Optik - International Journal for Light and Electron Optics 178 (2019) 636–644

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.com/locate/ijleo

Original research article

Chirped and chirp-free optical solitons with generalized anti-cubic nonlinearity by extended trial function scheme

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ARTICLE INFO

OCIS:
060.2310
060.4510
060.5530
190.3270
190.4370

Keywords:
Solitons
Generalized anti-cubic law
Extended trial function scheme

ABSTRACT

This paper secures optical soliton solutions with generalized anti-cubic nonlinearity by extended trial function scheme. Both chirped and chirp-free optical soliton solutions are recovered. Bright and singular soliton solutions are obtained from the scheme.

1. Introduction

Optical solitons, with a wide range of nonlinear forms of fiber, have been around for quite a long time [1–10]. In particular, anti-cubic (AC) nonlinearity was conceived during 2003 [8]. Several studies have been conducted with AC nonlinearity. These include conservation laws, resonant solitons, soliton perturbation, magneto-optic waveguides and many others. It is now time to turn the page and move on to a new chapter. This paper thus considers a generalized version of AC nonlinearity. The extended trial function scheme is applied to retrieve soliton solutions to the governing nonlinear Schrödinger's equation (NLSE). Both chirped as well as chirp-free soliton solutions are considered. Bright and singular soliton solutions are revealed with this scheme. The model is first introduced in the paper as a generalized form of AC nonlinearity. Subsequently, the soliton solution derivation details are carried out. These are explored in the rest of the paper.

2. Governing model

The NLSE with AC nonlinearity and spatio-temporal dispersion (STD) is: [2–4,8–10]

$$iq_t + aq_{xx} + bq_{xt} + (c_1\,|q|^{-4} + c_2\,|q|^2 + c_3\,|q|^4)q = 0, \tag{1}$$

where a, b, c_1, c_2 and c_3 are real-valued constants. The independent variables are x and t that represents spatial and temporal co-ordinates. Again the dependent variable is $q(x, t)$ that is a complex-valued function. In (1), if $c_1 = 0$ it collapses to NLSE with parabolic law or cubic-quintic law of nonlinearity that has been extensively studied. It is this c_1 that introduces the anti-cubic

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https://doi.org/10.1016/j.ijleo.2018.09.045
Received 8 July 2018; Accepted 13 September 2018

nonlinear term.

Now, let us introduce the model with generalized AC nonlinearity as:

$$iq_t + \mathrm{a}q_{xx} + \mathrm{b}q_{xt} + \{c_1\,|q|^{-(2n+2)} + c_2\,|q|^{2n} + c_3\,|q|^{2n+2}\}q = 0. \tag{2}$$

This study will now embark into the retrieval of chirp-free solitons and chirped solitons using extended trial function scheme (ETFS) [5–7] in the subsequent two subsections.

2.1. Chirp-free solitons

To secure chirp-free solitons, the starting hypothesis is

$$q(x,\,t) = g(s)e^{i\phi(x,t)}, \tag{3}$$

where $g(s)$ represents the shape of the pulse,

$$s = x - \mathrm{v}\,\mathrm{t}, \tag{4}$$

and

$$\phi = -\kappa x + \omega t + \theta. \tag{5}$$

The function $\phi(x, t)$ is the phase component of the soliton, κ is the soliton frequency, while ω is the wave number, θ is the phase constant and v is the velocity of the soliton. After inserting (3) into (2), and separating into real and imaginary parts respectively lead to

$$-a\kappa^2 g^{2n+2} - \omega g^{2n+2} + b\kappa\omega g^{2n+2} + c_1 + c_2 g^{4n+2} + c_3 g^{4n+4} + a g^{2n+1}g'' - \mathrm{b}\,\mathrm{v}g^{2n+1}g'' = 0, \tag{6}$$

and

$$(b\omega - 2a\kappa + v(b\kappa - 1))g^{2n+1}g' = 0. \tag{7}$$

From (7), the velocity of the soliton falls out to be

$$v = \frac{b\omega - 2a\kappa}{1 - b\kappa}. \tag{8}$$

Next, in order to recover closed form solutions, one employs the transformation given by

$$g(x,\,t) = \varphi^{\frac{1}{n+1}}(x,\,t), \tag{9}$$

that will carry out Eq. (6) to

$$c_1(n+1)^2 - (n+1)^2(a\kappa^2 + \omega - b\kappa\omega)\varphi^2 + c_3(n+1)^2\varphi^4 + c_2(n+1)^2\varphi^{\frac{4n+2}{n+1}} n(\mathrm{b}\,\mathrm{v} - a)(\varphi')^2 + (n+1)(a - \mathrm{b}\,\mathrm{v})\varphi\varphi'' = 0. \tag{10}$$

For integrability, one must select $c_2 = 0$. This leads to the modification of the model of study as:

$$iq_t + \mathrm{a}q_{xx} + \mathrm{b}q_{xt} + \{c_1\,|q|^{-(2n+2)} + c_3\,|q|^{2n+2}\}q = 0. \tag{11}$$

Consequently, Eq. (10) changes to

$$c_1(n+1)^2 - (n+1)^2(a\kappa^2 + \omega - b\kappa\omega)\varphi^2 + c_3(n+1)^2\varphi^4 + n(\mathrm{b}\,\mathrm{v} - a)(\varphi')^2 + (n+1)(a - \mathrm{b}\,\mathrm{v})\varphi\varphi'' = 0. \tag{12}$$

2.1.1. Extended trial function scheme

To start off, the start-up assumption for the solution structure to (12) is given by

$$\varphi = \sum_{j=0}^{\varsigma} \gamma_j \Phi^j, \tag{13}$$

where

$$(\Phi')^2 = \Theta(\Phi) = \frac{\Gamma(\Phi)}{\Upsilon(\Phi)} = \frac{\mu_\sigma\Phi^\sigma + \cdots + \mu_1\Phi + \mu_0}{\chi_\rho\Phi^\rho + \cdots + \chi_1\Phi + \chi_0}. \tag{14}$$

