. \\ & \left. + \vartheta_3 \operatorname{cs}^3[x - (a_1 - 2a_2\kappa - 8a_4\kappa^3 - 96a_6\kappa^5)t] \right\} \\ & \times \exp\left[i\left\{ -\kappa x + \left(\frac{\mathcal{H}_8 + a_6(15\aleph_0 + 15\aleph_1 - P\aleph_2(\vartheta_3^2\mathcal{H}_9 + 5P\aleph_2(\vartheta_3(1177\kappa^2 - 8649Q) + 1343P\aleph_2)))}{5\vartheta_3^3} \right) t + \theta \right\} \right]. \end{aligned} \tag{24}$$

Case 7: $P = 1/4, \quad Q = (1 - 2m^2)/2, \quad R = 1/4, \quad F(s) = \operatorname{ns} s \pm \operatorname{cs} s,$

$$\begin{aligned} q(x,t) = & \left\{ \frac{\sqrt{\aleph_0 + \aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}} + \frac{\aleph_2}{2}(\operatorname{ns}[x - (a_1 - 2a_2\kappa - 8a_4\kappa^3 - 96a_6\kappa^5)t] \pm \operatorname{cs}[x - (a_1 - 2a_2\kappa - 8a_4\kappa^3 - 96a_6\kappa^5)t]) \right. \\ & \left. + \vartheta_3(\operatorname{ns}[x - (a_1 - 2a_2\kappa - 8a_4\kappa^3 - 96a_6\kappa^5)t] \pm \operatorname{cs}[x - (a_1 - 2a_2\kappa - 8a_4\kappa^3 - 96a_6\kappa^5)t])^3 \right\} \\ & \times \exp\left[i\left\{ -\kappa x + \left(\frac{\mathcal{H}_8 + a_6(15\aleph_0 + 15\aleph_1 - P\aleph_2(\vartheta_3^2\mathcal{H}_9 + 5P\aleph_2(\vartheta_3(1177\kappa^2 - 8649Q) + 1343P\aleph_2)))}{5\vartheta_3^3} \right) t + \theta \right\} \right]. \end{aligned} \tag{25}$$

Case 8: $P = (1 - m^2)/4, \quad Q = (1 + m^2)/2, \quad R = (1 - m^2)/4, \quad F(s) = \operatorname{nc} s \pm \operatorname{sc} s,$

$$\begin{aligned} q(x,t) = & \left\{ \frac{\sqrt{\aleph_0 + \aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}} + \frac{\aleph_2}{2}(\operatorname{nc}[x - (a_1 - 2a_2\kappa - 8a_4\kappa^3 - 96a_6\kappa^5)t] \pm \operatorname{sc}[x - (a_1 - 2a_2\kappa - 8a_4\kappa^3 - 96a_6\kappa^5)t]) \right. \\ & \left. + \vartheta_3(\operatorname{nc}[x - (a_1 - 2a_2\kappa - 8a_4\kappa^3 - 96a_6\kappa^5)t] \pm \operatorname{sc}[x - (a_1 - 2a_2\kappa - 8a_4\kappa^3 - 96a_6\kappa^5)t])^3 \right\} \\ & \times \exp\left[i\left\{ -\kappa x + \left(\frac{\mathcal{H}_8 + a_6(15\aleph_0 + 15\aleph_1 - P\aleph_2(\vartheta_3^2\mathcal{H}_9 + 5P\aleph_2(\vartheta_3(1177\kappa^2 - 8649Q) + 1343P\aleph_2)))}{5\vartheta_3^3} \right) t + \theta \right\} \right]. \end{aligned} \tag{26}$$

Case 9: $P = m^2/4, \quad Q = (m^2 - 2)/2, \quad R = m^2/4, \quad F(s) = \operatorname{sn} s \pm i \operatorname{cn} s,$

$$q(x,t)=\Bigg\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}(\operatorname{sn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\pm i\operatorname{cn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])$$
$$+\vartheta_3(\operatorname{sn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\pm i\operatorname{cn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])^3\Bigg\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right]. \tag{27}$$

