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Highly dispersive optical solitons with cubic-quintic-septic law by *F*-expansion

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ABSTRACT

This paper successfully applied *F*-expansion algorithm to obtain highly dispersive optical solitons with cubic-quintic-septic nonlinearity. Bright, dark and singular solitons and their combinations thereof are listed. Their respective existence criteria are also indicated. Additional solutions such as periodic singular solutions as well as results in terms of Weierstrass elliptic function are also revealed from the scheme.

1. Introduction

Dispersive optical solitons are studied when dispersive effects are dominantly present. There are quite a few models that been studied in this context. One is the Schrödinger–Hirota's equation that is obtained from the familiar nonlinear Schrödinger's equation (NLSE) through Lie symmetry. Other models studied are cubic-quartic solitons [1–4], Fokas–Lenells equation, Biswas–Arshed equation [5,12]. This paper will address highly dispersive optical solitons where inter-modal dispersion (IMD), third-order dispersion (3OD), fourth-order dispersion (4OD), fifth-order dispersion (5OD) and sixth-order dispersion (6OD) terms are present in addition to the usual group velocity dispersion (GVD). The type of nonlinearity that will be studied in this paper is cubic-quintic-septic (CQS) type. This is an extension of Kerr type nonlinearity. In the past, highly dispersive solitons have been addressed with Kerr law, power law and quadratic-cubic law of nonlinearity [6–8]. There are a variety of mathematical formalisms that can handle such kind of problems [1–20]. This paper will pick up *F*-expansion scheme to handle the model. A wide variety of soliton solutions will emerge from the scheme and, as a byproduct, a plethora of additional solutions will fall out. These are all listed throughout the paper. After a quick intro to the model, the details are chalked out.

1.1. Governing model

The dimensionless form of NLSE with CQS law in presence of dispersion terms of all orders is [6–8]:

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$$iq_t + ia_1q_x + a_2q_{xx} + ia_3q_{xxx} + a_4q_{xxxx} + ia_5q_{xxxxx} + a_6q_{xxxxxx} + (b_1\ |q|^2 + b_2\ |q|^4 + b_3\ |q|^6)q = 0. \tag{1}$$

Here, in (1), $q(x, t)$ represents soliton molecules and other forms of nonlinear waves where x and t are independent spatial and temporal variables respectively. The first term represents linear temporal evolution, while a_1, a_2, a_3, a_4, a_5 and a_6 are coefficients of IMD, GVD, 3OD, 4OD, 5OD and 6OD respectively. Next, b_1, b_2 and b_3 together comprise CQS law. The complex-valued function $q(x, t)$ is the wave profile that represents the dependent variable. The coefficients are all real-valued constants and $i = \sqrt{-1}$.

2. Mathematical analysis

In order to launch the integration process, the starting hypothesis is:

$$q(x, t) = g(\varsigma)e^{i\phi(x,t)}, \tag{2}$$

where $g(\varsigma)$ gives wave structure with

$$\varsigma = x - \text{v t}, \tag{3}$$

where v is the speed of the wave and

$$\phi(x, t) = -\kappa x + \omega t + \theta. \tag{4}$$

From the phase part ϕ, κ is the soliton frequency, ω is its wave number and θ is the phase constant. After inserting (2) into (1) real part reads off to be:

$$\begin{aligned}&-(\omega + \kappa(-a_1 + \kappa(a_2 + \kappa(a_3 - \kappa(a_4 + a_5\kappa) + a_6\kappa^3))))g + b_1g^3 + b_2g^5 + b_3g^7\\&+(a_2 + 3a_3\kappa - 6a_4\kappa^2 - 10a_5\kappa^3 + 15a_6\kappa^4)g'' + (a_4 + 5a_5\kappa - 15a_6\kappa^2)g^{(4)} + a_6g^{(6)} = 0.\end{aligned} \tag{5}$$

Next, the imaginary part implies

$$(v - a_1 + 2a_2\kappa + 3a_3\kappa^2 - 4a_4\kappa^3 - 5a_5\kappa^4 + 6a_6\kappa^5)g + (-a_3 + 2\kappa(2a_4 + 5\kappa(a_5 - 2a_6\kappa)))g'' - (a_5 - 6a_6\kappa)g^{(4)} = 0. \tag{6}$$

From (6), the constraint condition that falls out is:

$$a_5 = 6a_6\kappa, \tag{7}$$

and then the other constraint condition comes out as:

$$a_3 = \frac{4\kappa(3a_4 + 5a_5\kappa)}{3}. \tag{8}$$

Thus the soliton speed reads as:

$$v = a_1 - 2a_2\kappa - 8\kappa^3(a_4 + 2a_5\kappa). \tag{9}$$

Consequently, by virtue of (7) and (8) the real part changes to:

$$\begin{aligned}&-\kappa(6\omega - 6a_1\kappa + 6a_2\kappa^2 + 18a_4\kappa^4 + 35a_5\kappa^5)g + 6b_1\kappa g^3 + 6b_2\kappa g^5 + 6b_3\kappa g^7\\&+3\kappa(2a_2 + 12a_4\kappa^2 + 25a_5\kappa^3)g'' + 3\kappa(2a_4 + 5a_5\kappa)g^{(4)} + a_5g^{(6)} = 0.\end{aligned} \tag{10}$$

The real part equation, indicated by (10) will be now analyzed by F-expansion scheme [10,11,16] in the subsequent section.

