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Highly dispersive optical solitons with Kerr law nonlinearity by *F*-expansion

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ABSTRACT

This paper employs *F*-expansion scheme to secure bright, dark, singular and other combo highly dispersive optical solitons when all orders of dispersion up to sixth order are taken into account. There are other solutions that naturally emerge from this scheme as a byproduct. These include Weierstrass function, singular periodic solutions, doubly periodic solutions and so on.

1. Introduction

Dispersive optical solitons is a very viable topic of study in the field of telecommunications engineering. There are several models that address the study of dispersive optical solitons. While the most visible and well known is the Schrödinger–Hirota equation, there are other models that addressed dispersive solitons such as cubic–quartic (CQ) solitons [1–5]. The concept of CQ solitons was introduced when group velocity dispersion (GVD) was low and so to replenish it, higher order dispersion terms were introduced to provide the necessary balance for the solitons to sustain. However, this paper studies highly dispersive solitons when a variety of dispersive effects are included. In addition to GVD, the other dispersion terms are inter-modal dispersion (IMD), third-order dispersion (3OD), fourth-order dispersion (4OD), fifth-order dispersion (5OD) and sixth-order dispersion (6OD). The governing nonlinear Schrödinger's equation (NLSE) is studied by the aid of *F*-expansion to retrieve solitons and other solutions to the model. This is a very powerful scheme that has been successfully applied to various forms of nonlinear evolution equations in fluid dynamics and other areas of applied sciences [6–9]. The detailed analysis is enumerated in the subsequent sections.

1.1. Governing model

The dimensionless form of NLSE with Kerr law nonlinearity in presence of dispersion terms of all orders is [1–5]:

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$$iq_t + ia_1 q_x + a_2 q_{xx} + ia_3 q_{xxx} + a_4 q_{xxxx} + ia_5 q_{xxxxx} + a_6 q_{xxxxxx} + b\,|q|^2 q = 0. \tag{1}$$

Here, in (1), $q(x, t)$ represents the soliton molecule with x and t being the independent spatial and temporal variables respectively. The first term represents linear temporal evolution, while a_1, a_2, a_3, a_4, a_5 and a_6 are coefficients of IMD, GVD, 3OD, 4OD, 5OD and 6OD respectively. Finally, b is the coefficient of self-phase modulation based on the type of nonlinearity. The complex-valued function $q(x, t)$ is the wave profile that is the dependent variable. The independent variables are x and t which represent spatial and temporal variables respectively. The coefficients are all real-valued constants while $i = \sqrt{-1}$.

2. Mathematical analysis

In order to tackle with (1), the starting hypothesis is selected as follows:

$$q(x, t) = g(s)e^{i\phi(x,t)}, \tag{2}$$

where $g(s)$ represents the shape of the pulse and

$$s = x - \mathrm{v}\,t, \tag{3}$$

where v is the velocity of the soliton and

$$\phi(x, t) = -\kappa x + \omega t + \theta. \tag{4}$$

From the phase component $\phi(x, t)$, κ is the soliton frequency, while ω is the soliton wave number and θ is the phase constant. Insert (2) into (1) and then decompose into real and imaginary parts respectively. Thus real part gives rise to

$$\begin{aligned}&-(\omega + \kappa(-a_1 + \kappa(a_2 + \kappa(a_3 - \kappa(a_4 + a_5\kappa) + a_6\kappa^3))))g + bg^3 + (a_2 + 3a_3\kappa - 6a_4\kappa^2 - 10a_5\kappa^3 + 15a_6\kappa^4)g''\\&+ (a_4 + 5a_5\kappa - 15a_6\kappa^2)g^{(4)} + a_6 g^{(6)} = 0,\end{aligned} \tag{5}$$

while imaginary part implies

$$(v - a_1 + 2a_2\kappa + 3a_3\kappa^2 - 4a_4\kappa^3 - 5a_5\kappa^4 + 6a_6\kappa^5)g + (-a_3 + 2\kappa(2a_4 + 5\kappa(a_5 - 2a_6\kappa)))g'' - (a_5 - 6a_6\kappa)g^{(4)} = 0. \tag{6}$$

From (6), setting the coefficients of the linearly independent functions to zero gives the velocity of the soliton as

$$v = a_1 - \kappa(2a_2 + 3a_3\kappa - 4a_4\kappa^2 - 5a_5\kappa^3 + 6a_6\kappa^4), \tag{7}$$

and the constraint conditions fall out as

$$2\kappa(2a_4 + 5\kappa(a_5 - 2a_6\kappa)) - a_3 = 0, \tag{8}$$

$$a_5 - 6a_6\kappa = 0. \tag{9}$$

The real part equation (5) will now be studied by F-expansion scheme in the following section.

3. *F*-Expansion scheme

F-expansion scheme [6–8] will be adopted to integrate the model (5). To begin, the solution to (5) is taken to be

$$g(s) = \sum_{j=0}^{N} \zeta_j F^j(s), \tag{10}$$

where ζ_j are constants to be determined and also $F = F(s)$ is a solution of

$$(F')^2 = PF^4 + QF^2 + R, \tag{11}$$

where P, Q and R are constants. Balancing g^3 with $g^{(6)}$ in (5) gives $N = 3$. This means that

$$g(s) = \zeta_0 + \zeta_1 F(s) + \zeta_2 F^2(s) + \zeta_3 F^3(s). \tag{12}$$

Putting (12) into (5), collecting the coefficients of F, and solving the resulting system one has

$$\begin{aligned}&\zeta_0 = 0, \quad \zeta_1 = \tfrac{1}{2}(2\mathcal{H}_0 + \sqrt{\mathcal{H}_1} + \sqrt{\mathcal{L}_1}), \quad \zeta_2 = 0, \quad \zeta_3 = \zeta_3,\\&\omega = \frac{P^2(\mathcal{L}_4 + a_6(\mathcal{L}_3 - 320\mathcal{L}_6 + \mathcal{L}_0(\mathcal{L}_2 + 1343(\mathcal{L}_5 + \sqrt{\mathcal{L}_1}\mathcal{L}_9)) - \sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7 + \mathcal{L}_8 + \mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}},\\&a_2 = \frac{1}{1343\zeta_3^2\mathcal{L}_{12}}\left(\mathcal{L}_{11} + a_6\left(\mathcal{H}_8 + \mathcal{L}_{13}(\mathcal{H}_{10} + 1580711P\sqrt{\mathcal{L}_1}) + \frac{3\zeta_3\mathcal{L}_{12}(\mathcal{H}_{11} + \mathcal{H}_9\sqrt{\mathcal{L}_1})}{1343}\right)\right),\\&a_4 = -5a_5\kappa + a_6\left(\mathcal{H}_{12} + \frac{83(1803649P\sqrt{\mathcal{L}_1} + \mathcal{L}_{12})}{1803649\zeta_3}\right), \quad b = -\frac{20160a_6P^3}{\zeta_3^2},\end{aligned} \tag{13}$$