Case 10: $P=m^2/4,\quad Q=(m^2-2)/2,\quad R=1/4,\quad F(s)=\frac{\operatorname{sn}s}{1\pm\operatorname{dn}s},$

$$q(x,t)=\Bigg\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\left(\frac{\operatorname{sn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{1\pm\operatorname{dn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}\right)$$
$$+\vartheta_3\left(\frac{\operatorname{sn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{1\pm\operatorname{dn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}\right)^3\Bigg\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right]. \tag{28}$$

Case 11: $P=-1/4,\quad Q=(m^2+1)/2,\quad R=(1-m^2)^2/4,\quad F(s)=m\operatorname{cn}s\pm\operatorname{dn}s,$

$$q(x,t)=\Bigg\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}$$
$$+\frac{\aleph_2}{2}(m\operatorname{cn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\pm\operatorname{dn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])$$
$$+\vartheta_3(m\operatorname{cn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\pm\operatorname{dn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])^3\Bigg\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right]. \tag{29}$$

Case 12: $P=(1-m^2)^2/4,\quad Q=(m^2+1)/2,\quad R=1/4,\quad F(s)=\operatorname{ds}s\pm\operatorname{cs}s,$

$$q(x,t)=\Bigg\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}(\operatorname{ds}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\pm\operatorname{cs}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])$$
$$+\vartheta_3(\operatorname{ds}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\pm\operatorname{cs}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])^3\Bigg\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right]. \tag{30}$$

Case 13: $P>0,\quad Q<0,\quad R=\frac{m^2Q^2}{(1+m^2)^2P},\quad F(s)=\sqrt{-\frac{m^2Q}{(1+m^2)P}}\operatorname{sn}\left(\sqrt{-\frac{Q}{1+m^2}}s\right),$

$$q(x,t)=\Bigg\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\left(\sqrt{-\frac{m^2Q}{(1+m^2)P}}\operatorname{sn}\left(\sqrt{-\frac{Q}{1+m^2}}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right)\right)$$
$$+\vartheta_3\left(\sqrt{-\frac{m^2Q}{(1+m^2)P}}\operatorname{sn}\left(\sqrt{-\frac{Q}{1+m^2}}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right)\right)^3\Bigg\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right]. \tag{31}$$

Case 14: $P<0,\quad Q>0,\quad R=\frac{(1-m^2)Q^2}{(m^2-2)^2P},\quad F(s)=\sqrt{-\frac{Q}{(2-m^2)P}}\operatorname{dn}\left(\sqrt{\frac{Q}{2-m^2}}s\right),$

$$q(x,t)=\Bigg\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\left(\sqrt{-\frac{Q}{(2-m^2)P}}\operatorname{dn}\left(\sqrt{\frac{Q}{2-m^2}}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right)\right)$$
$$+\vartheta_3\left(\sqrt{-\frac{Q}{(2-m^2)P}}\operatorname{dn}\left(\sqrt{\frac{Q}{2-m^2}}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right)\right)^3\Bigg\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right]. \tag{32}$$

Case 15: $P=1,\quad Q=m^2+2,\quad R=1-2m^2+m^4,\quad F(s)=\frac{\operatorname{dn}s\operatorname{cn}s}{\operatorname{sn}s},$

$$q(x,t)=\Bigg\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\left(\frac{\operatorname{dn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\operatorname{cn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{\operatorname{sn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}\right)$$
$$+\vartheta_3\left(\frac{\operatorname{dn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\operatorname{cn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{\operatorname{sn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}\right)^3\Bigg\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right]. \tag{33}$$

Case 16: $P=\frac{A^2(m-1)^2}{4},\quad Q=\frac{m^2+1}{2}+3m,\quad R=\frac{(m-1)^2}{4A^2},\quad F(s)=\frac{\operatorname{dn}s\operatorname{cn}s}{A(1+\operatorname{sn}s)(1+m\operatorname{sn}s)},$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\left(\frac{\mathrm{dn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\,\mathrm{cn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{A(1+\,\mathrm{sn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])(1+m\,\mathrm{sn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])}\right)+\vartheta_3\left(\frac{\mathrm{dn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\,\mathrm{cn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{A(1+\,\mathrm{sn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])(1+m\,\mathrm{sn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])}\right)^3\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right].\tag{34}$$

