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Mitigating Internet bottleneck with fractional temporal evolution of optical solitons having quadratic–cubic nonlinearity

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ABSTRACT

This paper studies fractional temporal evolution of optical solitons with quadratic–cubic nonlinearity that comes with a few perturbation terms. Khalil's conformable fractional derivative as well as Liu's extended trial function scheme are applied to retrieve these soliton solutions. The results are applicable to mitigate Internet bottleneck that is a growing problem in telecommunications industry.

1. Introduction

Optical soliton perturbation with fractional temporal evolution is one of the viable means to address a growing problem in telecommunication industry, namely the Internet bottleneck. This problem leads to slow Internet traffic and eventually

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blockage of the traffic. Several mechanisms have been proposed to address this concern. One of them is to choose time-dependent coefficients of dispersion and nonlinearity. A better and much more efficient method is to consider fractional temporal evolution which is what this paper will address.

The current work is to study the governing nonlinear Schrödinger's equation (NLSE) with quadratic–cubic (QC) nonlinearity in presence of perturbation terms and higher order dispersions such as third order dispersion (3OD) and fourth order dispersion (4OD). It is well known that NLSE as well as other forms of nonlinear evolution equations are addressed by a variety of mathematical methods [1–30]. In this paper, two approaches will reveal soliton solutions that will illustrate slow progress of solitons through optical fibers and other waveguides.

There are a variety of ways to define fractional temporal evolution such as Caputo derivative, Baleanu derivative and others. This paper will employ Khalil's conformable fractional derivative that will be first introduced. With this definition of temporal evolution, Kudryashov's method will lead to the soliton solutions of the perturbed NLSE. Finally, a second method that is known as Liu's extended trial equation method also will reveal additional forms of soliton solutions to the model. The rest of the paper is devoted to revisitation of these concepts and derivation of the soliton solutions.

1.1. Mathematical model

The governing resonant NLSE with conformable time fractional and perturbation terms that is studied in nonlinear optics is given in its dimensionless form as [7,8,28]

$$iD_t^{\alpha}q + aq_{xx} + \left(b_1\,|q| + b_2|q|^2\right)q = i\left\{\delta q_x - \gamma q_{xxx} - i\sigma q_{xxxx} + \lambda\left(|q|^2 q\right)_x + \theta\left(|q|^2\right)_x q\right\}, \qquad (1)$$

where D_t^{α} is the conformable derivative operator of order $\alpha \in (0, 1]$ in the t-direction. In Eq. (1), the independent variables x and t respectively represent spatial and temporal variables. The dependent variable $q(x, t)$ gives the complex-valued wave profile together with $i = \sqrt{-1}$. The coefficient of the real-valued constant a is known as the group velocity dispersion. The nonlinear terms are given by the coefficients of b_1 and b_2, which represent quadratic and cubic forms, respectively. On the right hand side δ is the coefficient of inter-modal dispersion which appears when the group velocity of light propagating through multi-mode optical fibers or other optical waveguides depends on the optical frequency as well as the propagation mode involved. The coefficients of γ and σ are 3OD and 4OD, respectively. These appear when GVD is negligably small and thus the higher-order dispersions creep in order to maintain the necessary balance between dispersion and nonlinearity for the sustainment of optical solitons. The coefficient of λ is due to self steepening that is included to eliminate formation of shock waves. Finally, θ represents the coefficient of nonlinear dispersion.

1.2. Khalil's conformable fractional derivative

Recently, Khalil et al. introduced a new simple well-behaved definition of the fractional derivative called conformable derivative. The conformable fractional derivative of order is defined by the following definition [22].

Definition: Let $f: [0, \infty) \rightarrow R$, then, the conformable derivative of f of order α is defined as

$$D_t^{\alpha}(f)(t) = \lim_{\varepsilon \to 0}\frac{f(t + \varepsilon t^{1-\alpha}) - f(t)}{\varepsilon}, \qquad (2)$$

for all $t > 0$, $\alpha \in (0, 1]$.

This new definition satisfies the properties in the following theorem:

Theorem 1. *Let $\alpha \in (0, 1]$, f, g be α-differentiable at a point t, then* [22]:

(i) $D_t^{\alpha}(af + bg) = aD_t^{\alpha}(f) + bD_t^{\alpha}(g)$, *for all* $a, b \in R$.
(ii) $D_t^{\alpha}(t^{\mu}) = \mu t^{\mu-\alpha}$, *for all* $\mu \in R$.
(iii) $D_t^{\alpha}(fg) = fD_t^{\alpha}(g) + gD_t^{\alpha}(f)$.
(iv) $D_t^{\alpha}\left(\frac{f}{g}\right) = \frac{gD_t^{\alpha}(f) - fD_t^{\alpha}(g)}{g^2}$.

If, in addition, f is differentiable, then $D_t^{\alpha}(f)(t) = t^{1-\alpha}\frac{df}{dt}$.

Abdeljawad [1] established the chain rule for conformable fractional derivatives as following theorem.

Theorem 2. *Let $f: (0, \infty) \rightarrow R$, be a function such that f is differentiable and also α-conformable differentiable. Let g be a differentiable function defined in the range of f. Then*

$$TD_t^{\alpha}(fog)(t) = t^{1-\alpha}g'(t)f'(g(t)), \qquad (3)$$

where prime denotes the classical derivatives with respect to t.

Later on, many researchers established exact traveling wave solutions of various nonlinear fractional evolution equations via this fractional derivative [2,10,16,17,23,27,30].