where

$$
\begin{aligned}
\mathcal{L}_0 &= 1803649P\mathcal{H}_1\\
\mathcal{L}_1 &= \mathcal{H}_6 + \frac{\mathcal{H}_3}{4\sqrt{\mathcal{H}_1}} - \frac{\sqrt[3]{\mathcal{H}_5-\sqrt{\mathcal{H}_2}}}{4029\sqrt[3]{2}P^3} - \frac{1008\sqrt[3]{2}\mathcal{H}_4}{1343P^3\sqrt[3]{\mathcal{H}_5-\sqrt{\mathcal{H}_2}}}\\
\mathcal{L}_2 &= 10\zeta_3^2P\sqrt[3]{\mathcal{H}_5-\sqrt{\mathcal{H}_2}}\,(1343(3024\mathrm{P\,R}-83\kappa^4)-541233Q^2-134358\kappa^2Q)\\
\mathcal{L}_3 &= -6506299430402P^4\mathcal{H}_1^2\sqrt[3]{\mathcal{H}_5-\sqrt{\mathcal{H}_2}}\\
\mathcal{L}_4 &= 4844601214P\sqrt{\mathcal{H}_1}\mathcal{H}_7\sqrt[3]{\mathcal{H}_5-\sqrt{\mathcal{H}_2}}\\
\mathcal{L}_5 &= \sqrt[3]{2}\,(\sqrt[3]{2}\,(\mathcal{H}_5-\sqrt{\mathcal{H}_2})^{2/3}-6048\zeta_3^4P^2(4611P^2R^2-1104PQ^2R-16Q^4))\\
\mathcal{L}_6 &= \zeta_3^4Q\sqrt[3]{\mathcal{H}_5-\sqrt{\mathcal{H}_2}}\,(3045Q-1177\kappa^2)(5441984Q^2-40431015\mathrm{P\,R})\\
\mathcal{L}_7 &= 2\zeta_3^2P\sqrt[3]{\mathcal{H}_5-\sqrt{\mathcal{H}_2}}\,(557345\kappa^4-23352084\mathrm{P\,R}+1907493Q^2+671790\kappa^2Q)\\
\mathcal{L}_8 &= 1343\sqrt[3]{2}\,(\sqrt[3]{2}\,(\mathcal{H}_5-\sqrt{\mathcal{H}_2})^{2/3}-6048\zeta_3^4P^2(4611P^2R^2-1104PQ^2R-16Q^4))\\
\mathcal{L}_9 &= 4\zeta_3P^2\sqrt[3]{\mathcal{H}_5-\sqrt{\mathcal{H}_2}}\,(3045Q-1177\kappa^2)\\
\mathcal{L}_{10} &= 3607298P^3\sqrt[3]{\mathcal{H}_5-\sqrt{\mathcal{H}_2}}\\
\mathcal{L}_{11} &= -2422300607\zeta_3^2\kappa P\sqrt{\mathcal{H}_1}\,(20a_5\kappa^2+3a_3)\\
\mathcal{L}_{12} &= 1803649P\sqrt{\mathcal{H}_1}\\
\mathcal{L}_{13} &= 3607298P^2\mathcal{H}_1
\end{aligned}
\tag{14}
$$

in which

$$
\begin{aligned}
\mathcal{H}_0 &= \frac{934\zeta_3Q}{1343P}\\
\mathcal{H}_1 &= \ell_1 + \frac{\sqrt[3]{\ell_3-\sqrt{\ell_4+\ell_5}}}{4029\sqrt[3]{2}P^3} + \frac{1008\,\ell_2}{1343P^3\sqrt[3]{\ell_3-\sqrt{\ell_4+\ell_5}}}\\
\mathcal{H}_2 &= \ell_4+\ell_5\\
\mathcal{H}_3 &= \frac{640\zeta_3^3Q(40431015\mathrm{P\,R}-5441984Q^2)}{2422300607P^3}\\
\mathcal{H}_4 &= \zeta_3^4P^2(-4611P^2R^2+1104PQ^2R+16Q^4)\\
\mathcal{H}_5 &= -93312\zeta_3^6P^3(1141749P^3R^3-1128194P^2Q^2R^2+294848PQ^4R-21312Q^6)\\
\mathcal{H}_6 &= \frac{56\zeta_3^2(36261\mathrm{P\,R}+9508Q^2)}{1803649P^2}\\
\mathcal{H}_7 &= \ell_6-61a_6\zeta_3^3\kappa^6+16a_5\zeta_3^3\kappa^5+2a_3\zeta_3^3\kappa^3+a_1\zeta_3^3\kappa\\
\mathcal{H}_8 &= 188320\zeta_3^3Q(40431015\mathrm{P\,R}-5441984Q^2)\\
\mathcal{H}_9 &= 2686P(111469\kappa^2+111965Q)\\
\mathcal{H}_{10} &= 3\zeta_3(111469\kappa^2+111965Q)\\
\mathcal{H}_{11} &= 5\zeta_3(1343(6715\kappa^4+42588\mathrm{P\,R})+99905925Q^2+23408490\kappa^2Q)\\
\mathcal{H}_{12} &= 15\kappa^2+\frac{43575Q}{1343}.
\end{aligned}
\tag{15}
$$

Also here

$$
\begin{aligned}
\ell_1 &= \frac{28\zeta_3^2(36261\mathrm{P\,R}+9508Q^2)}{1803649P^2}\\
\ell_2 &= \sqrt[3]{2}\zeta_3^4P^2(-4611P^2R^2+1104PQ^2R+16Q^4)\\
\ell_3 &= -93312\zeta_3^6P^3(1141749P^3R^3-1128194P^2Q^2R^2+294848PQ^4R-21312Q^6)\\
\ell_4 &= -206391214080\zeta_3^{12}P^6Q^6(30579312213P^3R^3-5666847084P^2Q^2R^2+530650080PQ^4R-19159424Q^6)\\
\ell_5 &= 174142586880\zeta_3^{12}P^{10}R^4(127450522724P^2R^2-173539586829PQ^2R+107366239041Q^4)\\
\ell_6 &= -\frac{5a_6\zeta_3^3(20145Q(2905\kappa^4+40356\mathrm{P\,R})+171587052\kappa^2\mathrm{P\,R}-1142904209Q^3+299717775\kappa^2Q^2)}{1803649}.
\end{aligned}
\tag{16}
$$

Thus, Eq. (1) has the following formal solution:

$$
q(x,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})F(s)+\zeta_3F^3(s)\right\}\exp[i(-\kappa x+\omega t+\theta)].
\tag{17}
$$

3.1. Jacobi elliptic function solutions

By employing the solutions of (11), one can derive Jacobi elliptic function solutions to the model in the forms:
Case 1: $P=m^2,\quad Q=-(1+m^2),\quad R=1,\quad F(s)=\mathrm{sn}\,s,$

$$q(x,\ t)= \left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\,\mathrm{sn}\,[x-\mathrm{v\,t}]+\zeta_3\,\mathrm{sn}^3[x-\mathrm{v\,t}]\right\} \times e^{i\left\{-\kappa x+\left(\frac{P^{2}(\mathcal{L}_{4}+a_{6}(\mathcal{L}_{3}-320\mathcal{L}_{6}+\mathcal{L}_{0}(\mathcal{L}_{2}+1343(\mathcal{L}_{5}+\sqrt{\mathcal{L}_{1}}\mathcal{L}_{9}))-\sqrt{\mathcal{L}_{1}}\mathcal{L}_{12}(\mathcal{L}_{7}+\mathcal{L}_{8}+\mathcal{L}_{1}\mathcal{L}_{10})))}{1343\zeta_{3}^{3}\sqrt{\mathcal{H}_{1}}\mathcal{L}_{10}}\right)t+\theta\right\}}. \tag{18}$$