Case 17: $P=-\frac{4}{m},\quad Q=6m-m^2-1,\quad R=-2m^3+m^4+m^2,\quad F(s)=\frac{m\,\mathrm{cn}\,s\,\mathrm{dn}\,s}{m\,\mathrm{sn}^2 s+1},$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\left(\frac{m\,\mathrm{cn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\,\mathrm{dn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{m\,\mathrm{sn}^2\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+1}\right)+\vartheta_3\left(\frac{m\,\mathrm{cn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\,\mathrm{dn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{m\,\mathrm{sn}^2\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+1}\right)^3\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right].\tag{35}$$

Case 18: $P=1/4,\quad Q=\frac{1-2m^2}{2},\quad R=1/4,\quad F(s)=\frac{\mathrm{sn}\,s}{1\pm\,\mathrm{cn}\,s},$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\left(\frac{\mathrm{sn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{1\pm\,\mathrm{cn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}\right)+\vartheta_3\left(\frac{\mathrm{sn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{1\pm\,\mathrm{cn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}\right)^3\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right].\tag{36}$$

Case 19: $P=\frac{1-m^2}{4},\quad Q=\frac{1+m^2}{2},\quad R=\frac{1-m^2}{4},\quad F(s)=\frac{\mathrm{cn}\,s}{1\pm\,\mathrm{sn}\,s},$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\left(\frac{\mathrm{cn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{1\pm\,\mathrm{sn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}\right)+\vartheta_3\left(\frac{\mathrm{cn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{1\pm\,\mathrm{sn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}\right)^3\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right].\tag{37}$$

Case 20: $P=\frac{2-m^2-2m_1}{4},\quad Q=\frac{m^2}{2}-1-3m_1,\quad R=\frac{2-m^2-2m_1}{4},\quad F(s)=\frac{m^2\,\mathrm{sn}\,s\,\mathrm{cn}\,s}{\mathrm{sn}^2 s+(1+m_1)\,\mathrm{dn}\,s-1-m_1},$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\left(\frac{m^2\,\mathrm{sn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\,\mathrm{cn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{\mathrm{sn}^2\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+m_2\,\mathrm{dn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]-m_2}\right)+\vartheta_3\left(\frac{m^2\,\mathrm{sn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\,\mathrm{cn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{\mathrm{sn}^2\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+m_2\,\mathrm{dn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]-m_2}\right)^3\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right],\tag{38}$$

where $m_2=1+m_1$.

Case 21: $P=\frac{2-m^2+2m_1}{4},\quad Q=\frac{m^2}{2}-1+3m_1,\quad R=\frac{2-m^2+2m_1}{4},\quad F(s)=\frac{m^2\,\mathrm{sn}\,s\,\mathrm{cn}\,s}{\mathrm{sn}^2 s+(-1+m_1)\,\mathrm{dn}\,s-1-m_1},$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\left(\frac{m^2\,\mathrm{sn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\,\mathrm{cn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{\mathrm{sn}^2\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+m_3\,\mathrm{dn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]-m_2}\right)+\vartheta_3\left(\frac{m^2\,\mathrm{sn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\,\mathrm{cn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{\mathrm{sn}^2\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+m_3\,\mathrm{dn}\,[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]-m_2}\right)^3\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right],\tag{39}$$

where $m_3=-1+m_1$.