Here $\gamma_0, \ldots, \gamma_\varsigma$; $\mu_0, \ldots, \mu_\sigma$ and $\chi_0, \ldots, \chi_\rho$ are unknown coefficients that will be fixed later such that γ_ς, μ_σ and χ_ρ are non-zero constants. Eq. (14) can be reformulated with an integral form as

$$\pm(s - s_0) = \int \frac{d\Phi}{\sqrt{\Theta(\Phi)}} = \int \sqrt{\frac{\Upsilon(\Phi)}{\Gamma(\Phi)}}\,d\Phi. \tag{15}$$

Balancing $(\varphi')^2$ or $\varphi\varphi''$ with φ^4 in Eq. (12) leads to

$$\varrho = \rho + 2\varsigma + 2. \tag{16}$$

When $\varrho = 4$, $\rho = 0$ and $\varsigma = 1$ in Eq. (16), ETFS admits the use of the finite expansion

$$\varphi = \gamma_0 + \gamma_1 \Phi. \tag{17}$$

Substituting (17) into (12), collecting the coefficients of Φ, and solving the resulting system one has

$$\begin{aligned}
&\mu_0 = \mu_0, \quad \mu_1 = \mu_1,\\
&\gamma_0 = \gamma_0, \quad \gamma_1 = \gamma_1,\\
&\chi_0 = \frac{n(n+2)(a - \mathrm{b}\,\mathrm{v})\gamma_1(2\mu_0\gamma_1 - \mu_1\gamma_0)}{2(n+1)^2(c_1(n+2) + c_3 n\gamma_0^4)},\\
&\mu_2 = \frac{\gamma_1(c_1(n+2)\mu_1 + c_3 n\gamma_0^3(5\mu_1\gamma_0 - 8\mu_0\gamma_1))}{2c_3 n\gamma_0^5 + 2c_1(n+2)\gamma_0},\\
&\mu_3 = \frac{2c_3 n\gamma_0\gamma_1^2(\mu_1\gamma_0 - 2\mu_0\gamma_1)}{c_3 n\gamma_0^4 + c_1(n+2)},\\
&\mu_4 = \frac{c_3 n\gamma_1^3(\mu_1\gamma_0 - 2\mu_0\gamma_1)}{2c_3 n\gamma_0^4 + 2c_1(n+2)},\\
&\omega = \frac{\mu_1(c_1(n+2) + \mathrm{a}\,\mathrm{n}(n+2)\kappa^2\gamma_0^2 - c_3 n\gamma_0^4) - 2n\mu_0\gamma_0\gamma_1(a(n+2)\kappa^2 - 2c_3\gamma_0^2)}{n(n+2)(b\kappa - 1)\gamma_0(\mu_1\gamma_0 - 2\mu_0\gamma_1)}.
\end{aligned} \tag{18}$$

Substituting this set into Eqs. (14) and (15) causes

$$\pm (s - s_0) = \Omega_1 \int \frac{d\Phi}{\sqrt{\Theta(\Phi)}}, \tag{19}$$

where

$$\Omega_1 = \sqrt{\frac{\chi_0}{\mu_4}}, \tag{20}$$

$$\Theta(\Phi) = \Phi^4 + \frac{\mu_3}{\mu_4}\Phi^3 + \frac{\mu_2}{\mu_4}\Phi^2 + \frac{\mu_1}{\mu_4}\Phi + \frac{\mu_0}{\mu_4}. \tag{21}$$

As a consequence, traveling wave solutions to the model are retrieved as follows:

For $\Theta(\Phi) = (\Phi - \zeta_1)^4$,

$$\begin{aligned}
q(x,t) = &\left\{\gamma_0 + \gamma_1\zeta_1 \pm \frac{\gamma_1\Omega_1}{x - \left\{\frac{b\omega - 2a\kappa}{1 - b\kappa}\right\}t - s_0}\right\}^{\frac{1}{n+1}}\\
&\times \exp\left[i\left\{-\kappa x + \left(\frac{\mu_1(c_1(n+2) + \mathrm{a}\,\mathrm{n}(n+2)\kappa^2\gamma_0^2 - c_3 n\gamma_0^4) - 2n\mu_0\gamma_0\gamma_1(a(n+2)\kappa^2 - 2c_3\gamma_0^2)}{n(n+2)(b\kappa - 1)\gamma_0(\mu_1\gamma_0 - 2\mu_0\gamma_1)}\right)t + \theta\right\}\right].
\end{aligned} \tag{22}$$

If $\Theta(\Phi) = (\Phi - \zeta_1)^3(\Phi - \zeta_2)$ and $\zeta_2 > \zeta_1$,

$$\begin{aligned}
q(x,t) = &\left\{\gamma_0 + \gamma_1\zeta_1 + \frac{4\gamma_1\Omega_1^2(\zeta_2 - \zeta_1)}{4\Omega_1^2 - \left[(\zeta_1 - \zeta_2)\left(x - \left\{\frac{b\omega - 2a\kappa}{1 - b\kappa}\right\}t - s_0\right)\right]^2}\right\}^{\frac{1}{n+1}}\\
&\times \exp\left[i\left\{-\kappa x + \left(\frac{\mu_1(c_1(n+2) + \mathrm{a}\,\mathrm{n}(n+2)\kappa^2\gamma_0^2 - c_3 n\gamma_0^4) - 2n\mu_0\gamma_0\gamma_1(a(n+2)\kappa^2 - 2c_3\gamma_0^2)}{n(n+2)(b\kappa - 1)\gamma_0(\mu_1\gamma_0 - 2\mu_0\gamma_1)}\right)t + \theta\right\}\right].
\end{aligned} \tag{23}$$