Case 2: $P=-m^2,\quad Q=2m^2-1,\quad R=1-m^2,\quad F(s)=\mathrm{cn}\,s,$

$$q(x,\ t)= \left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\,\mathrm{cn}\,[x-\mathrm{v\,t}]+\zeta_3\,\mathrm{cn}^3[x-\mathrm{v\,t}]\right\} \times e^{i\left\{-\kappa x+\left(\frac{P^{2}(\mathcal{L}_{4}+a_{6}(\mathcal{L}_{3}-320\mathcal{L}_{6}+\mathcal{L}_{0}(\mathcal{L}_{2}+1343(\mathcal{L}_{5}+\sqrt{\mathcal{L}_{1}}\mathcal{L}_{9}))-\sqrt{\mathcal{L}_{1}}\mathcal{L}_{12}(\mathcal{L}_{7}+\mathcal{L}_{8}+\mathcal{L}_{1}\mathcal{L}_{10})))}{1343\zeta_{3}^{3}\sqrt{\mathcal{H}_{1}}\mathcal{L}_{10}}\right)t+\theta\right\}}. \tag{19}$$

Case 3: $P=1,\quad Q=-(1+m^2),\quad R=m^2,\quad F(s)=\mathrm{ns}\,s,$

$$q(x,\ t)= \left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\,\mathrm{ns}\,[x-\mathrm{v\,t}]+\zeta_3\,\mathrm{ns}^3[x-\mathrm{v\,t}]\right\} \times e^{i\left\{-\kappa x+\left(\frac{P^{2}(\mathcal{L}_{4}+a_{6}(\mathcal{L}_{3}-320\mathcal{L}_{6}+\mathcal{L}_{0}(\mathcal{L}_{2}+1343(\mathcal{L}_{5}+\sqrt{\mathcal{L}_{1}}\mathcal{L}_{9}))-\sqrt{\mathcal{L}_{1}}\mathcal{L}_{12}(\mathcal{L}_{7}+\mathcal{L}_{8}+\mathcal{L}_{1}\mathcal{L}_{10})))}{1343\zeta_{3}^{3}\sqrt{\mathcal{H}_{1}}\mathcal{L}_{10}}\right)t+\theta\right\}}. \tag{20}$$

Case 4: $P=1,\quad Q=-(1+m^2),\quad R=m^2,\quad F(s)=\mathrm{dc}\,s,$

$$q(x,\ t)= \left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\,\mathrm{dc}\,[x-\mathrm{v\,t}]+\zeta_3\,\mathrm{dc}^3[x-\mathrm{v\,t}]\right\} \times e^{i\left\{-\kappa x+\left(\frac{P^{2}(\mathcal{L}_{4}+a_{6}(\mathcal{L}_{3}-320\mathcal{L}_{6}+\mathcal{L}_{0}(\mathcal{L}_{2}+1343(\mathcal{L}_{5}+\sqrt{\mathcal{L}_{1}}\mathcal{L}_{9}))-\sqrt{\mathcal{L}_{1}}\mathcal{L}_{12}(\mathcal{L}_{7}+\mathcal{L}_{8}+\mathcal{L}_{1}\mathcal{L}_{10})))}{1343\zeta_{3}^{3}\sqrt{\mathcal{H}_{1}}\mathcal{L}_{10}}\right)t+\theta\right\}}. \tag{21}$$

Case 5: $P=1-m^2,\quad Q=2-m^2,\quad R=1,\quad F(s)=\mathrm{sc}\ s,$

$$q(x,\ t)= \left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\mathrm{sc}\,[x-\mathrm{v\,t}]+\zeta_3\mathrm{sc}^3[x-\mathrm{v\,t}]\right\} \times e^{i\left\{-\kappa x+\left(\frac{P^{2}(\mathcal{L}_{4}+a_{6}(\mathcal{L}_{3}-320\mathcal{L}_{6}+\mathcal{L}_{0}(\mathcal{L}_{2}+1343(\mathcal{L}_{5}+\sqrt{\mathcal{L}_{1}}\mathcal{L}_{9}))-\sqrt{\mathcal{L}_{1}}\mathcal{L}_{12}(\mathcal{L}_{7}+\mathcal{L}_{8}+\mathcal{L}_{1}\mathcal{L}_{10})))}{1343\zeta_{3}^{3}\sqrt{\mathcal{H}_{1}}\mathcal{L}_{10}}\right)t+\theta\right\}}. \tag{22}$$

Case 6: $P=1,\quad Q=2-m^2,\quad R=1-m^2,\quad F(s)=\mathrm{cs}\,s,$

$$q(x,\ t)= \left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\,\mathrm{cs}\,[x-\mathrm{v\,t}]+\zeta_3\,\mathrm{cs}^3[x-\mathrm{v\,t}]\right\} \times e^{i\left\{-\kappa x+\left(\frac{P^{2}(\mathcal{L}_{4}+a_{6}(\mathcal{L}_{3}-320\mathcal{L}_{6}+\mathcal{L}_{0}(\mathcal{L}_{2}+1343(\mathcal{L}_{5}+\sqrt{\mathcal{L}_{1}}\mathcal{L}_{9}))-\sqrt{\mathcal{L}_{1}}\mathcal{L}_{12}(\mathcal{L}_{7}+\mathcal{L}_{8}+\mathcal{L}_{1}\mathcal{L}_{10})))}{1343\zeta_{3}^{3}\sqrt{\mathcal{H}_{1}}\mathcal{L}_{10}}\right)t+\theta\right\}}. \tag{23}$$

Case 7: $P=1/4,\quad Q=(1-2m^2)/2,\quad R=1/4,\quad F(s)=\mathrm{ns}\,s\ \pm\ \mathrm{cs}\,s,$

$$q(x,\ t)= \left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})(\,\mathrm{ns}\,[x-\mathrm{v\,t}]\pm\ \mathrm{cs}\,[x-\mathrm{v\,t}])+\zeta_3(\,\mathrm{ns}\,[x-\mathrm{v\,t}]\pm\ \mathrm{cs}\,[x-\mathrm{v\,t}])^3\right\} \times e^{i\left\{-\kappa x+\left(\frac{P^{2}(\mathcal{L}_{4}+a_{6}(\mathcal{L}_{3}-320\mathcal{L}_{6}+\mathcal{L}_{0}(\mathcal{L}_{2}+1343(\mathcal{L}_{5}+\sqrt{\mathcal{L}_{1}}\mathcal{L}_{9}))-\sqrt{\mathcal{L}_{1}}\mathcal{L}_{12}(\mathcal{L}_{7}+\mathcal{L}_{8}+\mathcal{L}_{1}\mathcal{L}_{10})))}{1343\zeta_{3}^{3}\sqrt{\mathcal{H}_{1}}\mathcal{L}_{10}}\right)t+\theta\right\}}. \tag{24}$$