Case 22: $P=\frac{C^2m^4-(B^2+C^2)m^2+B^2}{4},\quad Q=\frac{m^2+1}{2},\quad R=\frac{m^2-1}{4(C^2m^2-B^2)},\quad F(s)=\frac{\sqrt{\frac{B^2-C^2}{B^2-C^2m^2}}+\operatorname{sn}s}{B\operatorname{cn}s+C\operatorname{dn}s},$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\left(\frac{\sqrt{\frac{B^2-C^2}{B^2-C^2m^2}}+\operatorname{sn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{B\operatorname{cn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+C\operatorname{dn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}\right)+\vartheta_3\left(\frac{\sqrt{\frac{B^2-C^2}{B^2-C^2m^2}}+\operatorname{sn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{B\operatorname{cn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+C\operatorname{dn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}\right)^3\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right]. \tag{40}$$

Case 23: $P=\frac{B^2+C^2m^2}{4},\quad Q=\frac{1}{2}-m^2,\quad R=\frac{1}{4(B^2+C^2m^2)},\quad F(s)=\frac{\sqrt{\frac{C^2m^2+B^2-C^2}{B^2+C^2m^2}}+\operatorname{cn}s}{B\operatorname{sn}s+C\operatorname{dn}s},$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\left(\frac{\sqrt{\frac{C^2m^2+B^2-C^2}{B^2+C^2m^2}}+\operatorname{cn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{B\operatorname{sn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+C\operatorname{dn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}\right)+\vartheta_3\left(\frac{\sqrt{\frac{C^2m^2+B^2-C^2}{B^2+C^2m^2}}+\operatorname{cn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{B\operatorname{sn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+C\operatorname{dn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}\right)^3\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right]. \tag{41}$$

Case 24: $P=\frac{B^2+C^2}{4},\quad Q=\frac{m^2}{2}-1,\quad R=\frac{m^4}{4(B^2+C^2)},\quad F(s)=\frac{\sqrt{\frac{B^2+C^2-C^2m^2}{B^2+C^2}}+\operatorname{dn}s}{B\operatorname{sn}s+C\operatorname{cn}s},$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\left(\frac{\sqrt{\frac{B^2+C^2-C^2m^2}{B^2+C^2}}+\operatorname{dn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{B\operatorname{sn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+C\operatorname{cn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}\right)+\vartheta_3\left(\frac{\sqrt{\frac{B^2+C^2-C^2m^2}{B^2+C^2}}+\operatorname{dn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{B\operatorname{sn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+C\operatorname{cn}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}\right)^3\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right]. \tag{42}$$

3.2. Weïerstrass elliptic function solutions

Upon recapitulating, Weierstrass elliptic function is defined as [15]:

$$\wp(z;\omega_1,\omega_2)=\frac{1}{z^2}+\sum_{\{m,n\}\neq\{0,0\}}\left(\frac{1}{(z+2m\omega_1+2n\omega_2)^2}-\frac{1}{(2m\omega_1+2n\omega_2)^2}\right). \tag{43}$$

Now, by the help of the solutions of (11) given in [12], one can recover Weierstrass elliptic function solutions as:

Case 25: $g_2=\frac{4}{3}(Q^2-3PR),\quad g_3=\frac{4Q}{27}(-2Q^2+9PR),\quad F(s)=\sqrt{\frac{1}{P}\left[\wp(s;g_2,g_3)-\frac{1}{3}Q\right]},$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\sqrt{\frac{1}{P}\left[\wp(x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t;g_2,g_3)-\frac{1}{3}Q\right]}\right.$$
$$\left.+\vartheta_3\left(\sqrt{\frac{1}{P}\left[\wp(x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t;g_2,g_3)-\frac{1}{3}Q\right]}\right)^3\right\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right]. \tag{44}$$

Case 26: $g_2=\frac{4}{3}(Q^2-3\text{PR}),\quad g_3=\frac{4Q}{27}(-2Q^2+9\text{PR}),\quad F(s)=\sqrt{\frac{3R}{3\wp(s;g_2,g_3)-Q}},$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\sqrt{\frac{3R}{3\wp(x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t;g_2,g_3)-Q}}\right.$$
$$\left.+\vartheta_3\left(\sqrt{\frac{3R}{3\wp(x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t;g_2,g_3)-Q}}\right)^3\right\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right]. \tag{45}$$