However, when $\Theta(\Phi) = (\Phi - \zeta_1)^2(\Phi - \zeta_2)^2$,

$$\begin{aligned}
q(x,t) = &\left\{\gamma_0 + \gamma_1\zeta_2 + \frac{\gamma_1(\zeta_2 - \zeta_1)}{\exp\left[\frac{\zeta_1 - \zeta_2}{\Omega_1}\left(x - \left\{\frac{b\omega - 2a\kappa}{1 - b\kappa}\right\}t - s_0\right)\right] - 1}\right\}^{\frac{1}{n+1}}\\
&\times \exp\left[i\left\{-\kappa x + \left(\frac{\mu_1(c_1(n+2) + \mathrm{a}\,\mathrm{n}(n+2)\kappa^2\gamma_0^2 - c_3 n\gamma_0^4) - 2n\mu_0\gamma_0\gamma_1(a(n+2)\kappa^2 - 2c_3\gamma_0^2)}{n(n+2)(b\kappa - 1)\gamma_0(\mu_1\gamma_0 - 2\mu_0\gamma_1)}\right)t + \theta\right\}\right],
\end{aligned} \tag{24}$$

and

$$\begin{aligned}
q(x,t) = &\left\{\gamma_0 + \gamma_1\zeta_1 + \frac{\gamma_1(\zeta_1 - \zeta_2)}{\exp\left[\frac{\zeta_1 - \zeta_2}{\Omega_1}\left(x - \left\{\frac{b\omega - 2a\kappa}{1 - b\kappa}\right\}t - s_0\right)\right] - 1}\right\}^{\frac{1}{n+1}}\\
&\times \exp\left[i\left\{-\kappa x + \left(\frac{\mu_1(c_1(n+2) + \mathrm{a}\,\mathrm{n}(n+2)\kappa^2\gamma_0^2 - c_3 n\gamma_0^4) - 2n\mu_0\gamma_0\gamma_1(a(n+2)\kappa^2 - 2c_3\gamma_0^2)}{n(n+2)(b\kappa - 1)\gamma_0(\mu_1\gamma_0 - 2\mu_0\gamma_1)}\right)t + \theta\right\}\right].
\end{aligned} \tag{25}$$

Whenever $\Theta(\Phi)=(\Phi-\zeta_1)^2(\Phi-\zeta_2)(\Phi-\zeta_3)$ and $\zeta_1>\zeta_2>\zeta_3$,

$$q(x,t)=\left\{\gamma_0+\gamma_1\zeta_1-\frac{2\gamma_1(\zeta_1-\zeta_2)(\zeta_1-\zeta_3)}{2\zeta_1-\zeta_2-\zeta_3+(\zeta_3-\zeta_2)\cosh\left[\frac{\sqrt{(\zeta_1-\zeta_2)(\zeta_1-\zeta_3)}}{\Omega_1}\left(x-\left\{\frac{b\omega-2a\kappa}{1-b\kappa}\right\}t\right)\right]}\right\}^{\frac{1}{n+1}}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mu_1(c_1(n+2)+a\,n(n+2)\kappa^2\gamma_0^2-c_3n\gamma_0^4)-2n\mu_0\gamma_0\gamma_1(a(n+2)\kappa^2-2c_3\gamma_0^2)}{n(n+2)(b\kappa-1)\gamma_0(\mu_1\gamma_0-2\mu_0\gamma_1)}\right)t+\theta\right\}\right]. \tag{26}$$

Finally, if $\Theta(\Phi)=(\Phi-\zeta_1)(\Phi-\zeta_2)(\Phi-\zeta_3)(\Phi-\zeta_4)$ and $\zeta_1>\zeta_2>\zeta_3>\zeta_4$,

$$q(x,t)=\left\{\gamma_0+\gamma_1\zeta_2+\frac{\gamma_1(\zeta_1-\zeta_2)(\zeta_4-\zeta_2)}{\zeta_4-\zeta_2+(\zeta_1-\zeta_4)\,\mathrm{sn}^2\left[\pm\frac{\sqrt{(\zeta_1-\zeta_3)(\zeta_2-\zeta_4)}}{2\Omega_1}\left(x-\left\{\frac{b\omega-2a\kappa}{1-b\kappa}\right\}t-s_0\right),k\right]}\right\}^{\frac{1}{n+1}}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mu_1(c_1(n+2)+a\,n(n+2)\kappa^2\gamma_0^2-c_3n\gamma_0^4)-2n\mu_0\gamma_0\gamma_1(a(n+2)\kappa^2-2c_3\gamma_0^2)}{n(n+2)(b\kappa-1)\gamma_0(\mu_1\gamma_0-2\mu_0\gamma_1)}\right)t+\theta\right\}\right], \tag{27}$$

where

$$k^2=\frac{(\zeta_2-\zeta_3)(\zeta_1-\zeta_4)}{(\zeta_1-\zeta_3)(\zeta_2-\zeta_4)}. \tag{28}$$