Case 8: $P=(1-m^2)/4,\quad Q=(1+m^2)/2,\quad R=(1-m^2)/4,\quad F(s)=\mathrm{nc}\,s\ \pm\ \mathrm{sc}\ s,$

$$q(x,\ t)= \left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})(\,\mathrm{nc}\,[x-\mathrm{v\,t}]\pm\mathrm{sc}[x-\mathrm{v\,t}])+\zeta_3(\,\mathrm{nc}\,[x-\mathrm{v\,t}]\pm\mathrm{sc}[x-\mathrm{v\,t}])^3\right\} \times e^{i\left\{-\kappa x+\left(\frac{P^{2}(\mathcal{L}_{4}+a_{6}(\mathcal{L}_{3}-320\mathcal{L}_{6}+\mathcal{L}_{0}(\mathcal{L}_{2}+1343(\mathcal{L}_{5}+\sqrt{\mathcal{L}_{1}}\mathcal{L}_{9}))-\sqrt{\mathcal{L}_{1}}\mathcal{L}_{12}(\mathcal{L}_{7}+\mathcal{L}_{8}+\mathcal{L}_{1}\mathcal{L}_{10})))}{1343\zeta_{3}^{3}\sqrt{\mathcal{H}_{1}}\mathcal{L}_{10}}\right)t+\theta\right\}}. \tag{25}$$

Case 9: $P=m^2/4,\quad Q=(m^2-2)/2,\quad R=m^2/4,\quad F(s)=\mathrm{sn}\,s\ \pm\ i\,\mathrm{cn}\,s,$

$$q(x,\ t)= \left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})(\,\mathrm{sn}\,[x-\mathrm{v\,t}]\pm i\,\mathrm{cn}\,[x-\mathrm{v\,t}])+\zeta_3(\,\mathrm{sn}\,[x-\mathrm{v\,t}]\pm i\,\mathrm{cn}\,[x-\mathrm{v\,t}])^3\right\} \times e^{i\left\{-\kappa x+\left(\frac{P^{2}(\mathcal{L}_{4}+a_{6}(\mathcal{L}_{3}-320\mathcal{L}_{6}+\mathcal{L}_{0}(\mathcal{L}_{2}+1343(\mathcal{L}_{5}+\sqrt{\mathcal{L}_{1}}\mathcal{L}_{9}))-\sqrt{\mathcal{L}_{1}}\mathcal{L}_{12}(\mathcal{L}_{7}+\mathcal{L}_{8}+\mathcal{L}_{1}\mathcal{L}_{10})))}{1343\zeta_{3}^{3}\sqrt{\mathcal{H}_{1}}\mathcal{L}_{10}}\right)t+\theta\right\}}. \tag{26}$$

Case 10: $P=m^2/4,\quad Q=(m^2-2)/2,\quad R=1/4,\quad F(s)=\frac{\mathrm{sn}\,s}{1\pm\,\mathrm{dn}\,s},$

$$q(x,\ t)= \left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\left(\frac{\mathrm{sn}\,[x-\mathrm{v\,t}]}{1\pm\,\mathrm{dn}\,[x-\mathrm{v\,t}]}\right)+\zeta_3\left(\frac{\mathrm{sn}\,[x-\mathrm{v\,t}]}{1\pm\,\mathrm{dn}\,[x-\mathrm{v\,t}]}\right)^3\right\} \times e^{i\left\{-\kappa x+\left(\frac{P^{2}(\mathcal{L}_{4}+a_{6}(\mathcal{L}_{3}-320\mathcal{L}_{6}+\mathcal{L}_{0}(\mathcal{L}_{2}+1343(\mathcal{L}_{5}+\sqrt{\mathcal{L}_{1}}\mathcal{L}_{9}))-\sqrt{\mathcal{L}_{1}}\mathcal{L}_{12}(\mathcal{L}_{7}+\mathcal{L}_{8}+\mathcal{L}_{1}\mathcal{L}_{10})))}{1343\zeta_{3}^{3}\sqrt{\mathcal{H}_{1}}\mathcal{L}_{10}}\right)t+\theta\right\}}. \tag{27}$$

Case 11: $P=-1/4,\quad Q=(m^2+1)/2,\quad R=(1-m^2)^2/4,\quad F(s)=m\,\mathrm{cn}\,s\ \pm\ \mathrm{dn}\,s,$

$$q(x,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})(m\,\mathrm{cn}[x-\mathrm{v}\,t]\pm\mathrm{dn}[x-\mathrm{v}\,t])+\zeta_3(m\,\mathrm{cn}[x-\mathrm{v}\,t]\pm\mathrm{dn}[x-\mathrm{v}\,t])^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}. \tag{28}$$

Case 12: $P=(1-m^2)^2/4,\quad Q=(m^2+1)/2,\quad R=1/4,\quad F(s)=\mathrm{ds}\,s\pm\mathrm{cs}\,s,$

$$q(x,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})(\mathrm{ds}[x-\mathrm{v}\,t]\pm\mathrm{cs}[x-\mathrm{v}\,t])+\zeta_3(\mathrm{ds}[x-\mathrm{v}\,t]\pm\mathrm{cs}[x-\mathrm{v}\,t])^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}. \tag{29}$$

Case 13: $P>0,\quad Q<0,\quad R=\frac{m^2Q^2}{(1+m^2)^2P},\quad F(s)=\sqrt{-\frac{m^2Q}{(1+m^2)P}}\,\mathrm{sn}\left(\sqrt{-\frac{Q}{1+m^2}}\,s\right),$

$$q(x,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\left(\sqrt{-\frac{m^2Q}{(1+m^2)P}}\,\mathrm{sn}\left(\sqrt{-\frac{Q}{1+m^2}}[x-\mathrm{v}\,t]\right)\right)+\zeta_3\left(\sqrt{-\frac{m^2Q}{(1+m^2)P}}\,\mathrm{sn}\left(\sqrt{-\frac{Q}{1+m^2}}[x-\mathrm{v}\,t]\right)\right)^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}. \tag{30}$$

Case 14: $P<0,\quad Q>0,\quad R=\frac{(1-m^2)Q^2}{(m^2-2)^2P},\quad F(s)=\sqrt{-\frac{Q}{(2-m^2)P}}\,\mathrm{dn}\left(\sqrt{\frac{Q}{2-m^2}}\,s\right),$

$$q(x,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\left(\sqrt{-\frac{Q}{(2-m^2)P}}\,\mathrm{dn}\left(\sqrt{\frac{Q}{2-m^2}}[x-\mathrm{v}\,t]\right)\right)+\zeta_3\left(\sqrt{-\frac{Q}{(2-m^2)P}}\,\mathrm{dn}\left(\sqrt{\frac{Q}{2-m^2}}[x-\mathrm{v}\,t]\right)\right)^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}. \tag{31}$$

Case 15: $P=1,\quad Q=m^2+2,\quad R=1-2m^2+m^4,\quad F(s)=\frac{\mathrm{dn}\,s\,\mathrm{cn}\,s}{\mathrm{sn}\,s},$

$$q(x,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\left(\frac{\mathrm{dn}[x-\mathrm{v}\,t]\,\mathrm{cn}[x-\mathrm{v}\,t]}{\mathrm{sn}[x-\mathrm{v}\,t]}\right)+\zeta_3\left(\frac{\mathrm{dn}[x-\mathrm{v}\,t]\,\mathrm{cn}[x-\mathrm{v}\,t]}{\mathrm{sn}[x-\mathrm{v}\,t]}\right)^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}. \tag{32}$$