3. *F*-expansion scheme

To proceed, one assumes a solution of (10) is given in the form

$$g(\varsigma) = \sum_{j=0}^{N} \varrho_jF^j(\varsigma), \tag{11}$$

where ϱ_j are constants to be fixed and also $F = F(\varsigma)$ provides

$$(F')^2 = \text{P}F^4 + \text{Q}F^2 + R, \tag{12}$$

along with constants P, Q and R. Balancing g^7 with $g^{(6)}$ in (10) yields $N = 1$. Thus (11) becomes

$$g(\varsigma) = \varrho_0 + \varrho_1F(\varsigma). \tag{13}$$

Plugging (13) into (10), compiling the coefficients of F, and handing the resulting system one gets

$$\begin{aligned}\varrho_0 &= 0, \quad \varrho_1 = \sqrt{\frac{\ell_2 + 3b_1\kappa P}{3b_2\kappa^3 + 5b_2\kappa Q}},\\ \omega &= -\frac{9b_1^2\kappa P\,\ell_3 + 3b_1\,\ell_2\,\ell_3 + b_2P(5Q + 3\kappa^2)(6\kappa(a_2\,\ell_4 - 2a_1(3\kappa^3 + 5\kappa Q)) + a_5\,\ell_5)}{12b_2\kappa P(5Q + 3\kappa^2)^2},\\ a_4 &= -\frac{9b_1^2\kappa P + b_2P(5Q + 3\kappa^2)(6a_2\kappa + a_5(91Q^2 + 252\mathrm{P\,R} + 150\kappa^2Q + 75\kappa^4)) + 3b_1\,\ell_2}{12b_2\kappa P(5Q + 3\kappa^2)^2},\\ b_3 &= \frac{30a_5(9b_1\kappa P(6b_1^2\kappa + b_2(5Q + 3\kappa^2)(6a_2\kappa - a_5\,\ell_1)) + \ell_2\,(b_2(5Q + 3\kappa^2)(a_5\,\ell_1 - 6a_2\kappa) - 18b_1^2\kappa))}{P(6a_2\kappa - a_5\,\ell_1)^3},\end{aligned} \tag{14}$$

where

$$\begin{aligned}\ell_1 &= 259Q^2 - 252\mathrm{P\,R} + 210\kappa^2Q + 15\kappa^4\\ \ell_2 &= \sqrt{\kappa P^2(9b_1^2\kappa + 2b_2(5Q + 3\kappa^2)(6a_2\kappa - a_5\ell_1))}\\ \ell_3 &= Q^2 + 12\mathrm{P\,R} + 6\kappa^2Q - 3\kappa^4\\ \ell_4 &= -9Q^2 + 10\kappa^2Q + 3(4\mathrm{P\,R} + \kappa^4)\\ \ell_5 &= 81Q^4 + 24PQ^2R + 3024P^2R^2 + 180\kappa^2Q(3Q^2 + 4\mathrm{P\,R}) - 6\kappa^4(23Q^2 + 156\mathrm{P\,R}) - 100\kappa^6Q - 15\kappa^8.\end{aligned} \tag{15}$$

In view of these results one has the following solution of the model (1):

$$q(x, t) = \sqrt{\frac{\ell_2 + 3b_1\kappa P}{3b_2\kappa^3 + 5b_2\kappa Q}}\,F(\varsigma)\exp[i(-\kappa x + \omega t + \theta)]. \tag{16}$$

3.1. Jacobi elliptic function solutions

Jacobi elliptic function solutions for the model using the solutions of (12) are revealed as below:

Case 1: $P = m^2, \quad Q = -(1 + m^2), \quad R = 1, \quad F(\varsigma) = \mathrm{sn}\,\varsigma,$

$$\begin{aligned}q(x, t) =& \sqrt{\frac{\ell_2 + 3b_1\kappa P}{3b_2\kappa^3 + 5b_2\kappa Q}}\,\mathrm{sn}\,[x - (a_1 - 2a_2\kappa - 8\kappa^3(a_4 + 2a_5\kappa))t]\\ &\times\exp\left[i\left\{-\kappa x - \left(\frac{9b_1^2\kappa P\,\ell_3 + 3b_1\,\ell_2\,\ell_3 + b_2P(5Q + 3\kappa^2)(6\kappa(a_2\,\ell_4 - 2a_1(3\kappa^3 + 5\kappa Q)) + a_5\,\ell_5)}{12b_2\kappa P(5Q + 3\kappa^2)^2}\right)t + \theta\right\}\right].\end{aligned} \tag{17}$$

Case 2: $P = -m^2, \quad Q = 2m^2 - 1, \quad R = 1 - m^2, \quad F(\varsigma) = \mathrm{cn}\,\varsigma,$

$$\begin{aligned}q(x, t) =& \sqrt{\frac{\ell_2 + 3b_1\kappa P}{3b_2\kappa^3 + 5b_2\kappa Q}}\,\mathrm{cn}\,[x - (a_1 - 2a_2\kappa - 8\kappa^3(a_4 + 2a_5\kappa))t]\\ &\times\exp\left[i\left\{-\kappa x - \left(\frac{9b_1^2\kappa P\,\ell_3 + 3b_1\,\ell_2\,\ell_3 + b_2P(5Q + 3\kappa^2)(6\kappa(a_2\,\ell_4 - 2a_1(3\kappa^3 + 5\kappa Q)) + a_5\,\ell_5)}{12b_2\kappa P(5Q + 3\kappa^2)^2}\right)t + \theta\right\}\right].\end{aligned} \tag{18}$$