One needs to explain that ζ_j for $j=1,\ldots,4$ are the roots of

$$\Theta(\Phi)=0. \tag{29}$$

For $\gamma_0=-\gamma_1\zeta_1$ and $s_0=0$, the solutions (22)–(26) can be reduced to plane wave solutions

$$q(x,t)=\left\{\pm\frac{\gamma_1\Omega_1}{x-\left\{\frac{b\omega-2a\kappa}{1-b\kappa}\right\}t}\right\}^{\frac{1}{n+1}}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mu_1(c_1(n+2)+a\,n(n+2)\kappa^2\gamma_0^2-c_3n\gamma_0^4)-2n\mu_0\gamma_0\gamma_1(a(n+2)\kappa^2-2c_3\gamma_0^2)}{n(n+2)(b\kappa-1)\gamma_0(\mu_1\gamma_0-2\mu_0\gamma_1)}\right)t+\theta\right\}\right], \tag{30}$$

$$q(x,t)=\left\{\frac{4\gamma_1\Omega_1^2(\zeta_2-\zeta_1)}{4\Omega_1^2-\left[(\zeta_1-\zeta_2)\left(x-\left\{\frac{b\omega-2a\kappa}{1-b\kappa}\right\}t\right)\right]^2}\right\}^{\frac{1}{n+1}}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mu_1(c_1(n+2)+a\,n(n+2)\kappa^2\gamma_0^2-c_3n\gamma_0^4)-2n\mu_0\gamma_0\gamma_1(a(n+2)\kappa^2-2c_3\gamma_0^2)}{n(n+2)(b\kappa-1)\gamma_0(\mu_1\gamma_0-2\mu_0\gamma_1)}\right)t+\theta\right\}\right], \tag{31}$$

singular soliton solutions

$$q(x,t)=\left\{\frac{\gamma_1(\zeta_2-\zeta_1)}{2}\left(1\mp\coth\left[\frac{\zeta_1-\zeta_2}{2\Omega_1}\left(x-\left\{\frac{b\omega-2a\kappa}{1-b\kappa}\right\}t\right)\right]\right)\right\}^{\frac{1}{n+1}}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mu_1(c_1(n+2)+a\,n(n+2)\kappa^2\gamma_0^2-c_3n\gamma_0^4)-2n\mu_0\gamma_0\gamma_1(a(n+2)\kappa^2-2c_3\gamma_0^2)}{n(n+2)(b\kappa-1)\gamma_0(\mu_1\gamma_0-2\mu_0\gamma_1)}\right)t+\theta\right\}\right], \tag{32}$$

and bright soliton solution

$$q(x,t)=\left\{\frac{\mathcal{P}_1}{\left(\mathcal{R}_1+\cosh\left[\mathcal{S}_1\left(x-\left\{\frac{b\omega-2a\kappa}{1-b\kappa}\right\}t\right)\right]\right)^{\frac{1}{n+1}}}\right\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{\mu_1(c_1(n+2)+a\,n(n+2)\kappa^2\gamma_0^2-c_3n\gamma_0^4)-2n\mu_0\gamma_0\gamma_1(a(n+2)\kappa^2-2c_3\gamma_0^2)}{n(n+2)(b\kappa-1)\gamma_0(\mu_1\gamma_0-2\mu_0\gamma_1)}\right)t+\theta\right\}\right], \tag{33}$$

where

$$\mathcal{P}_1=\left(\frac{2\gamma_1(\zeta_1-\zeta_2)(\zeta_1-\zeta_3)}{\zeta_3-\zeta_2}\right)^{\frac{1}{n+1}}, \tag{34}$$

$$\mathcal{R}_1 = \frac{2\zeta_1 - \zeta_2 - \zeta_3}{\zeta_3 - \zeta_2}, \tag{35}$$

$$\mathcal{S}_1 = \frac{\sqrt{(\zeta_1 - \zeta_2)(\zeta_1 - \zeta_3)}}{\Omega_1}. \tag{36}$$

Here, amplitude of the soliton is given by $\mathcal{P}_1$ while inverse width is indicated by $\mathcal{S}_1$ The recovered solitons imply the constraint $\gamma_1 < 0$. On the other hand, if $\gamma_0 = -\gamma_1\zeta_2$ and $s_0 = 0$, Jacobi elliptic function solutions (27) can be reduced to

$$q(x,t) = \left\{\frac{\mathcal{P}_2}{\left(\mathcal{R}_2 + \mathrm{sn}^2\left[\mathcal{S}_j\left(x - \left\{\frac{b\omega - 2a\kappa}{1 - b\kappa}\right\}t\right), \frac{(\zeta_2-\zeta_3)(\zeta_1-\zeta_4)}{(\zeta_1-\zeta_3)(\zeta_2-\zeta_4)}\right]\right)^{\frac{1}{n+1}}}\right\} \times \exp\left[i\left\{-\kappa x + \left(\frac{\mu_1(c_1(n+2) + a\,n(n+2)\kappa^2\gamma_0^2 - c_3 n\gamma_0^4) - 2n\mu_0\gamma_0\gamma_1(a(n+2)\kappa^2 - 2c_3\gamma_0^2)}{n(n+2)(b\kappa - 1)\gamma_0(\mu_1\gamma_0 - 2\mu_0\gamma_1)}\right)t + \theta\right\}\right], \tag{37}$$

where

$$\mathcal{P}_2 = \left(\frac{\gamma_1(\zeta_1 - \zeta_2)(\zeta_4 - \zeta_2)}{\zeta_1 - \zeta_4}\right)^{\frac{1}{n+1}}, \tag{38}$$

$$\mathcal{R}_2 = \frac{\zeta_4 - \zeta_2}{\zeta_1 - \zeta_4}, \tag{39}$$

$$\mathcal{S}_j = \frac{(-1)^j\sqrt{(\zeta_1 - \zeta_3)(\zeta_2 - \zeta_4)}}{2\Omega_1} \quad \text{for} \quad j = 2, 3. \tag{40}$$

Remark 1. When the modulus $k \to 1$, singular optical soliton solutions emerge as

$$q(x,t) = \left\{\frac{\mathcal{P}_2}{\left(\mathcal{R}_2 + \tanh^2\left[\mathcal{S}_j\left(x - \left\{\frac{b\omega - 2a\kappa}{1 - b\kappa}\right\}t\right)\right]\right)^{\frac{1}{n+1}}}\right\} \times \exp\left[i\left\{-\kappa x + \left(\frac{\mu_1(c_1(n+2) + a\,n(n+2)\kappa^2\gamma_0^2 - c_3 n\gamma_0^4) - 2n\mu_0\gamma_0\gamma_1(a(n+2)\kappa^2 - 2c_3\gamma_0^2)}{n(n+2)(b\kappa - 1)\gamma_0(\mu_1\gamma_0 - 2\mu_0\gamma_1)}\right)t + \theta\right\}\right], \tag{41}$$

where $\zeta_3 = \zeta_4$.