Case 16: $P=\frac{A^2(m-1)^2}{4},\quad Q=\frac{m^2+1}{2}+3m,\quad R=\frac{(m-1)^2}{4A^2},\quad F(s)=\frac{\mathrm{dn}\,s\,\mathrm{cn}\,s}{A(1+\mathrm{sn}\,s)(1+m\,\mathrm{sn}\,s)},$

$$q(x,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\left(\frac{\mathrm{dn}[x-\mathrm{v}\,t]\,\mathrm{cn}[x-\mathrm{v}\,t]}{A(1+\mathrm{sn}[x-\mathrm{v}\,t])(1+m\,\mathrm{sn}[x-\mathrm{v}\,t])}\right)+\zeta_3\left(\frac{\mathrm{dn}[x-\mathrm{v}\,t]\,\mathrm{cn}[x-\mathrm{v}\,t]}{A(1+\mathrm{sn}[x-\mathrm{v}\,t])(1+m\,\mathrm{sn}[x-\mathrm{v}\,t])}\right)^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}. \tag{33}$$

Case 17: $P=-\frac{4}{m},\quad Q=6m-m^2-1,\quad R=-2m^3+m^4+m^2,\quad F(s)=\frac{m\,\mathrm{cn}\,s\,\mathrm{dn}\,s}{m\,\mathrm{sn}^2s+1},$

$$q(x,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\left(\frac{m\,\mathrm{cn}[x-\mathrm{v}\,t]\,\mathrm{dn}[x-\mathrm{v}\,t]}{m\,\mathrm{sn}^2[x-\mathrm{v}\,t]+1}\right)+\zeta_3\left(\frac{m\,\mathrm{cn}[x-\mathrm{v}\,t]\,\mathrm{dn}[x-\mathrm{v}\,t]}{m\,\mathrm{sn}^2[x-\mathrm{v}\,t]+1}\right)^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}. \tag{34}$$

Case 18: $P=1/4,\quad Q=\frac{1-2m^2}{2},\quad R=1/4,\quad F(s)=\frac{\mathrm{sn}\,s}{1\pm\mathrm{cn}\,s},$

$$q(x,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\left(\frac{\mathrm{sn}[x-\mathrm{v}\,t]}{1\pm\mathrm{cn}[x-\mathrm{v}\,t]}\right)+\zeta_3\left(\frac{\mathrm{sn}[x-\mathrm{v}\,t]}{1\pm\mathrm{cn}[x-\mathrm{v}\,t]}\right)^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}. \tag{35}$$

Case 19: $P=\frac{1-m^2}{4},\quad Q=\frac{1+m^2}{2},\quad R=\frac{1-m^2}{4},\quad F(s)=\frac{\mathrm{cn}\,s}{1\pm\mathrm{sn}\,s},$

$$q(x,t)=\left\{\frac{1}{2}\left(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1}\right)\left(\frac{\mathrm{cn}[x-\mathrm{v}\,t]}{1\pm\mathrm{sn}[x-\mathrm{v}\,t]}\right)+\zeta_3\left(\frac{\mathrm{cn}[x-\mathrm{v}\,t]}{1\pm\mathrm{sn}[x-\mathrm{v}\,t]}\right)^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}. \tag{36}$$

Case 20: $P=\frac{2-m^2-2m_1}{4},\quad Q=\frac{m^2}{2}-1-3m_1,\quad R=\frac{2-m^2-2m_1}{4},\quad F(s)=\frac{m^2\,\mathrm{sn}\,s\,\mathrm{cn}\,s}{\mathrm{sn}^2s+(1+m_1)\mathrm{dn}\,s-1-m_1},$

$$q(x,t)=\left\{\frac{1}{2}\left(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1}\right)\left(\frac{m^2\mathrm{sn}[x-\mathrm{v}\,t]\mathrm{cn}[x-\mathrm{v}\,t]}{\mathrm{sn}^2[x-\mathrm{v}\,t]+m_2\mathrm{dn}[x-\mathrm{v}\,t]-m_2}\right)+\zeta_3\left(\frac{m^2\mathrm{sn}[x-\mathrm{v}\,t]\mathrm{cn}[x-\mathrm{v}\,t]}{\mathrm{sn}^2[x-\mathrm{v}\,t]+m_2\mathrm{dn}[x-\mathrm{v}\,t]-m_2}\right)^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}, \tag{37}$$

where $m_2=1+m_1$.

Case 21: $P=\frac{2-m^2+2m_1}{4},\quad Q=\frac{m^2}{2}-1+3m_1,\quad R=\frac{2-m^2+2m_1}{4},\quad F(s)=\frac{m^2\,\mathrm{sn}\,s\,\mathrm{cn}\,s}{\mathrm{sn}^2s+(-1+m_1)\mathrm{dn}\,s-1-m_1},$

$$q(x,t)=\left\{\frac{1}{2}\left(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1}\right)\left(\frac{m^2\mathrm{sn}[x-\mathrm{v}\,t]\mathrm{cn}[x-\mathrm{v}\,t]}{\mathrm{sn}^2[x-\mathrm{v}\,t]+m_3\mathrm{dn}[x-\mathrm{v}\,t]-m_2}\right)+\zeta_3\left(\frac{m^2\mathrm{sn}[x-\mathrm{v}\,t]\mathrm{cn}[x-\mathrm{v}\,t]}{\mathrm{sn}^2[x-\mathrm{v}\,t]+m_3\mathrm{dn}[x-\mathrm{v}\,t]-m_2}\right)^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}, \tag{38}$$

where $m_3=-1+m_1$.

Case 22: $P=\frac{C^2m^4-(B^2+C^2)m^2+B^2}{4},\quad Q=\frac{m^2+1}{2},\quad R=\frac{m^2-1}{4(C^2m^2-B^2)},\quad F(s)=\frac{\sqrt{\frac{B^2-C^2}{B^2-C^2m^2}}+\mathrm{sn}\,s}{B\,\mathrm{cn}\,s+C\,\mathrm{dn}\,s},$

$$q(x,t)=\left\{\frac{1}{2}\left(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1}\right)\left(\frac{\sqrt{\frac{B^2-C^2}{B^2-C^2m^2}}+\mathrm{sn}[x-\mathrm{v}\,t]}{B\,\mathrm{cn}[x-\mathrm{v}\,t]+C\,\mathrm{dn}[x-\mathrm{v}\,t]}\right)+\zeta_3\left(\frac{\sqrt{\frac{B^2-C^2}{B^2-C^2m^2}}+\mathrm{sn}[x-\mathrm{v}\,t]}{B\,\mathrm{cn}[x-\mathrm{v}\,t]+C\,\mathrm{dn}[x-\mathrm{v}\,t]}\right)^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}. \tag{39}$$