Case 3: $P = 1, \quad Q = -(1 + m^2), \quad R = m^2, \quad F(\varsigma) = \mathrm{ns}\,\varsigma,$

$$\begin{aligned}q(x, t) =& \sqrt{\frac{\ell_2 + 3b_1\kappa P}{3b_2\kappa^3 + 5b_2\kappa Q}}\,\mathrm{ns}\,[x - (a_1 - 2a_2\kappa - 8\kappa^3(a_4 + 2a_5\kappa))t]\\ &\times\exp\left[i\left\{-\kappa x - \left(\frac{9b_1^2\kappa P\,\ell_3 + 3b_1\,\ell_2\,\ell_3 + b_2P(5Q + 3\kappa^2)(6\kappa(a_2\,\ell_4 - 2a_1(3\kappa^3 + 5\kappa Q)) + a_5\,\ell_5)}{12b_2\kappa P(5Q + 3\kappa^2)^2}\right)t + \theta\right\}\right].\end{aligned} \tag{19}$$

Case 4: $P = 1, \quad Q = -(1 + m^2), \quad R = m^2, \quad F(\varsigma) = \mathrm{dc}\,\varsigma,$

$$\begin{aligned}q(x, t) =& \sqrt{\frac{\ell_2 + 3b_1\kappa P}{3b_2\kappa^3 + 5b_2\kappa Q}}\,\mathrm{dc}\,[x - (a_1 - 2a_2\kappa - 8\kappa^3(a_4 + 2a_5\kappa))t]\\ &\times\exp\left[i\left\{-\kappa x - \left(\frac{9b_1^2\kappa P\,\ell_3 + 3b_1\,\ell_2\,\ell_3 + b_2P(5Q + 3\kappa^2)(6\kappa(a_2\,\ell_4 - 2a_1(3\kappa^3 + 5\kappa Q)) + a_5\,\ell_5)}{12b_2\kappa P(5Q + 3\kappa^2)^2}\right)t + \theta\right\}\right].\end{aligned} \tag{20}$$

Case 5: $P = 1 - m^2, \quad Q = 2 - m^2, \quad R = 1, \quad F(\varsigma) = \mathrm{sc}\varsigma,$

$$\begin{aligned}q(x, t) =& \sqrt{\frac{\ell_2 + 3b_1\kappa P}{3b_2\kappa^3 + 5b_2\kappa Q}}\,\mathrm{sc}[x - (a_1 - 2a_2\kappa - 8\kappa^3(a_4 + 2a_5\kappa))t]\\ &\times\exp\left[i\left\{-\kappa x - \left(\frac{9b_1^2\kappa P\,\ell_3 + 3b_1\,\ell_2\,\ell_3 + b_2P(5Q + 3\kappa^2)(6\kappa(a_2\,\ell_4 - 2a_1(3\kappa^3 + 5\kappa Q)) + a_5\,\ell_5)}{12b_2\kappa P(5Q + 3\kappa^2)^2}\right)t + \theta\right\}\right].\end{aligned} \tag{21}$$

Case 6: $P = 1, \quad Q = 2 - m^2, \quad R = 1 - m^2, \quad F(\varsigma) = \mathrm{cs}\,\varsigma,$

$$\begin{aligned}q(x, t) =& \sqrt{\frac{\ell_2 + 3b_1\kappa P}{3b_2\kappa^3 + 5b_2\kappa Q}}\,\mathrm{cs}\,[x - (a_1 - 2a_2\kappa - 8\kappa^3(a_4 + 2a_5\kappa))t]\\ &\times\exp\left[i\left\{-\kappa x - \left(\frac{9b_1^2\kappa P\,\ell_3 + 3b_1\,\ell_2\,\ell_3 + b_2P(5Q + 3\kappa^2)(6\kappa(a_2\,\ell_4 - 2a_1(3\kappa^3 + 5\kappa Q)) + a_5\,\ell_5)}{12b_2\kappa P(5Q + 3\kappa^2)^2}\right)t + \theta\right\}\right].\end{aligned} \tag{22}$$

Case 7: $P = 1/4, \quad Q = (1-2m^2)/2, \quad R = 1/4, \quad F(\varsigma) = \mathrm{ns}\,\varsigma \pm \mathrm{cs}\,\varsigma,$

$$q(x,t) = \sqrt{\frac{\ell_2 + 3b_1\kappa P}{3b_2\kappa^3 + 5b_2\kappa Q}}\left(\mathrm{ns}\left[x - (a_1 - 2a_2\kappa - 8\kappa^3(a_4 + 2a_5\kappa))t\right] \pm \mathrm{cs}\left[x - (a_1 - 2a_2\kappa - 8\kappa^3(a_4 + 2a_5\kappa))t\right]\right)$$
$$\times \exp\left[i\left\{-\kappa x - \left(\frac{9b_1^2\kappa P\ell_3 + 3b_1\ell_2\ell_3 + b_2P(5Q + 3\kappa^2)(6\kappa(a_2\ell_4 - 2a_1(3\kappa^3 + 5\kappa Q)) + a_5\ell_5)}{12b_2\kappa P(5Q + 3\kappa^2)^2}\right)t + \theta\right\}\right]. \tag{23}$$

Case 8: $P = (1-m^2)/4, \quad Q = (1+m^2)/2, \quad R = (1-m^2)/4, \quad F(\varsigma) = \mathrm{nc}\,\varsigma \pm \mathrm{sc}\,\varsigma,$

$$q(x,t) = \sqrt{\frac{\ell_2 + 3b_1\kappa P}{3b_2\kappa^3 + 5b_2\kappa Q}}\left(\mathrm{nc}\left[x - (a_1 - 2a_2\kappa - 8\kappa^3(a_4 + 2a_5\kappa))t\right] \pm \mathrm{sc}\left[x - (a_1 - 2a_2\kappa - 8\kappa^3(a_4 + 2a_5\kappa))t\right]\right)$$
$$\times \exp\left[i\left\{-\kappa x - \left(\frac{9b_1^2\kappa P\ell_3 + 3b_1\ell_2\ell_3 + b_2P(5Q + 3\kappa^2)(6\kappa(a_2\ell_4 - 2a_1(3\kappa^3 + 5\kappa Q)) + a_5\ell_5)}{12b_2\kappa P(5Q + 3\kappa^2)^2}\right)t + \theta\right\}\right]. \tag{24}$$