Remark 2. However, if $k \to 0$, periodic singular solutions are

$$q(x,t) = \left\{\frac{\mathcal{P}_2}{\left(\mathcal{R}_2 + \sin^2\left[\mathcal{S}_j\left(x - \left\{\frac{b\omega - 2a\kappa}{1 - b\kappa}\right\}t\right)\right]\right)^{\frac{1}{n+1}}}\right\} \times \exp\left[i\left\{-\kappa x + \left(\frac{\mu_1(c_1(n+2) + a\,n(n+2)\kappa^2\gamma_0^2 - c_3 n\gamma_0^4) - 2n\mu_0\gamma_0\gamma_1(a(n+2)\kappa^2 - 2c_3\gamma_0^2)}{n(n+2)(b\kappa - 1)\gamma_0(\mu_1\gamma_0 - 2\mu_0\gamma_1)}\right)t + \theta\right\}\right], \tag{42}$$

where $\zeta_2 = \zeta_3$.

2.2. Chirped solitons

In order to procure chirped solitons, the start-up assumption is

$$q(x,t) = g(s)e^{i[\chi(s) - \omega t]}, \tag{43}$$

where $g(s)$ is the amplitude function,

$$s = x - \mathrm{v}\,\mathrm{t}, \tag{44}$$

and v is the wave velocity. Also, $\chi(s)$ is the phase function and ω is the frequency of the wave oscillation.

Next, insert (43) into (2) and then split into real and imaginary parts. Real part yields

$$\omega g^2 + c_1 g^{-2n} + c_2 g^{2n+2} + c_3 g^{2n+4} + (v + b\omega)g^2\chi' - (a - \mathrm{b}\,\mathrm{v})g^2(\chi')^2 + (a - \mathrm{b}\,\mathrm{v})gg'' = 0, \tag{45}$$

while imaginary part gives

$$(v + b\omega)gg' - 2(a - \mathrm{b\,v})gg'\chi' - (a - \mathrm{b\,v})g^2\chi'' = 0, \tag{46}$$

where $g' = dg/ds$, $g'' = d^2g/ds^2$, $\chi' = d\chi/ds$ and $\chi'' = d^2\chi/ds^2$. Now, integrating (46), with zero constant of integration, gives rise to

$$\chi' = \frac{v + b\omega}{2(a - \mathrm{b\,v})}, \tag{47}$$

and then the corresponding chirp defined by

$$\delta\omega = -\frac{\partial}{\partial x}[\chi(s) - \omega t] = -\chi'(s), \tag{48}$$

can be written as

$$\delta\omega(x, t) = -\frac{v + b\omega}{2(a - \mathrm{b\,v})}. \tag{49}$$

Substituting (47) into (45) yields

$$4c_1(a - \mathrm{b\,v}) + (v^2 - 2\mathrm{b\,v}\omega + \omega(4a + b^2\omega))g^{2n+2} + 4c_2(a - \mathrm{b\,v})g^{4n+2} + 4c_3(a - \mathrm{b\,v})g^{4n+4} + 4(a - \mathrm{b\,v})^2g^{2n+1}g'' = 0. \tag{50}$$

To get a closed form analytic solution, one utilizes a transformation formula

$$g(x, t) = \varphi^{\frac{1}{n+1}}(x, t), \tag{51}$$

that will carry (50) into

$$4c_1(n + 1)^2(a - \mathrm{b\,v}) + (n + 1)^2(4a\omega + (v - b\omega)^2)\varphi^2 + 4c_2(n + 1)^2(a - \mathrm{b\,v})\varphi^{\frac{4n+2}{n+1}} + 4c_3(n + 1)^2(a - \mathrm{b\,v})\varphi^4 - 4n(a - \mathrm{b\,v})^2(\varphi')^2 + 4(n + 1)(a - \mathrm{b\,v})^2\varphi\varphi'' = 0. \tag{52}$$

For chirped solitons, one needs to choose $c_2 = 0$, as well, for permitting integrability. In this case, Eq. (52) changes to

$$4c_1(n + 1)^2(a - \mathrm{b\,v}) + (n + 1)^2(4a\omega + (v - b\omega)^2)\varphi^2 + 4c_3(n + 1)^2(a - \mathrm{b\,v})\varphi^4 - 4n(a - \mathrm{b\,v})^2(\varphi')^2 + 4(n + 1)(a - \mathrm{b\,v})^2\varphi\varphi'' = 0. \tag{53}$$