Case 23: $P=\frac{B^2+C^2m^2}{4},\quad Q=\frac{1}{2}-m^2,\quad R=\frac{1}{4(B^2+C^2m^2)},\quad F(s)=\frac{\sqrt{\frac{C^2m^2+B^2-C^2}{B^2+C^2m^2}}+\mathrm{cn}\,s}{B\,\mathrm{sn}\,s+C\,\mathrm{dn}\,s},$

$$q(x,t)=\left\{\frac{1}{2}\left(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1}\right)\left(\frac{\sqrt{\frac{C^2m^2+B^2-C^2}{B^2+C^2m^2}}+\mathrm{cn}[x-\mathrm{v}\,t]}{B\,\mathrm{sn}[x-\mathrm{v}\,t]+C\,\mathrm{dn}[x-\mathrm{v}\,t]}\right)+\zeta_3\left(\frac{\sqrt{\frac{C^2m^2+B^2-C^2}{B^2+C^2m^2}}+\mathrm{cn}[x-\mathrm{v}\,t]}{B\,\mathrm{sn}[x-\mathrm{v}\,t]+C\,\mathrm{dn}[x-\mathrm{v}\,t]}\right)^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}. \tag{40}$$

Case 24: $P=\frac{B^2+C^2}{4},\quad Q=\frac{m^2}{2}-1,\quad R=\frac{m^4}{4(B^2+C^2)},\quad F(s)=\frac{\sqrt{\frac{B^2+C^2-C^2m^2}{B^2+C^2}}+\mathrm{dn}\,s}{B\,\mathrm{sn}\,s+C\,\mathrm{cn}\,s},$

$$q(x,t)=\left\{\frac{1}{2}\left(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1}\right)\left(\frac{\sqrt{\frac{B^2+C^2-C^2m^2}{B^2+C^2}}+\mathrm{dn}[x-\mathrm{v}\,t]}{B\,\mathrm{sn}[x-\mathrm{v}\,t]+C\,\mathrm{cn}[x-\mathrm{v}\,t]}\right)+\zeta_3\left(\frac{\sqrt{\frac{B^2+C^2-C^2m^2}{B^2+C^2}}+\mathrm{dn}[x-\mathrm{v}\,t]}{B\,\mathrm{sn}[x-\mathrm{v}\,t]+C\,\mathrm{cn}[x-\mathrm{v}\,t]}\right)^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}. \tag{41}$$

3.2. Weierstrass elliptic function solutions

Upon recapitulating, Weierstrass elliptic function with a complex variable u and a pair of complex periods ω_1, ω_2 is defined as [11]:

$$\wp(u;\omega_1,\omega_2)=\frac{1}{u^2}+\sum_{m,n}{}'\left(\frac{1}{(u-m\omega_1-n\omega_2)^2}-\frac{1}{(m\omega_1+n\omega_2)^2}\right), \tag{42}$$

where ′ implies that terms with zero denominators are omitted.

By using the solutions of (11) given in [8], one can reveal Weierstrass elliptic function solutions as listed below:

Case 25: $g_2=\frac{4}{3}(Q^2-3PR),\quad g_3=\frac{4Q}{27}(-2Q^2+9PR),\quad F(s)=\sqrt{\frac{1}{P}\left[\wp(s;g_2,g_3)-\frac{1}{3}Q\right]},$

$$q(x,t)=\left\{\tfrac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\sqrt{\tfrac{1}{P}\left[\wp(x-\mathrm{v}\,t;g_2,g_3)-\tfrac{1}{3}Q\right]}+\zeta_3\left(\sqrt{\tfrac{1}{P}\left[\wp(x-\mathrm{v}\,t;g_2,g_3)-\tfrac{1}{3}Q\right]}\right)^3\right\}$$
$$\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}. \quad (43)$$

Case 26: $g_2=\frac{4}{3}(Q^2-3\mathrm{PR}),\quad g_3=\frac{4Q}{27}(-2Q^2+9\mathrm{PR}),\quad F(s)=\sqrt{\frac{3R}{3\wp(s;g_2,g_3)-Q}}$,

$$q(x,t)=\left\{\tfrac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\sqrt{\frac{3R}{3\wp(x-\mathrm{v}\,t;g_2,g_3)-Q}}+\zeta_3\left(\sqrt{\frac{3R}{3\wp(x-\mathrm{v}\,t;g_2,g_3)-Q}}\right)^3\right\}$$
$$\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}. \quad (44)$$

Case 27: $g_2=-\frac{5\mathrm{QD}+4Q^2+33\mathrm{PQR}}{12},\quad g_3=\frac{21Q^2D-63\mathrm{PRD}+20Q^3-27\mathrm{PQR}}{216}$,
$F(s)=\frac{\sqrt{12R\wp(s;g_2,g_3)+2R(2Q+D)}}{12\wp(s;g_2,g_3)+D}$,

$$q(x,t)=\left\{\tfrac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\frac{\sqrt{12R\wp(x-\mathrm{v}\,t;g_2,g_3)+2R(2Q+D)}}{12\wp(x-\mathrm{v}\,t;g_2,g_3)+D}\right.$$
$$\left.+\zeta_3\left(\frac{\sqrt{12R\wp(x-\mathrm{v}\,t;g_2,g_3)+2R(2Q+D)}}{12\wp(x-\mathrm{v}\,t;g_2,g_3)+D}\right)^3\right\}$$
$$\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}. \quad (45)$$

Case 28: $g_2=\frac{1}{12}Q^2+\mathrm{PR},\quad g_3=\frac{1}{216}Q(36\mathrm{PR}-Q^2),\quad F(s)=\frac{\sqrt{R}\,[6\wp(s;g_2,g_3)+Q]}{3\wp'(s;g_2,g_3)}$,

$$q(x,t)=\left\{\tfrac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\frac{\sqrt{R}\,[6\wp(x-\mathrm{v}\,t;g_2,g_3)+Q]}{3\wp'(x-\mathrm{v}\,t;g_2,g_3)}+\zeta_3\left(\frac{\sqrt{R}\,[6\wp(x-\mathrm{v}\,t;g_2,g_3)+Q]}{3\wp'(x-\mathrm{v}\,t;g_2,g_3)}\right)^3\right\}$$
$$\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}. \quad (46)$$

Case 29: $g_2=\frac{1}{12}Q^2+\mathrm{PR},\quad g_3=\frac{1}{216}Q(36\mathrm{PR}-Q^2),\quad F(s)=\frac{3\wp'(s;g_2,g_3)}{\sqrt{P}\,[6\wp(s;g_2,g_3)+Q]}$,

$$q(x,t)=\left\{\tfrac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\frac{3\wp'(x-\mathrm{v}\,t;g_2,g_3)}{\sqrt{P}\,[6\wp(x-\mathrm{v}\,t;g_2,g_3)+Q]}+\zeta_3\left(\frac{3\wp'(x-\mathrm{v}\,t;g_2,g_3)}{\sqrt{P}\,[6\wp(x-\mathrm{v}\,t;g_2,g_3)+Q]}\right)^3\right\}$$
$$\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}. \quad (47)$$

Case 30: $R=\frac{5Q^2}{36P},\quad g_2=\frac{2Q^2}{9},\quad g_3=\frac{Q^3}{54},\quad F(s)=\frac{Q\sqrt{-15Q/2P}\,\wp(s;g_2,g_3)}{3\wp(s;g_2,g_3)+Q}$,

$$q(x,t)=\left\{\tfrac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\frac{Q\sqrt{-15Q/2P}\,\wp(x-\mathrm{v}\,t;g_2,g_3)}{3\wp(x-\mathrm{v}\,t;g_2,g_3)+Q}+\zeta_3\left(\frac{Q\sqrt{-15Q/2P}\,\wp(x-\mathrm{v}\,t;g_2,g_3)}{3\wp(x-\mathrm{v}\,t;g_2,g_3)+Q}\right)^3\right\}$$
$$\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}. \quad (48)$$