Case 27: $g_2=-\frac{5\text{QD}+4Q^2+33\text{PQR}}{12},\quad g_3=\frac{21Q^2D-63\text{PRD}+20Q^3-27\text{PQR}}{216},\ F(s)=\frac{\sqrt{12R\wp(s;g_2,g_3)+2R(2Q+D)}}{12\wp(s;g_2,g_3)+D},$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\frac{\sqrt{12R\wp(x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t;g_2,g_3)+2R(2Q+D)}}{12\wp(x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t;g_2,g_3)+D}\right.$$
$$\left.+\vartheta_3\left(\frac{\sqrt{12R\wp(x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t;g_2,g_3)+2R(2Q+D)}}{12\wp(x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t;g_2,g_3)+D}\right)^3\right\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right]. \tag{46}$$

Case 28: $g_2=\frac{1}{12}Q^2+\text{PR},\quad g_3=\frac{1}{216}Q(36\text{PR}-Q^2),\quad F(s)=\frac{\sqrt{R}\,[6\wp(s;g_2,g_3)+Q]}{3\wp'(s;g_2,g_3)},$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\frac{\sqrt{R}\,[6\wp(x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t;g_2,g_3)+Q]}{3\wp'(x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t;g_2,g_3)}\right.$$
$$\left.+\vartheta_3\left(\frac{\sqrt{R}\,[6\wp(x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t;g_2,g_3)+Q]}{3\wp'(x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t;g_2,g_3)}\right)^3\right\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right]. \tag{47}$$

Case 29: $g_2=\frac{1}{12}Q^2+\text{PR},\quad g_3=\frac{1}{216}Q(36\text{PR}-Q^2),\quad F(s)=\frac{3\wp'(s;g_2,g_3)}{\sqrt{P}\,[6\wp(s;g_2,g_3)+Q]},$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\frac{3\wp'(x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t;g_2,g_3)}{\sqrt{P}\,[6\wp(x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t;g_2,g_3)+Q]}\right.$$
$$\left.+\vartheta_3\left(\frac{3\wp'(x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t;g_2,g_3)}{\sqrt{P}\,[6\wp(x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t;g_2,g_3)+Q]}\right)^3\right\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right]. \tag{48}$$

Case 30: $R=\frac{5Q^2}{36P},\quad g_2=\frac{2Q^2}{9},\quad g_3=\frac{Q^3}{54},\quad F(s)=\frac{Q\sqrt{-15Q/2P}\,\wp(s;g_2,g_3)}{3\wp(s;g_2,g_3)+Q},$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\frac{Q\sqrt{-15Q/2P}\,\wp(x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t;g_2,g_3)}{3\wp(x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t;g_2,g_3)+Q}\right.$$
$$\left.+\vartheta_3\left(\frac{Q\sqrt{-15Q/2P}\,\wp(x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t;g_2,g_3)}{3\wp(x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t;g_2,g_3)+Q}\right)^3\right\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right]. \tag{49}$$

3.3. Soliton and other solutions

When the modulus $m\to 1$, bright, dark and singular solitons, the combined solitons, and complexiton solutions are derived as:

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\tanh[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right.$$
$$\left.+\vartheta_3\tanh^3[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right], \tag{50}$$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\operatorname{sech}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right.$$
$$\left.+\vartheta_3\operatorname{sech}^3[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right], \tag{51}$$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\coth[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right.$$
$$\left.+\vartheta_3\coth^3[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right], \tag{52}$$

Solutions (50)–(52) are dark, bright and singular solitons respectively.

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\sinh[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right.$$
$$\left.+\vartheta_3\sinh^3[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right], \tag{53}$$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\operatorname{csch}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right.$$
$$\left.+\vartheta_3\operatorname{csch}^3[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right], \tag{54}$$

Solution (54) is the second form of singular soliton solution to the model.