Case 9: $P = m^2/4, \quad Q = (m^2-2)/2, \quad R = m^2/4, \quad F(\varsigma) = \mathrm{sn}\,\varsigma \pm i\,\mathrm{cn}\,\varsigma,$

$$q(x,t) = \sqrt{\frac{\ell_2 + 3b_1\kappa P}{3b_2\kappa^3 + 5b_2\kappa Q}}\left(\mathrm{sn}\left[x - (a_1 - 2a_2\kappa - 8\kappa^3(a_4 + 2a_5\kappa))t\right] \pm i\,\mathrm{cn}\left[x - (a_1 - 2a_2\kappa - 8\kappa^3(a_4 + 2a_5\kappa))t\right]\right)$$
$$\times \exp\left[i\left\{-\kappa x - \left(\frac{9b_1^2\kappa P\ell_3 + 3b_1\ell_2\ell_3 + b_2P(5Q + 3\kappa^2)(6\kappa(a_2\ell_4 - 2a_1(3\kappa^3 + 5\kappa Q)) + a_5\ell_5)}{12b_2\kappa P(5Q + 3\kappa^2)^2}\right)t + \theta\right\}\right]. \tag{25}$$

Case 10: $P = m^2/4, \quad Q = (m^2-2)/2, \quad R = 1/4, \quad F(\varsigma) = \frac{\mathrm{sn}\,\varsigma}{1 \pm \mathrm{dn}\,\varsigma},$

$$q(x,t) = \sqrt{\frac{\ell_2 + 3b_1\kappa P}{3b_2\kappa^3 + 5b_2\kappa Q}}\frac{\mathrm{sn}\left[x - (a_1 - 2a_2\kappa - 8\kappa^3(a_4 + 2a_5\kappa))t\right]}{1 \pm \mathrm{dn}\left[x - (a_1 - 2a_2\kappa - 8\kappa^3(a_4 + 2a_5\kappa))t\right]}$$
$$\times \exp\left[i\left\{-\kappa x - \left(\frac{9b_1^2\kappa P\ell_3 + 3b_1\ell_2\ell_3 + b_2P(5Q + 3\kappa^2)(6\kappa(a_2\ell_4 - 2a_1(3\kappa^3 + 5\kappa Q)) + a_5\ell_5)}{12b_2\kappa P(5Q + 3\kappa^2)^2}\right)t + \theta\right\}\right]. \tag{26}$$

Case 11: $P = -1/4, \quad Q = (m^2+1)/2, \quad R = (1-m^2)^2/4, \quad F(\varsigma) = m\,\mathrm{cn}\,\varsigma \pm \mathrm{dn}\,\varsigma,$

$$q(x,t) = \sqrt{\frac{\ell_2 + 3b_1\kappa P}{3b_2\kappa^3 + 5b_2\kappa Q}}\left(m\,\mathrm{cn}\left[x - (a_1 - 2a_2\kappa - 8\kappa^3(a_4 + 2a_5\kappa))t\right] \pm \mathrm{dn}\left[x - (a_1 - 2a_2\kappa - 8\kappa^3(a_4 + 2a_5\kappa))t\right]\right)$$
$$\times \exp\left[i\left\{-\kappa x - \left(\frac{9b_1^2\kappa P\ell_3 + 3b_1\ell_2\ell_3 + b_2P(5Q + 3\kappa^2)(6\kappa(a_2\ell_4 - 2a_1(3\kappa^3 + 5\kappa Q)) + a_5\ell_5)}{12b_2\kappa P(5Q + 3\kappa^2)^2}\right)t + \theta\right\}\right]. \tag{27}$$

Case 12: $P = (1-m^2)^2/4, \quad Q = (m^2+1)/2, \quad R = 1/4, \quad F(\varsigma) = \mathrm{ds}\,\varsigma \pm \mathrm{cs}\,\varsigma,$

$$q(x,t) = \sqrt{\frac{\ell_2 + 3b_1\kappa P}{3b_2\kappa^3 + 5b_2\kappa Q}}\left(\mathrm{ds}\left[x - (a_1 - 2a_2\kappa - 8\kappa^3(a_4 + 2a_5\kappa))t\right] \pm \mathrm{cs}\left[x - (a_1 - 2a_2\kappa - 8\kappa^3(a_4 + 2a_5\kappa))t\right]\right)$$
$$\times \exp\left[i\left\{-\kappa x - \left(\frac{9b_1^2\kappa P\ell_3 + 3b_1\ell_2\ell_3 + b_2P(5Q + 3\kappa^2)(6\kappa(a_2\ell_4 - 2a_1(3\kappa^3 + 5\kappa Q)) + a_5\ell_5)}{12b_2\kappa P(5Q + 3\kappa^2)^2}\right)t + \theta\right\}\right]. \tag{28}$$