2.2.1. *Extended trial function scheme*

Balancing $(\varphi')^2$ or $\varphi\varphi''$ with φ^4 in Eq. (53) one has

$$\varrho = \rho + 2\varsigma + 2. \tag{54}$$

When $\varrho = 4$, $\rho = 0$ and $\varsigma = 1$ in Eq. (54), ETFS admits the use of the finite expansion

$$\varphi = \gamma_0 + \gamma_1\Phi. \tag{55}$$

Substituting (55) into (53), collecting the coefficients of Φ, and solving the resulting system one has

$$\begin{aligned}
&\mu_2 = \mu_2, \quad \mu_3 = \mu_3,\\
&\gamma_0 = \gamma_0, \quad \gamma_1 = \gamma_1,\\
&\chi_0 = -\frac{(n+2)(a - \mathrm{b\,v})\mu_3}{4c_3(n+1)^2\gamma_0\gamma_1},\\
&\mu_0 = \frac{c_3 n\gamma_0^3(4\mu_2\gamma_1 - 5\mu_3\gamma_0) - c_1(n+2)\mu_3}{4c_3 n\gamma_0\gamma_1^3},\\
&\mu_1 = \frac{2\gamma_0(\mu_2\gamma_1 - \mu_3\gamma_0)}{\gamma_1^2}, \quad \mu_4 = \frac{\mu_3\gamma_1}{4\gamma_0},\\
&\omega = -\frac{(n+2)(2a - \mathrm{b\,v})\mu_3 + 2\sqrt{(n+2)(a - \mathrm{b\,v})\mu_3(\mu_3(a(n+2) - 6b^2c_3\gamma_0^2) + 4b^2c_3\mu_2\gamma_0\gamma_1)}}{b^2(n+2)\mu_3}.
\end{aligned} \tag{56}$$

Substituting this set into Eqs. (14) and (15) causes

$$\pm(s - s_0) = \Omega_2\int\frac{d\Phi}{\sqrt{\Theta(\Phi)}}, \tag{57}$$

where

$$\Omega_2 = \sqrt{\frac{\chi_0}{\mu_4}}, \tag{58}$$

$$\Theta(\Phi) = \Phi^4 + \frac{\mu_3}{\mu_4}\Phi^3 + \frac{\mu_2}{\mu_4}\Phi^2 + \frac{\mu_1}{\mu_4}\Phi + \frac{\mu_0}{\mu_4}. \tag{59}$$

As a result, traveling wave solutions to the model are acquired as below:

For $\Theta(\Phi) = (\Phi - \zeta_1)^4$,

$$q(x,t)=\left\{\gamma_0+\gamma_1\zeta_1\pm\frac{\gamma_1\Omega_2}{x-\mathrm{v}\,t-s_0}\right\}^{\frac{1}{n+1}}\times\exp\left[i\left\{\chi[x-\mathrm{v}\,t]+\left(\frac{(n+2)(2a-\mathrm{b}\,\mathrm{v})\mu_3+2\sqrt{(n+2)(a-\mathrm{b}\,\mathrm{v})\mu_3(\mu_3(a(n+2)-6b^2c_3\gamma_0^2)+4b^2c_3\mu_2\gamma_0\gamma_1)}}{b^2(n+2)\mu_3}\right)t\right\}\right].\tag{60}$$

If $\Theta(\Phi)=(\Phi-\zeta_1)^3(\Phi-\zeta_2)$ and $\zeta_2>\zeta_1$,

$$q(x,t)=\left\{\gamma_0+\gamma_1\zeta_1+\frac{4\gamma_1\Omega_2^2(\zeta_2-\zeta_1)}{4\Omega_2^2-[(\zeta_1-\zeta_2)(x-\mathrm{v}\,t-s_0)]^2}\right\}^{\frac{1}{n+1}}\times\exp\left[i\left\{\chi[x-\mathrm{v}\,t]+\left(\frac{(n+2)(2a-\mathrm{b}\,\mathrm{v})\mu_3+2\sqrt{(n+2)(a-\mathrm{b}\,\mathrm{v})\mu_3(\mu_3(a(n+2)-6b^2c_3\gamma_0^2)+4b^2c_3\mu_2\gamma_0\gamma_1)}}{b^2(n+2)\mu_3}\right)t\right\}\right].\tag{61}$$

However, when $\Theta(\Phi)=(\Phi-\zeta_1)^2(\Phi-\zeta_2)^2$,

$$q(x,t)=\left\{\gamma_0+\gamma_1\zeta_2+\frac{\gamma_1(\zeta_2-\zeta_1)}{\exp\left[\frac{\zeta_1-\zeta_2}{\Omega_2}(x-\mathrm{v}\,t-s_0)\right]-1}\right\}^{\frac{1}{n+1}}\times\exp\left[i\left\{\chi[x-\mathrm{v}\,t]+\left(\frac{(n+2)(2a-\mathrm{b}\,\mathrm{v})\mu_3+2\sqrt{(n+2)(a-\mathrm{b}\,\mathrm{v})\mu_3(\mu_3(a(n+2)-6b^2c_3\gamma_0^2)+4b^2c_3\mu_2\gamma_0\gamma_1)}}{b^2(n+2)\mu_3}\right)t\right\}\right],\tag{62}$$

and

$$q(x,t)=\left\{\gamma_0+\gamma_1\zeta_1+\frac{\gamma_1(\zeta_1-\zeta_2)}{\exp\left[\frac{\zeta_1-\zeta_2}{\Omega_2}(x-\mathrm{v}\,t-s_0)\right]-1}\right\}^{\frac{1}{n+1}}\times\exp\left[i\left\{\chi[x-\mathrm{v}\,t]+\left(\frac{(n+2)(2a-\mathrm{b}\,\mathrm{v})\mu_3+2\sqrt{(n+2)(a-\mathrm{b}\,\mathrm{v})\mu_3(\mu_3(a(n+2)-6b^2c_3\gamma_0^2)+4b^2c_3\mu_2\gamma_0\gamma_1)}}{b^2(n+2)\mu_3}\right)t\right\}\right].\tag{63}$$