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}\right.$$
$$+\frac{\aleph_2}{2}(\coth[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\pm\operatorname{csch}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])$$
$$\left.+\vartheta_3(\coth[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\pm\operatorname{csch}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])^3\right\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right], \tag{55}$$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}\right.$$
$$+\frac{\aleph_2}{2}(\cosh[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\pm\sinh[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])$$
$$\left.+\vartheta_3(\cosh[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\pm\sinh[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])^3\right\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right], \tag{56}$$

Solution (55) and (56) also represents singular solitons.

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}\right.$$
$$+\frac{\aleph_2}{2}(\tanh[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\pm i\operatorname{sech}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])$$
$$\left.+\vartheta_3(\tanh[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\pm i\operatorname{sech}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])^3\right\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right], \tag{57}$$

Then, solution (57) represents complexiton solutions.

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\left(\frac{\tanh[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{1\pm\operatorname{sech}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}\right)+\vartheta_3\left(\frac{\tanh[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{1\pm\operatorname{sech}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}\right)^3\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right], \tag{58}$$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\aleph_2\operatorname{sech}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+8\vartheta_3\operatorname{sech}^3[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right], \tag{59}$$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\aleph_2\operatorname{csch}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+8\vartheta_3\operatorname{csch}^3[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right], \tag{60}$$

Next, (59) and (60) are bright and singular solitons respectively.

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\left(\sqrt{-\frac{Q}{2P}}\tanh\left(\sqrt{-\frac{Q}{2}}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right)\right)+\vartheta_3\left(\sqrt{-\frac{Q}{2P}}\tanh\left(\sqrt{-\frac{Q}{2}}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right)\right)^3\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right], \tag{61}$$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\left(\sqrt{-\frac{Q}{P}}\operatorname{sech}(\sqrt{Q}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])\right)+\vartheta_3\left(\sqrt{-\frac{Q}{P}}\operatorname{sech}(\sqrt{Q}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])\right)^3\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right], \tag{62}$$

Again (61) and (62) are dark and bright solitons respectively. The solution (61) is valid for $Q < 0$ and $P > 0$ while the solution (62) exists for $Q > 0$ and $P < 0$.

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\aleph_2\operatorname{csch}2[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+8\vartheta_3\operatorname{csch}^3 2[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right], \tag{63}$$

Eq. (63) represents singular solitons.

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2A}\exp(-2[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])+\frac{\vartheta_3}{A^3}\exp(-6[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right], \tag{64}$$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\operatorname{sech}2[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+\vartheta_3\operatorname{sech}^3 2[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right], \tag{65}$$

Eq. (65) is another form of bright solitons.

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\left(\frac{\tanh[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{1\pm\operatorname{sech}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}\right)\right.$$
$$\left.+\vartheta_3\left(\frac{\tanh[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{1\pm\operatorname{sech}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}\right)^3\right\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right], \tag{66}$$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\left(\frac{\operatorname{sech}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{1\pm\tanh[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}\right)\right.$$
$$\left.+\vartheta_3\left(\frac{\operatorname{sech}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{1\pm\tanh[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}\right)^3\right\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right], \tag{67}$$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\coth\left[\frac{x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t}{2}\right]\right.$$
$$\left.+\vartheta_3\coth^3\left[\frac{x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t}{2}\right]\right\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right], \tag{68}$$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}-\frac{\aleph_2}{2}\tanh\left[\frac{x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t}{2}\right]\right.$$
$$\left.-\vartheta_3\tanh^3\left[\frac{x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t}{2}\right]\right\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right], \tag{69}$$

Finally, (68) and (69) respectively represent singular and dark solitons to the model.