Case 13: $P > 0, \quad Q < 0, \quad R = \frac{m^2Q^2}{(1+m^2)^2P}, \quad F(\varsigma) = \sqrt{-\frac{m^2Q}{(1+m^2)P}}\,\mathrm{sn}\left(\sqrt{-\frac{Q}{1+m^2}}\varsigma\right),$

$$q(x,t) = \sqrt{\frac{\ell_2 + 3b_1\kappa P}{3b_2\kappa^3 + 5b_2\kappa Q}}\sqrt{-\frac{m^2Q}{(1+m^2)P}}\,\mathrm{sn}\left(\sqrt{-\frac{Q}{1+m^2}}\left[x - (a_1 - 2a_2\kappa - 8\kappa^3(a_4 + 2a_5\kappa))t\right]\right)$$
$$\times \exp\left[i\left\{-\kappa x - \left(\frac{9b_1^2\kappa P\ell_3 + 3b_1\ell_2\ell_3 + b_2P(5Q + 3\kappa^2)(6\kappa(a_2\ell_4 - 2a_1(3\kappa^3 + 5\kappa Q)) + a_5\ell_5)}{12b_2\kappa P(5Q + 3\kappa^2)^2}\right)t + \theta\right\}\right]. \tag{29}$$

Case 14: $P < 0, \quad Q > 0, \quad R = \frac{(1-m^2)Q^2}{(m^2-2)^2P}, \quad F(\varsigma) = \sqrt{-\frac{Q}{(2-m^2)P}}\,\mathrm{dn}\left(\sqrt{\frac{Q}{2-m^2}}\varsigma\right),$

$$q(x,t) = \sqrt{\frac{\ell_2 + 3b_1\kappa P}{3b_2\kappa^3 + 5b_2\kappa Q}}\sqrt{-\frac{Q}{(2-m^2)P}}\,\mathrm{dn}\left(\sqrt{\frac{Q}{2-m^2}}\left[x - (a_1 - 2a_2\kappa - 8\kappa^3(a_4 + 2a_5\kappa))t\right]\right)$$
$$\times \exp\left[i\left\{-\kappa x - \left(\frac{9b_1^2\kappa P\ell_3 + 3b_1\ell_2\ell_3 + b_2P(5Q + 3\kappa^2)(6\kappa(a_2\ell_4 - 2a_1(3\kappa^3 + 5\kappa Q)) + a_5\ell_5)}{12b_2\kappa P(5Q + 3\kappa^2)^2}\right)t + \theta\right\}\right]. \tag{30}$$

Case 15: $P = 1, \quad Q = m^2 + 2, \quad R = 1 - 2m^2 + m^4, \quad F(\varsigma) = \frac{\mathrm{dn}\,\varsigma\,\mathrm{cn}\,\varsigma}{\mathrm{sn}\,\varsigma},$

$$q(x,t) = \sqrt{\frac{\ell_2 + 3b_1\kappa P}{3b_2\kappa^3 + 5b_2\kappa Q}}\frac{\mathrm{dn}\left[x - (a_1 - 2a_2\kappa - 8\kappa^3(a_4 + 2a_5\kappa))t\right]\mathrm{cn}\left[x - (a_1 - 2a_2\kappa - 8\kappa^3(a_4 + 2a_5\kappa))t\right]}{\mathrm{sn}\left[x - (a_1 - 2a_2\kappa - 8\kappa^3(a_4 + 2a_5\kappa))t\right]}$$
$$\times \exp\left[i\left\{-\kappa x - \left(\frac{9b_1^2\kappa P\ell_3 + 3b_1\ell_2\ell_3 + b_2P(5Q + 3\kappa^2)(6\kappa(a_2\ell_4 - 2a_1(3\kappa^3 + 5\kappa Q)) + a_5\ell_5)}{12b_2\kappa P(5Q + 3\kappa^2)^2}\right)t + \theta\right\}\right]. \tag{31}$$

Case 16: $P = \frac{A^2(m-1)^2}{4}, \quad Q = \frac{m^2+1}{2} + 3m, \quad R = \frac{(m-1)^2}{4A^2}, \quad F(\varsigma) = \frac{\mathrm{dn}\,\varsigma\,\mathrm{cn}\,\varsigma}{A(1 + \mathrm{sn}\,\varsigma)(1 + m\,\mathrm{sn}\,\varsigma)},$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\times\frac{\mathrm{dn}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]\,\mathrm{cn}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]}{A(1+\,\mathrm{sn}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t])(1+m\,\mathrm{sn}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t])}\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right].\tag{32}$$

Case 17: $P=-\frac{4}{m},\quad Q=6m-m^2-1,\quad R=-2m^3+m^4+m^2,\quad F(\varsigma)=\frac{m\,\mathrm{cn}\,\varsigma\,\mathrm{dn}\,\varsigma}{m\,\mathrm{sn}^2\varsigma+1},$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\frac{m\,\mathrm{cn}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]\,\mathrm{dn}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]}{m\,\mathrm{sn}^2\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]+1}\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right].\tag{33}$$

Case 18: $P=1/4,\quad Q=\frac{1-2m^2}{2},\quad R=1/4,\quad F(\varsigma)=\frac{\mathrm{sn}\,\varsigma}{1\pm\,\mathrm{cn}\,\varsigma},$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\frac{\mathrm{sn}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]}{1\pm\,\mathrm{cn}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]}\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right].\tag{34}$$

Case 19: $P=\frac{1-m^2}{4},\quad Q=\frac{1+m^2}{2},\quad R=\frac{1-m^2}{4},\quad F(\varsigma)=\frac{\mathrm{cn}\,\varsigma}{1\pm\,\mathrm{sn}\,\varsigma},$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\frac{\mathrm{cn}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]}{1\pm\,\mathrm{sn}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]}\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right].\tag{35}$$

Case 20: $P=\frac{2-m^2-2m_1}{4},\quad Q=\frac{m^2}{2}-1-3m_1,\quad R=\frac{2-m^2-2m_1}{4},\quad F(\varsigma)=\frac{m^2\,\mathrm{sn}\,\varsigma\,\mathrm{cn}\,\varsigma}{\mathrm{sn}^2\varsigma+(1+m_1)\,\mathrm{dn}\,\varsigma-1-m_1},$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\times\frac{m^2\,\mathrm{sn}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]\,\mathrm{cn}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]}{\mathrm{sn}^2\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]+(1+m_1)\,\mathrm{dn}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]-1-m_1}\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right],\tag{36}$$

where $m_1=\sqrt{1-m^2}$.