Whenever $\Theta(\Phi)=(\Phi-\zeta_1)^2(\Phi-\zeta_2)(\Phi-\zeta_3)$ and $\zeta_1>\zeta_2>\zeta_3$,

$$q(x,t)=\left\{\gamma_0+\gamma_1\zeta_1-\frac{2\gamma_1(\zeta_1-\zeta_2)(\zeta_1-\zeta_3)}{2\zeta_1-\zeta_2-\zeta_3+(\zeta_3-\zeta_2)\cosh\left[\frac{\sqrt{(\zeta_1-\zeta_2)(\zeta_1-\zeta_3)}}{\Omega_2}(x-\mathrm{v}\,t)\right]}\right\}^{\frac{1}{n+1}}\times\exp\left[i\left\{\chi[x-\mathrm{v}\,t]+\left(\frac{(n+2)(2a-\mathrm{b}\,\mathrm{v})\mu_3+2\sqrt{(n+2)(a-\mathrm{b}\,\mathrm{v})\mu_3(\mu_3(a(n+2)-6b^2c_3\gamma_0^2)+4b^2c_3\mu_2\gamma_0\gamma_1)}}{b^2(n+2)\mu_3}\right)t\right\}\right].\tag{64}$$

Finally, if $\Theta(\Phi)=(\Phi-\zeta_1)(\Phi-\zeta_2)(\Phi-\zeta_3)(\Phi-\zeta_4)$ and $\zeta_1>\zeta_2>\zeta_3>\zeta_4$,

$$q(x,t)=\{\gamma_0+\gamma_1\zeta_2+\frac{\gamma_1(\zeta_1-\zeta_2)(\zeta_4-\zeta_2)}{\zeta_4-\zeta_2+(\zeta_1-\zeta_4)\,\mathrm{sn}^2[\pm\frac{\sqrt{(\zeta_1-\zeta_3)(\zeta_2-\zeta_4)}}{2\Omega_2}(x-\mathrm{v}\,t-s_0),k]}\}^{\frac{1}{n+1}}\times\exp[i\{\chi[x-\mathrm{v}\,t]+(\frac{(n+2)(2a-\mathrm{b}\,\mathrm{v})\mu_3+2\sqrt{(n+2)(a-\mathrm{b}\,\mathrm{v})\mu_3(\mu_3(a(n+2)-6b^2c_3\gamma_0^2)+4b^2c_3\mu_2\gamma_0\gamma_1)}}{b^2(n+2)\mu_3})t\}],\tag{65}$$

where

$$k^2=\frac{(\zeta_2-\zeta_3)(\zeta_1-\zeta_4)}{(\zeta_1-\zeta_3)(\zeta_2-\zeta_4)}.\tag{66}$$

One needs to explain that ζ_j for $j=1,\ldots,4$ are the roots of

$$\Theta(\Phi)=0.\tag{67}$$

For $\gamma_0=-\gamma_1\zeta_1$ and $s_0=0$, the solutions (60)–(64) can be reduced to plane wave solutions

$$q(x,t)=\left\{\pm\frac{\gamma_1\Omega_2}{x-\mathrm{v}\,t}\right\}^{\frac{1}{n+1}}\times\exp\left[i\left\{\chi[x-\mathrm{v}\,t]+\left(\frac{(n+2)(2a-\mathrm{b}\,\mathrm{v})\mu_3+2\sqrt{(n+2)(a-\mathrm{b}\,\mathrm{v})\mu_3(\mu_3(a(n+2)-6b^2c_3\gamma_0^2)+4b^2c_3\mu_2\gamma_0\gamma_1)}}{b^2(n+2)\mu_3}\right)t\right\}\right],\tag{68}$$

$$q(x,t)=\left\{\frac{4\gamma_1\Omega_2^2(\zeta_2-\zeta_1)}{4\Omega_2^2-[(\zeta_1-\zeta_2)(x-\mathrm{v}\,t)]^2}\right\}^{\frac{1}{n+1}}\times\exp\left[i\left\{\chi[x-\mathrm{v}\,t]+\left(\frac{(n+2)(2a-b\mathrm{v})\mu_3+2\sqrt{(n+2)(a-b\mathrm{v})\mu_3(\mu_3(a(n+2)-6b^2c_3\gamma_0^2)+4b^2c_3\mu_2\gamma_0\gamma_1)}}{b^2(n+2)\mu_3}\right)t\right\}\right], \tag{69}$$

singular soliton solutions

$$q(x,t)=\left\{\frac{\gamma_1(\zeta_2-\zeta_1)}{2}\left(1\mp\coth\left[\frac{\zeta_1-\zeta_2}{2\Omega_2}(x-\mathrm{v}\,t)\right]\right)\right\}^{\frac{1}{n+1}}\times\exp\left[i\left\{\chi[x-\mathrm{v}\,t]+\left(\frac{(n+2)(2a-b\mathrm{v})\mu_3+2\sqrt{(n+2)(a-b\mathrm{v})\mu_3(\mu_3(a(n+2)-6b^2c_3\gamma_0^2)+4b^2c_3\mu_2\gamma_0\gamma_1)}}{b^2(n+2)\mu_3}\right)t\right\}\right], \tag{70}$$

and bright soliton solution

$$q(x,t)=\left\{\frac{\mathcal{P}_3}{(\mathcal{R}_3+\cosh[\mathcal{S}_4(x-\mathrm{v}\,t)])^{\frac{1}{n+1}}}\right\}\times\exp\left[i\left\{\chi[x-\mathrm{v}\,t]+\left(\frac{(n+2)(2a-b\mathrm{v})\mu_3+2\sqrt{(n+2)(a-b\mathrm{v})\mu_3(\mu_3(a(n+2)-6b^2c_3\gamma_0^2)+4b^2c_3\mu_2\gamma_0\gamma_1)}}{b^2(n+2)\mu_3}\right)t\right\}\right], \tag{71}$$

where

$$\mathcal{P}_3=\left(\frac{2\gamma_1(\zeta_1-\zeta_2)(\zeta_1-\zeta_3)}{\zeta_3-\zeta_2}\right)^{\frac{1}{n+1}}, \tag{72}$$

$$\mathcal{R}_3=\frac{2\zeta_1-\zeta_2-\zeta_3}{\zeta_3-\zeta_2}, \tag{73}$$

$$\mathcal{S}_4=\frac{\sqrt{(\zeta_1-\zeta_2)(\zeta_1-\zeta_3)}}{\Omega_2}. \tag{74}$$