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2(B+C)}\exp[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right.$$
$$\left.+\frac{\vartheta_3}{(B+C)^3}\exp 3[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right], \tag{70}$$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}\right.$$
$$+\frac{\aleph_2}{2}\left(\frac{\sqrt{\frac{B^2}{B^2+C^2}}+\operatorname{sech}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{B\tanh[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+C\operatorname{sech}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}\right)$$
$$\left.+\vartheta_3\left(\frac{\sqrt{\frac{B^2}{B^2+C^2}}+\operatorname{sech}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}{B\tanh[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+C\operatorname{sech}[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}\right)^3\right\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right]. \tag{71}$$

3.4. Trigonometric function solutions

However, if $m \to 0$, periodic waves, periodic singular waves and a combination of such solutions fall out as follows:

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\sin[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right.$$
$$\left.+\vartheta_3\sin^3[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right], \tag{72}$$

$$
\begin{aligned}
q(x,t)=&\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\cos[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right.\\
&\left.+\vartheta_3\cos^3[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right\}\\
&\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right],
\end{aligned}
\tag{73}
$$

$$
\begin{aligned}
q(x,t)=&\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\csc[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right.\\
&\left.+\vartheta_3\csc^3[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right\}\\
&\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right],
\end{aligned}
\tag{74}
$$

$$
\begin{aligned}
q(x,t)=&\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\sec[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right.\\
&\left.+\vartheta_3\sec^3[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right\}\\
&\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right],
\end{aligned}
\tag{75}
$$

$$
\begin{aligned}
q(x,t)=&\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\tan[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right.\\
&\left.+\vartheta_3\tan^3[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right\}\\
&\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right],
\end{aligned}
\tag{76}
$$

$$
\begin{aligned}
q(x,t)=&\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\cot[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right.\\
&\left.+\vartheta_3\cot^3[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right\}\\
&\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right],
\end{aligned}
\tag{77}
$$

$$
\begin{aligned}
q(x,t)=&\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}\right.\\
&+\frac{\aleph_2}{2}(\csc[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\pm\cot[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])\\
&\left.+\vartheta_3(\csc[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\pm\cot[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])^3\right\}\\
&\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right],
\end{aligned}
\tag{78}
$$

$$
\begin{aligned}
q(x,t)=&\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}\right.\\
&+\frac{\aleph_2}{2}(\sec[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\pm\tan[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])\\
&\left.+\vartheta_3(\sec[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\pm\tan[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])^3\right\}\\
&\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right],
\end{aligned}
\tag{79}
$$

$$
\begin{aligned}
q(x,t)=&\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}\right.\\
&+\frac{\aleph_2}{2}(\sin[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\pm i\cos[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])\\
&\left.+\vartheta_3(\sin[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\pm i\cos[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])^3\right\}\\
&\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right],
\end{aligned}
\tag{80}
$$

$$
\begin{aligned}
q(x,t)=&\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{4}\sin[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right.\\
&\left.+\frac{\vartheta_3}{8}\sin^3[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]\right\}\\
&\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right],
\end{aligned}
\tag{81}
$$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2A}(\sec[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]-\tan[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])+\frac{\vartheta_3}{A^3}(\sec[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]-\tan[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t])^3\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right], \tag{82}$$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\left(\frac{\sin[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+\sqrt{\frac{B^2-C^2}{B^2}}}{B\cos[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+C}\right)+\vartheta_3\left(\frac{\sin[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+\sqrt{\frac{B^2-C^2}{B^2}}}{B\cos[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+C}\right)^3\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right], \tag{83}$$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\left(\frac{\cos[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+\sqrt{\frac{B^2-C^2}{B^2}}}{B\sin[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+C}\right)+\vartheta_3\left(\frac{\cos[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+\sqrt{\frac{B^2-C^2}{B^2}}}{B\sin[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+C}\right)^3\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right], \tag{84}$$

$$q(x,t)=\left\{\frac{\sqrt{\aleph_0+\aleph_1}}{24\sqrt{35\vartheta_3}P^{3/2}}+\frac{\aleph_2}{2}\left(\frac{2}{B\sin[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+C\cos[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}\right)+\vartheta_3\left(\frac{2}{B\sin[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]+C\cos[x-(a_1-2a_2\kappa-8a_4\kappa^3-96a_6\kappa^5)t]}\right)^3\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{\mathcal{H}_8+a_6(15\aleph_0+15\aleph_1-P\aleph_2(\vartheta_3^2\mathcal{H}_9+5P\aleph_2(\vartheta_3(1177\kappa^2-8649Q)+1343P\aleph_2)))}{5\vartheta_3^3}\right)t+\theta\right\}\right]. \tag{85}$$