3.3. Soliton and other solutions

When the modulus $m\to 1$, bright, dark and singular solitons, the combined solitons, and complexiton solutions are procured as:

$$q(x,t)=\left\{\tfrac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\tanh[x-\mathrm{v}\,t]+\zeta_3\tanh^3[x-\mathrm{v}\,t]\right\}$$
$$\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}, \quad (49)$$

$$q(x,t)=\left\{\tfrac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\,\mathrm{sech}\,[x-\mathrm{v}\,t]+\zeta_3\,\mathrm{sech}^3[x-\mathrm{v}\,t]\right\}$$
$$\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}, \quad (50)$$

$$q(x,t)=\left\{\tfrac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\coth[x-\mathrm{v}\,t]+\zeta_3\coth^3[x-\mathrm{v}\,t]\right\}$$
$$\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}, \quad (51)$$

Solutions (49)–(51) are dark, bright and singular solitons respectively.

$$q(x,\,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\sinh[x-\mathrm{v}\,t]+\zeta_3\sinh^3[x-\mathrm{v}\,t]\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}},\tag{52}$$

$$q(x,\,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\,\mathrm{csch}\,[x-\mathrm{v}\,t]+\zeta_3\,\mathrm{csch}^3[x-\mathrm{v}\,t]\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}},\tag{53}$$

Solution (53) is the second form of singular soliton solution to the model.

$$q(x,\,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})(\coth[x-\mathrm{v}\,t]\pm\,\mathrm{csch}\,[x-\mathrm{v}\,t])+\zeta_3(\coth[x-\mathrm{v}\,t]\pm\,\mathrm{csch}\,[x-\mathrm{v}\,t])^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}},\tag{54}$$

$$q(x,\,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})(\cosh[x-\mathrm{v}\,t]\pm\sinh[x-\mathrm{v}\,t])+\zeta_3(\cosh[x-\mathrm{v}\,t]\pm\sinh[x-\mathrm{v}\,t])^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}},\tag{55}$$

Solution (54) and (55) also represents singular solitons.

$$q(x,\,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})(\tanh[x-\mathrm{v}\,t]\pm i\,\mathrm{sech}\,[x-\mathrm{v}\,t])+\zeta_3(\tanh[x-\mathrm{v}\,t]\pm i\,\mathrm{sech}\,[x-\mathrm{v}\,t])^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}},\tag{56}$$

Then, solution (56) represents complexiton solutions.

$$q(x,\,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\left(\frac{\tanh[x-\mathrm{v}\,t]}{1\pm\,\mathrm{sech}\,[x-\mathrm{v}\,t]}\right)+\zeta_3\left(\frac{\tanh[x-\mathrm{v}\,t]}{1\pm\,\mathrm{sech}\,[x-\mathrm{v}\,t]}\right)^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}},\tag{57}$$

$$q(x,\,t)=\{(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\,\mathrm{sech}\,[x-\mathrm{v}\,t]+8\zeta_3\,\mathrm{sech}^3[x-\mathrm{v}\,t]\}\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}},\tag{58}$$

$$q(x,\,t)=\{(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\,\mathrm{csch}\,[x-\mathrm{v}\,t]+8\zeta_3\,\mathrm{csch}^3[x-\mathrm{v}\,t]\}\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}},\tag{59}$$

Next, (58) and (59) are bright and singular solitons respectively.

$$q(x,\,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\left(\sqrt{-\frac{Q}{2P}}\tanh\left(\sqrt{-\frac{Q}{2}}[x-\mathrm{v}\,t]\right)\right)+\zeta_3\left(\sqrt{-\frac{Q}{2P}}\tanh\left(\sqrt{-\frac{Q}{2}}[x-\mathrm{v}\,t]\right)\right)^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}},\tag{60}$$

$$q(x,\,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\left(\sqrt{-\frac{Q}{P}}\,\mathrm{sech}\,(\sqrt{Q}[x-\mathrm{v}\,t])\right)+\zeta_3\left(\sqrt{-\frac{Q}{P}}\,\mathrm{sech}\,(\sqrt{Q}[x-\mathrm{v}\,t])\right)^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}},\tag{61}$$

Again (60) and (61) are dark and bright solitons respectively.

$$q(x,\,t)=\{(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\,\mathrm{csch}\,2[x-\mathrm{v}\,t]+8\zeta_3\,\mathrm{csch}^3 2[x-\mathrm{v}\,t]\}\times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}},\tag{62}$$

Eq. (62) represents singular solitons.

$$q(x,\,t)=\left\{\frac{1}{2A}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\exp(-2[x-\mathrm{v}\,\mathrm{t}])+\frac{\zeta_3}{A^3}\exp(-6[x-\mathrm{v}\,\mathrm{t}])\right\} \times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}, \tag{63}$$

$$q(x,\,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\,\mathrm{sech}\,2[x-\mathrm{v}\,\mathrm{t}]+\zeta_3\,\mathrm{sech}^3 2[x-\mathrm{v}\,\mathrm{t}]\right\} \times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}, \tag{64}$$

Eq. (64) is another form of bright solitons.

$$q(x,\,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\left(\frac{\tanh[x-\mathrm{v}\,\mathrm{t}]}{1\pm\mathrm{sech}\,[x-\mathrm{v}\,\mathrm{t}]}\right)+\zeta_3\left(\frac{\tanh[x-\mathrm{v}\,\mathrm{t}]}{1\pm\mathrm{sech}\,[x-\mathrm{v}\,\mathrm{t}]}\right)^3\right\} \times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}, \tag{65}$$

$$q(x,\,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\left(\frac{\mathrm{sech}\,[x-\mathrm{v}\,\mathrm{t}]}{1\pm\tanh[x-\mathrm{v}\,\mathrm{t}]}\right)+\zeta_3\left(\frac{\mathrm{sech}\,[x-\mathrm{v}\,\mathrm{t}]}{1\pm\tanh[x-\mathrm{v}\,\mathrm{t}]}\right)^3\right\} \times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}, \tag{66}$$

$$q(x,\,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\coth\left[\frac{x-\mathrm{v}\,\mathrm{t}}{2}\right]+\zeta_3\coth^3\left[\frac{x-\mathrm{v}\,\mathrm{t}}{2}\right]\right\} \times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}, \tag{67}$$

$$q(x,\,t)=\left\{-\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\tanh\left[\frac{x-\mathrm{v}\,\mathrm{t}}{2}\right]-\zeta_3\tanh^3\left[\frac{x-\mathrm{v}\,\mathrm{t}}{2}\right]\right\} \times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}, \tag{68}$$

Finally, (67) and (68) respectively represent singular and dark solitons to the model.