Here, the amplitude $\mathcal{P}_3$ and the inverse width $\mathcal{S}_4$ of the soliton are located in (72) and (74) respectively. The derived solitons introduce the constraint $\gamma_1 < 0$. On the other hand, if $\gamma_0 = -\gamma_1\zeta_2$ and $s_0 = 0$, the solutions given by (65) can be reduced to

$$q(x,t)=\left\{\frac{\mathcal{P}_4}{\left(\mathcal{R}_4+\mathrm{sn}^2\left[\mathcal{S}_j(x-\mathrm{v}\,t),\frac{(\zeta_2-\zeta_3)(\zeta_1-\zeta_4)}{(\zeta_1-\zeta_3)(\zeta_2-\zeta_4)}\right]\right)^{\frac{1}{n+1}}}\right\}\times\exp\left[i\left\{\chi[x-\mathrm{v}\,t]+\left(\frac{(n+2)(2a-b\mathrm{v})\mu_3+2\sqrt{(n+2)(a-b\mathrm{v})\mu_3(\mu_3(a(n+2)-6b^2c_3\gamma_0^2)+4b^2c_3\mu_2\gamma_0\gamma_1)}}{b^2(n+2)\mu_3}\right)t\right\}\right], \tag{75}$$

where

$$\mathcal{P}_4=\left(\frac{\gamma_1(\zeta_1-\zeta_2)(\zeta_4-\zeta_2)}{\zeta_1-\zeta_4}\right)^{\frac{1}{n+1}}, \tag{76}$$

$$\mathcal{R}_4=\frac{\zeta_4-\zeta_2}{\zeta_1-\zeta_4}, \tag{77}$$

$$\mathcal{S}_j=\frac{(-1)^j\sqrt{(\zeta_1-\zeta_3)(\zeta_2-\zeta_4)}}{2\Omega_2}\quad\text{for}\quad j=5,6. \tag{78}$$

Remark 3. When the modulus $k \to 1$, singular optical soliton solutions emerge as

$$q(x,t)=\left\{\frac{\mathcal{P}_4}{(\mathcal{R}_4+\tanh^2[\mathcal{S}_j(x-\mathrm{v}\,t)])^{\frac{1}{n+1}}}\right\}\times\exp\left[i\left\{\chi[x-\mathrm{v}\,t]+\left(\frac{(n+2)(2a-b\mathrm{v})\mu_3+2\sqrt{(n+2)(a-b\mathrm{v})\mu_3(\mu_3(a(n+2)-6b^2c_3\gamma_0^2)+4b^2c_3\mu_2\gamma_0\gamma_1)}}{b^2(n+2)\mu_3}\right)t\right\}\right], \tag{79}$$

where $\zeta_3 = \zeta_4$.

Remark 4. However, if $k \to 0$, periodic singular solutions are

$$q(x,\ t)= \left\{\frac{\mathcal{P}_4}{(\mathcal{R}_4+\sin^2[\mathcal{S}_j(x-\mathrm{v}\,\mathrm{t})])^{\frac{1}{n+1}}}\right\} \times \exp\left[i\left\{\chi[x-\mathrm{v}\,\mathrm{t}]+\left(\frac{(n+2)(2a-\mathrm{b}\,\mathrm{v})\mu_3+2\sqrt{(n+2)(a-\mathrm{b}\,\mathrm{v})\mu_3(\mu_3(a(n+2)-6b^2c_3\gamma_0^2)+4b^2c_3\mu_2\gamma_0\gamma_1)}}{b^2(n+2)\mu_3}\right)t\right\}\right], \tag{80}$$

where $\zeta_2 = \zeta_3$.

Remark 5. It is interesting to note that the chirping corresponding to Eqs. (60)–(65), (68)–(71), (75), (79) and (80) is same and this chirping is given as

$$\delta\omega(x,\ t) = -\frac{v}{2(a-\mathrm{b}\,\mathrm{v})}+\frac{(n+2)(2a-\mathrm{b}\,\mathrm{v})\mu_3+2\sqrt{(n+2)(a-\mathrm{b}\,\mathrm{v})\mu_3(\mu_3(a(n+2)-6b^2c_3\gamma_0^2)+4b^2c_3\mu_2\gamma_0\gamma_1)}}{2b(n+2)(a-\mathrm{b}\,\mathrm{v})\mu_3}. \tag{81}$$

3. Conclusions

This paper recovered chirped and chirp-free bright and singular optical solitons with generalized AC nonlinearity. The ETFS was the integration methodology adopted. The results are being reported for the first time. It is clearly noted that this algorithm fails to retrieve dark soliton solutions to the model. Later, additional integration schemes, such as Lie symmetry analysis or Kudryashov's method and others will be adopted to secure dark solitons and other forms of waves.

This is a newly introduced model. Therefore, there is a lot of work that lies ahead with this model. A few of them are locating its conservation laws, study of soliton cooling effect, application to magneto-optic waveguides and optical couplers, study of optical metamaterials with this model, just to name a few. Furthermore, this model can be studied with stochastic perturbation terms as well as time-dependent coefficients. The results of those research activities are awaited at this time and will be gradually reported with time.

Acknowledgements

The research work of fourth author (MB) was supported by Qatar National Research Fund (QNRF) under the grant number NPRP 8-028-1-001. The authors also declare that there is no conflict of interest.

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