Case 21: $P=\frac{2-m^2+2m_1}{4},\quad Q=\frac{m^2}{2}-1+3m_1,\quad R=\frac{2-m^2+2m_1}{4},\quad F(\varsigma)=\frac{m^2\,\mathrm{sn}\,\varsigma\,\mathrm{cn}\,\varsigma}{\mathrm{sn}^2\varsigma+(-1+m_1)\,\mathrm{dn}\,\varsigma-1-m_1},$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\times\frac{m^2\,\mathrm{sn}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]\,\mathrm{cn}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]}{\mathrm{sn}^2\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]+(-1+m_1)\,\mathrm{dn}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]-1-m_1}\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right].\tag{37}$$

Case 22: $P=\frac{C^2m^4-(B^2+C^2)m^2+B^2}{4},\quad Q=\frac{m^2+1}{2},\quad R=\frac{m^2-1}{4(C^2m^2-B^2)},\quad F(\varsigma)=\frac{\sqrt{\frac{B^2-C^2}{B^2-C^2m^2}}+\,\mathrm{sn}\,\varsigma}{B\,\mathrm{cn}\,\varsigma+C\,\mathrm{dn}\,\varsigma},$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\frac{\sqrt{\frac{B^2-C^2}{B^2-C^2m^2}}+\,\mathrm{sn}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]}{B\,\mathrm{cn}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]+C\,\mathrm{dn}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]}\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right].\tag{38}$$

Case 23: $P=\frac{B^2+C^2m^2}{4},\quad Q=\frac{1}{2}-m^2,\quad R=\frac{1}{4(B^2+C^2m^2)},\quad F(\varsigma)=\frac{\sqrt{\frac{C^2m^2+B^2-C^2}{B^2+C^2m^2}}+\,\mathrm{cn}\,\varsigma}{B\,\mathrm{sn}\,\varsigma+C\,\mathrm{dn}\,\varsigma},$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\frac{\sqrt{\frac{C^2m^2+B^2-C^2}{B^2+C^2m^2}}+\,\mathrm{cn}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]}{B\,\mathrm{sn}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]+C\,\mathrm{dn}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]}\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right].\tag{39}$$

Case 24: $P=\frac{B^2+C^2}{4},\quad Q=\frac{m^2}{2}-1,\quad R=\frac{m^4}{4(B^2+C^2)},\quad F(\varsigma)=\frac{\sqrt{\frac{B^2+C^2-C^2m^2}{B^2+C^2}}+\,\mathrm{dn}\,\varsigma}{B\,\mathrm{sn}\,\varsigma+C\,\mathrm{cn}\,\varsigma},$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\frac{\sqrt{\frac{B^2+C^2-C^2m^2}{B^2+C^2}}+\mathrm{dn}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]}{B\,\mathrm{sn}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]+C\,\mathrm{cn}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right]. \quad (40)$$

3.2. Weïerstrass elliptic function solutions

Upon recapitulating, Weierstrass elliptic function is described as [14]:

$$\wp(z;\omega_1,\omega_2)=\frac{1}{z^2}+\sum_{\{m,n\}\neq\{0,0\}}\left(\frac{1}{(z+2m\omega_1+2n\omega_2)^2}-\frac{1}{(2m\omega_1+2n\omega_2)^2}\right). \quad (41)$$

Weierstrass elliptic function solutions to the model by the help of the solutions of (12) in [15] are acquired as follows:

Case 25: $g_2=\frac{4}{3}(Q^2-3\mathrm{PR}),\quad g_3=\frac{4Q}{27}(-2Q^2+9\mathrm{PR}),\quad F(\varsigma)=\sqrt{\frac{1}{P}\left[\wp(\varsigma;g_2,g_3)-\frac{1}{3}Q\right]},$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\sqrt{\frac{1}{P}\left[\wp([x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t];g_2,g_3)-\frac{1}{3}Q\right]}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right]. \quad (42)$$

Case 26: $g_2=\frac{4}{3}(Q^2-3\mathrm{PR}),\quad g_3=\frac{4Q}{27}(-2Q^2+9\mathrm{PR}),\quad F(\varsigma)=\sqrt{\frac{3R}{3\wp(\varsigma;g_2,g_3)-Q}},$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\sqrt{\frac{3R}{3\wp([x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t];g_2,g_3)-Q}}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right]. \quad (43)$$

Case 27: $g_2=-\frac{5\mathrm{QD}+4Q^2+33\mathrm{PQR}}{12},\quad g_3=\frac{21Q^2D-63\mathrm{PRD}+20Q^3-27\mathrm{PQR}}{216},\ F(\varsigma)=\frac{\sqrt{12R\wp(\varsigma;g_2,g_3)+2R(2Q+D)}}{12\wp(\varsigma;g_2,g_3)+D},$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\frac{\sqrt{12R\wp([x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t];g_2,g_3)+2R(2Q+D)}}{12\wp([x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t];g_2,g_3)+D}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right]. \quad (44)$$

Case 28: $g_2=\frac{1}{12}Q^2+\mathrm{PR},\quad g_3=\frac{1}{216}Q(36\mathrm{PR}-Q^2),\quad F(\varsigma)=\frac{\sqrt{R}\,[6\wp(\varsigma;g_2,g_3)+Q]}{3\wp'(\varsigma;g_2,g_3)},$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\frac{\sqrt{R}\,[6\wp([x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t];g_2,g_3)+Q]}{3\wp'([x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t];g_2,g_3)}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right]. \quad (45)$$