4. Conclusions

This paper reports highly dispersive optical solitons with QC nonlinearity. Bright, dark, singular and combo optical soliton solutions are retrieved by using F-expansion scheme. Several periodic solutions and other solutions in terms of Weierstrass elliptic function are also recovered. The results of the paper are extremely promising to venture further into the model. Later, this model will be studied using additional schemes that are available in the literature. Moreover, this model will be extended to other optoelectronic devices such as optical metamaterials, optical couplers, DWDM systems and several such. Research work in those arenas are under way and the results will be soon reported once they are available. This is just a foot in the door.

Conflict of interest

The authors also declare that there is no conflict of interest.

Acknowledgements

The research work of the fourth author (MRB) was supported by the grant NPRP 8-028-1-001 from QNRF and he is thankful for it.

References

[1] A. Bansal, A. Biswas, Q. Zhou, M.M. Babatin, Lie symmetry analysis for cubic-quartic nonlinear Schrödinger's equation, Optik 169 (2018) 12–15.
[2] A. Biswas, H. Triki, Q. Zhou, S.P. Moshokoa, M.Z. Ullah, M. Belic, Cubic-quartic optical solitons in Kerr and power law media, Optik 144 (2017) 357–362.
[3] A. Biswas, A.H. Kara, M.Z. Ullah, Q. Zhou, H. Triki, M. Belic, Conservation laws for cubic-quartic optical solitons in Kerr and power law media, Optik 145 (2017)

650–654.
[4] A. Biswas, S. Arshed, Application of semi-inverse variational principle to cubic-quartic optical solitons with Kerr and power law nonlinearity, Optik 172 (2018) 847–850.
[5] A. Biswas, S. Arshed, Optical solitons in presence of higher order dispersions and absence of self-phase modulation, Optik 174 (2018) 452–459.
[6] A. Biswas, J. Vega-Guzman, M.F. Mahmood, S. Khan, M. Ekici, Q. Zhou, S.P. Moshokoa, M.R. Belic, Highly Dispersive Optical Solitons by Undetermined Coefficients, (2019) In Press.
[7] A. Biswas, M. Ekici, A. Sonmezoglu, M.R. Belic, Highly dispersive optical solitons with kerr law nonlinearity by *F*-expansion, Optik 181 (2019) 1028–1038.
[8] A. Das, A. Biswas, M. Ekici, S. Khan, Q. Zhou, S.P. Moshokoa, Suppressing internet bottleneck with fractional temporal evolution of cubic-quartic optical solitons, Optik 182 (2019) 303–307.
[9] H.T. Chen, H.Q. Zhang, New double periodic and multiple soliton solutions of the generalized (2 + 1)-dimensional Boussinesq equation, Chaos Solitons Fractals 20 (4) (2004) 765–769.
[10] A. Ebaid, E.H. Aly, Exact solutions for the transformed reduced Ostrovsky equation via the *F*-expansion method in terms of Weierstrass-elliptic and Jacobian-elliptic functions, Wave Motion 49 (2) (2012) 296–308.
[11] M. Ekici, A. Sonmezoglu, Optical solitons with Biswas-Arshed equation by extended trial function method, Optik 177 (2019) 13–20.
[12] Z. Yan, An improved algebra method and its applications in nonlinear wave equations, MM Res. Preprints 22 (2003) 264–274.
[13] D. Zhang, Doubly periodic solutions of the modified Kawahara equation, Chaos Solitons Fractals 25 (5) (2005) 1155–1160.
[14] Q. Zhou, Q. Zhu, A. Biswas, Optical solitons in birefringent fibers with parabolic law nonlinearity, Opt. Appl. 41 (3) (2014) 399–409.
[15] D.F. Lawden, Elliptic Functions and Applications, Springer Verlag, New York, NY, USA, 1989.