$$q(x,\,t)=\left\{\frac{1}{2(B+C)}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\exp[x-\mathrm{v}\,\mathrm{t}]+\frac{\zeta_3}{(B+C)^3}\exp 3[x-\mathrm{v}\,\mathrm{t}]\right\} \times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}, \tag{69}$$

$$q(x,\,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\left(\frac{\sqrt{\frac{B^2}{B^2+C^2}}+\mathrm{sech}\,[x-\mathrm{v}\,\mathrm{t}]}{B\tanh[x-\mathrm{v}\,\mathrm{t}]+C\,\mathrm{sech}\,[x-\mathrm{v}\,\mathrm{t}]}\right)+\zeta_3\left(\frac{\sqrt{\frac{B^2}{B^2+C^2}}+\mathrm{sech}\,[x-\mathrm{v}\,\mathrm{t}]}{B\tanh[x-\mathrm{v}\,\mathrm{t}]+C\,\mathrm{sech}\,[x-\mathrm{v}\,\mathrm{t}]}\right)^3\right\} \times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}. \tag{70}$$

3.4. Trigonometric function solutions

However, if $m\to 0$, periodic waves, periodic singular waves and a combination of such solutions appear as follows:

$$q(x,\,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\sin[x-\mathrm{v}\,\mathrm{t}]+\zeta_3\sin^3[x-\mathrm{v}\,\mathrm{t}]\right\} \times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}, \tag{71}$$

$$q(x,\,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\cos[x-\mathrm{v}\,\mathrm{t}]+\zeta_3\cos^3[x-\mathrm{v}\,\mathrm{t}]\right\} \times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}, \tag{72}$$

$$q(x,\,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\csc[x-\mathrm{v}\,\mathrm{t}]+\zeta_3\csc^3[x-\mathrm{v}\,\mathrm{t}]\right\} \times e^{i\left\{-\kappa x+\left(\frac{P^2(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}, \tag{73}$$

$$q(x,\,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\sec[x-\mathrm{v}\,\mathrm{t}]+\zeta_3\sec^3[x-\mathrm{v}\,\mathrm{t}]\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^{2}(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}},\tag{74}$$

$$q(x,\,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\tan[x-\mathrm{v}\,\mathrm{t}]+\zeta_3\tan^3[x-\mathrm{v}\,\mathrm{t}]\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^{2}(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}},\tag{75}$$

$$q(x,\,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\cot[x-\mathrm{v}\,\mathrm{t}]+\zeta_3\cot^3[x-\mathrm{v}\,\mathrm{t}]\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^{2}(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}},\tag{76}$$

$$q(x,\,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})(\csc[x-\mathrm{v}\,\mathrm{t}]\pm\cot[x-\mathrm{v}\,\mathrm{t}])+\zeta_3(\csc[x-\mathrm{v}\,\mathrm{t}]\pm\cot[x-\mathrm{v}\,\mathrm{t}])^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^{2}(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}},\tag{77}$$

$$q(x,\,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})(\sec[x-\mathrm{v}\,\mathrm{t}]\pm\tan[x-\mathrm{v}\,\mathrm{t}])+\zeta_3(\sec[x-\mathrm{v}\,\mathrm{t}]\pm\tan[x-\mathrm{v}\,\mathrm{t}])^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^{2}(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}},\tag{78}$$

$$q(x,\,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})(\sin[x-\mathrm{v}\,\mathrm{t}]\pm i\cos[x-\mathrm{v}\,\mathrm{t}])+\zeta_3(\sin[x-\mathrm{v}\,\mathrm{t}]\pm i\cos[x-\mathrm{v}\,\mathrm{t}])^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^{2}(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}},\tag{79}$$

$$q(x,\,t)=\left\{\frac{1}{4}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\sin[x-\mathrm{v}\,\mathrm{t}]+\frac{\zeta_3}{8}\sin^3[x-\mathrm{v}\,\mathrm{t}]\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^{2}(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}},\tag{80}$$

$$q(x,\,t)=\left\{\frac{1}{2A}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})(\sec[x-\mathrm{v}\,\mathrm{t}]-\tan[x-\mathrm{v}\,\mathrm{t}])+\frac{\zeta_3}{A^3}(\sec[x-\mathrm{v}\,\mathrm{t}]-\tan[x-\mathrm{v}\,\mathrm{t}])^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^{2}(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}},\tag{81}$$

$$q(x,\,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\left(\frac{\sin[x-\mathrm{v}\,\mathrm{t}]+\sqrt{\frac{B^2-C^2}{B^2}}}{B\cos[x-\mathrm{v}\,\mathrm{t}]+C}\right)+\zeta_3\left(\frac{\sin[x-\mathrm{v}\,\mathrm{t}]+\sqrt{\frac{B^2-C^2}{B^2}}}{B\cos[x-\mathrm{v}\,\mathrm{t}]+C}\right)^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^{2}(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}},\tag{82}$$

$$q(x,\,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\left(\frac{\cos[x-\mathrm{v}\,\mathrm{t}]+\sqrt{\frac{B^2-C^2}{B^2}}}{B\sin[x-\mathrm{v}\,\mathrm{t}]+C}\right)+\zeta_3\left(\frac{\cos[x-\mathrm{v}\,\mathrm{t}]+\sqrt{\frac{B^2-C^2}{B^2}}}{B\sin[x-\mathrm{v}\,\mathrm{t}]+C}\right)^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^{2}(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}},\tag{83}$$

$$q(x,\,t)=\left\{\frac{1}{2}(2\mathcal{H}_0+\sqrt{\mathcal{H}_1}+\sqrt{\mathcal{L}_1})\left(\frac{2}{B\sin[x-\mathrm{v}\,\mathrm{t}]+C\cos[x-\mathrm{v}\,\mathrm{t}]}\right)+\zeta_3\left(\frac{2}{B\sin[x-\mathrm{v}\,\mathrm{t}]+C\cos[x-\mathrm{v}\,\mathrm{t}]}\right)^3\right\}\times e^{i\left\{-\kappa x+\left(\frac{P^{2}(\mathcal{L}_4+a_6(\mathcal{L}_3-320\mathcal{L}_6+\mathcal{L}_0(\mathcal{L}_2+1343(\mathcal{L}_5+\sqrt{\mathcal{L}_1}\mathcal{L}_9))-\sqrt{\mathcal{L}_1}\mathcal{L}_{12}(\mathcal{L}_7+\mathcal{L}_8+\mathcal{L}_1\mathcal{L}_{10})))}{1343\zeta_3^3\sqrt{\mathcal{H}_1}\mathcal{L}_{10}}\right)t+\theta\right\}}.\tag{84}$$

4. Conclusions

This paper studied highly dispersive optical solitons with Kerr law nonlinearity by *F*-expansion scheme. Bright, dark, singular as well as their combo soliton solutions and complexitons are retrieved from the algorithm. Additional solutions are also retrieved from the scheme that are listed to present a wide spectrum of solutions that fall out of the model. This profound success with the model paves way for further future research activities. In future, this model will be extended to the case of birefringent fibers [10] and moving further along this model will be proposed with DWDM topology. Additional work is still pending for such highly dispersive NLSE. One must study soliton perturbation theory, quasi-stationary solitons, stochastic perturbation as well as Lie symmetry analysis and other such issues must be addressed. Such results will be surely and sequentially reported.

Conflict of interest

The authors also declare that there is no conflict of interest.

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