Case 29: $g_2=\frac{1}{12}Q^2+\mathrm{PR},\quad g_3=\frac{1}{216}Q(36\mathrm{PR}-Q^2),\quad F(\varsigma)=\frac{3\wp'(\varsigma;g_2,g_3)}{\sqrt{P}\,[6\wp(\varsigma;g_2,g_3)+Q]},$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\frac{3\wp'([x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t];g_2,g_3)}{\sqrt{P}\,[6\wp([x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t];g_2,g_3)+Q]}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right]. \quad (46)$$

Case 30: $R=\frac{5Q^2}{36P},\quad g_2=\frac{2Q^2}{9},\quad g_3=\frac{Q^3}{54},\quad F(\varsigma)=\frac{Q\sqrt{-15Q/2P}\,\wp(\varsigma;g_2,g_3)}{3\wp(\varsigma;g_2,g_3)+Q},$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\frac{Q\sqrt{-15Q/2P}\,\wp([x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t];g_2,g_3)}{3\wp([x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t];g_2,g_3)+Q}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right]. \quad (47)$$

3.3. Soliton and other solutions

In limiting case, if the modulus $m\to 1$, the following bright, dark and singular solitons, the combined solitons, and complexiton solutions are secured:

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\tanh[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \quad (48)$$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\operatorname{sech}[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \quad (49)$$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\coth[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \quad (50)$$

Solutions (48)–(50) are dark, bright and singular solitons respectively.

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\sinh[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \quad (51)$$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\operatorname{csch}[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \quad (52)$$

Solution (52) is the second form of singular soliton solution to the model.

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}(\coth[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]\pm\operatorname{csch}[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t])\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \quad (53)$$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}(\cosh[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]\pm\sinh[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t])\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \quad (54)$$

Solution (53) and (54) also represents singular solitons.

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}(\tanh[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]\pm i\operatorname{sech}[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t])\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \quad (55)$$

Then, solution (55) represents complexiton solutions.

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\frac{\tanh[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]}{1\pm\operatorname{sech}[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]}\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \quad (56)$$

$$q(x,t)=2\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\operatorname{sech}[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \quad (57)$$

$$q(x,t)=2\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\operatorname{csch}[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \quad (58)$$

Next, (57) and (58) are bright and singular solitons respectively.

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\sqrt{-\frac{Q}{2P}}\tanh\left(\sqrt{-\frac{Q}{2}}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]\right)$$
$$\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \tag{59}$$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\sqrt{-\frac{Q}{P}}\operatorname{sech}(\sqrt{Q}\,[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t])$$
$$\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \tag{60}$$

Again (59) and (60) are dark and bright solitons respectively. The dark soliton exists for $Q<0$ and $P>0$ while the bright soliton is valid for $Q>0$ and $P<0$.

$$q(x,t)=2\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\operatorname{csch}2[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]$$
$$\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \tag{61}$$

Equation (61) represents singular solitons.

$$q(x,t)=\frac{1}{A}\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\exp(-2[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t])$$
$$\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \tag{62}$$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\operatorname{sech}2[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]$$
$$\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \tag{63}$$

Equation (63) is another form of bright solitons.

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\frac{\tanh[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]}{1\pm\operatorname{sech}[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \tag{64}$$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\frac{\operatorname{sech}[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]}{1\pm\tanh[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \tag{65}$$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\coth\left[\frac{x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t}{2}\right]$$
$$\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \tag{66}$$

$$q(x,t)=-\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\tanh\left[\frac{x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t}{2}\right]$$
$$\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \tag{67}$$

Finally, (66) and (67) respectively represent singular and dark solitons to the model.

$$q(x,t)=\frac{1}{B+C}\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\exp[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]$$
$$\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \tag{68}$$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\frac{\sqrt{\frac{B^2}{B^2+C^2}}+\operatorname{sech}[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]}{B\tanh[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]+C\operatorname{sech}[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right]. \tag{69}$$

3.4. Trigonometric function solutions

However, when $m \to 0$, periodic waves, periodic singular waves and a combination of such solutions appear as:

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\sin[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t] \times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \tag{70}$$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\cos[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t] \times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \tag{71}$$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\csc[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t] \times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \tag{72}$$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\sec[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t] \times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \tag{73}$$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\tan[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t] \times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \tag{74}$$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\cot[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t] \times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \tag{75}$$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}(\csc[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]\pm\cot[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]) \times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \tag{76}$$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}(\sec[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]\pm\tan[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]) \times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \tag{77}$$

$$q(x,t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}(\sin[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]\pm i\cos[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]) \times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \tag{78}$$

$$q(x,t)=\frac{1}{2}\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\sin[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t] \times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \tag{79}$$

$$q(x,t)=\frac{1}{A}\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}(\sec[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]-\tan[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]) \times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\ell_3+3b_1\ell_2\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \tag{80}$$

$$q(x,\ t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\,\frac{\sqrt{\frac{B^2-C^2}{B^2}}+\sin[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]}{B\cos[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]+C}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\,\ell_3+3b_1\,\ell_2\,\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\,\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\,\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \tag{81}$$

$$q(x,\ t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\,\frac{\sqrt{\frac{B^2-C^2}{B^2}}+\cos[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]}{B\sin[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]+C}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\,\ell_3+3b_1\,\ell_2\,\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\,\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\,\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right], \tag{82}$$

$$q(x,\ t)=\sqrt{\frac{\ell_2+3b_1\kappa P}{3b_2\kappa^3+5b_2\kappa Q}}\,\frac{2}{B\sin[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]+C\cos[x-(a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa))t]}$$
$$\times\exp\left[i\left\{-\kappa x-\left(\frac{9b_1^2\kappa P\,\ell_3+3b_1\,\ell_2\,\ell_3+b_2P(5Q+3\kappa^2)(6\kappa(a_2\,\ell_4-2a_1(3\kappa^3+5\kappa Q))+a_5\,\ell_5)}{12b_2\kappa P(5Q+3\kappa^2)^2}\right)t+\theta\right\}\right]. \tag{83}$$

4. Conclusions

This paper studied highly dispersive optical solitons with CQS nonlinearity. The *F*-expansion algorithm yielded a plethora of solutions. They are of various types including soliton solutions. Bright, dark and singular as well as their combination types of solitons are obtained. Other solution types included are in terms of Weierstrass elliptic functions, periodic singular solutions and complexitons, just to list a few. The results appeared all across the spectrum. This project thus stands on a strong footing to carry out additional studies with the model. Later, this model will be studied in birefringent fibers, DWDM systems, magneto-optic waveguides, optical couplers, metamaterials and other such devices. In addition, the current model will be further analyzed with fractional temporal evolution or stochastic coefficients as well as studies will be conducted with additional laws of nonlinearity. All such studies will yield marvelous and novel results that will be gradually disclosed.

Conflict of interest

The authors also declare that there is no conflict of interest.

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References

[1] A. Bansal, A. Biswas, Q. Zhou, M.M. Babatin, Lie symmetry analysis for cubic-quartic nonlinear Schrödinger's equation, Optik 169 (2018) 12–15.
[2] A. Biswas, H. Triki, Q. Zhou, S.P. Moshokoa, M.Z. Ullah, M. Belic, Cubic-quartic optical solitons in Kerr and power law media, Optik 144 (2017) 357–362.
[3] A. Biswas, A.H. Kara, M.Z. Ullah, Q. Zhou, H. Triki, M. Belic, Conservation laws for cubic-quartic optical solitons in Kerr and power law media, Optik 145 (2017) 650–654.
[4] A. Biswas, S. Arshed, Application of semi-inverse variational principle to cubic-quartic optical solitons with Kerr and power law nonlinearity, Optik 172 (2018) 847–850.
[5] A. Biswas, S. Arshed, Optical solitons in presence of higher order dispersions and absence of self-phase modulation, Optik 174 (2018) 452–459.
[6] A. Biswas, J. Vega-Guzman, M.F. Mahmood, Q. Zhou, S. Khan, S.P. Moshokoa, M.R. Belic, Highly Dispersive Optical Solitons by Undetermined Coefficients, (2019) (In Press).
[7] A. Biswas, M. Ekici, A. Sonmezoglu, M.R. Belic, Highly Dispersive Optical Solitons With Kerr Law Nonlinearity by *F*-Expansion, Optik 181 (2019) 1028–1038.
[8] A. Biswas, M. Ekici, A. Sonmezoglu, M.R. Belic, Highly Dispersive Optical Solitons With Quadratic-Cubic Law by *F*-Expansion, (2019) (In Press).
[9] A. Das, A. Biswas, M. Ekici, S. Khan, Q. Zhou, S.P. Moshokoa, Suppressing Internet Bottleneck With Fractional Temporal Evolution of Cubic-Quartic Optical Solitons, Optik 182 (2019) 303–307.
[10] H.T. Chen, H.Q. Zhang, New double periodic and multiple soliton solutions of the generalized (2 + 1)-dimensional Boussinesq equation, Chaos Solitons Fractals 20 (4) (2004) 765–769.
[11] A. Ebaid, E.H. Aly, Exact solutions for the transformed reduced Ostrovsky equation via the *F*-expansion method in terms of Weierstrass-elliptic and Jacobian-elliptic functions, Wave Motion 49 (2) (2012) 296–308.
[12] M. Ekici, A. Sonmezoglu, Optical solitons with Biswas-Arshed equation by extended trial function method, Optik 177 (2019) 13–20.
[13] M.G. Hafez, New travelling wave solutions of the (1 + 1)-dimensional cubic nonlinear Schrödinger equation using novel G'/G-expansion method, Beni-Suef Univ. J. Basic Appl. Sci. 5 (2) (2016) 109–118.
[14] D.F. Lawden, Elliptic Functions and Applications, Springer Verlag, New York, NY, USA, 1989.
[15] Z. Yan, An improved algebra method and its applications in nonlinear wave equations, MM Res. Preprints 22 (2003) 264–274.
[16] D. Zhang, Doubly periodic solutions of the modified Kawahara equation, Chaos Solitons Fractals 25 (5) (2005) 1155–1160.
[17] Q. Zhou, Q. Zhu, A. Biswas, Optical solitons in birefringent fibers with parabolic law nonlinearity, Opt. Appl. 41 (3) (2014) 399–409.
[18] Q. Zhou, Q. Zhu, L. Moraru, A. Biswas, Optical solitons in photonic crystal fibers with spatially inhomogeneous nonlinearities, Optoelectron. Adv. Mater.-Rapid Commun. 16 (11–12) (2014) 995–997.
[19] Q. Zhou, Q. Zhu, Y. Liu, H. Yu, P. Yao, A. Biswas, Thirring optical solitons in birefringent fibers with spatio-temporal dispersion and Kerr nonlinearity, Laser Phys. 25 (1) (2015) 015402.
[20] Q. Zhou, Q. Zhu, H. Yu, Y. Liu, C. Wei, P. Yao, A.H. Bhrawy, A. Biswas, Bright, dark and singular optical solitons in a cascaded system, Laser Phys. 25 (2) (2015) 025402.