2. Soliton solutions

In order to solve Eq. (1), initial assumption is:

$$q(x,t)=g(s)e^{i\phi(x,t)}, \tag{4}$$

where $g(s)$ represents the shape of the pulse and

$$s=x-\left(\frac{\nu}{\alpha}\right)t^{\alpha}, \tag{5}$$

and the phase component is defined as

$$\phi(x,t)=-\kappa x+\left(\frac{\omega}{\alpha}\right)t^{\alpha}+\theta_0. \tag{6}$$

Here κ is the soliton frequency, ω is the wave number of the soliton and θ_0 is the phase constant. Also, in (5), ν represents the speed of the soliton. Substituting (4) into (1) and decomposing into real and imaginary parts, give

$$\sigma g^{(iv)}-P_2g''+P_1g-\left(b_1g+(b_2-\lambda\kappa)g^2\right)g=0, \tag{7}$$

and

$$(\nu+2a\kappa+\delta+3\gamma\kappa^2+4\sigma\kappa^3)g'-(\gamma+4\sigma\kappa)g''+\left(3\lambda+2\theta\right)g^2g'=0. \tag{8}$$

Here the notations $g'=dg/ds$ and $g''=d^2g/ds^2$, etc. are adopted. Here, in Eq. (7)

$$P_1=\omega+\delta\kappa+a\kappa^2+\gamma\kappa^3+\sigma\kappa^4, \tag{9}$$

$$P_2=a+3\gamma\kappa+6\sigma\kappa^3. \tag{10}$$

From Eq. (8), setting the coefficients of linearly independent functions to zero gives:

$$\nu=-2a\kappa-\delta-3\gamma\kappa^2-4\sigma\kappa^4, \tag{11}$$

$$\gamma+4\sigma\kappa=0, \tag{12}$$

$$3\lambda+2\theta=0. \tag{13}$$

Thus, Eq. (11) gives the speed of the soliton in presence of the perturbation terms while relations (12) and (13) are the constraints on the perturbation parameters.

2.1. Modified Kudryashov's method

This subsection will apply the modified Kudryashov approach [20,21] to handle the resonant NLSE with conformable time fractional and perturbation terms. Assume that the solution of Eq. (7) is given as

$$g(s)=a_0+\sum_{i=1}^{N}a_iQ^i(s), \tag{14}$$

where unknowns $a_i\,(i=0,1,\ldots N)$ are identified later, so that $a_N\neq 0$, N is a positive integer and $Q(s)$ satisfies new auxiliary equation

$$Q'(s)=\left(Q^2(s)-Q(s)\right)\ln(A),\quad A\neq 0,1, \tag{15}$$

where the solution of Eq. (15) is

$$Q(s)=\frac{1}{1+dA^s}. \tag{16}$$

By considering the homogeneous balance between $g^{(iv)}$ and g^3 in Eq. (7) yields $N=2$. Assume the formal solution of Eq. (7) through Eq. (14)

$$g(s)=a_0+a_1Q(s)+a_2Q^2(s). \tag{17}$$

By substituting Eq. (17) along with its second and fourth derivatives into Eq. (7) and equating coefficients of like powers of $Q(s)$, we obtain a nonlinear algebraic system. Solving these system, we acquire the following solution sets:

$$a_2=\pm\frac{2\sqrt{30\ (b_2-\lambda\ \kappa)\sigma}\ln^2A}{-b_2+\lambda\ \kappa}, \tag{18}$$

$$a_1=\pm\frac{2\sqrt{30(b_2-\lambda\ \kappa)\sigma}\ln^2A}{b_2-\lambda\ \kappa}, \tag{19}$$

$$a_0 = 0, \tag{20}$$

$$b_1 = \mp \frac{\sqrt{30(b_2 - \lambda\ \kappa)\sigma}\left(-P_2 + 5\ \sigma\ \ln^2 A\right)}{10\ \ \sigma}, \tag{21}$$

$$\omega = -a\ \kappa^2 - \delta\ \kappa - \gamma\ \kappa^3 - \sigma\ \kappa^4 + P_2 \ln^2 A - \sigma\ \ln^4 A. \tag{22}$$

Using Eqs. (17)–(22), (4)–(6) and auxiliary equation (16), we can find the following exact solutions of Eq. (1) as follows

$$\begin{aligned} q_{1,2}(x,t) = & \frac{2\sqrt{30(b_2-\lambda\kappa)\sigma}\ln^2 A}{(b_2-\lambda\kappa)} \\ & \times \left(\pm \frac{1}{\left(1 + d\left[\cosh\left(\left(x - \left(\frac{-2a\kappa - \delta - 3\gamma\kappa^2 - 4\sigma\kappa^4}{\alpha}\right)t^\alpha\right)\ln A\right) + \sinh\left(\left(x - \left(\frac{-2a\kappa - \delta - 3\gamma\kappa^2 - 4\sigma\kappa^4}{\alpha}\right)t^\alpha\right)\ln A\right)\right]\right)} \right. \\ & \left. \pm \frac{1}{\left(1 + d\ \left[\cosh\left(\left(x - \left(\frac{-2a\kappa - \delta - 3\gamma\kappa^2 - 4\sigma\kappa^4}{\alpha}\right)t^\alpha\right)\ln A\right) + \sinh\left(\left(x - \left(\frac{-2a\kappa - \delta - 3\gamma\kappa^2 - 4\sigma\kappa^4}{\alpha}\right)t^\alpha\right)\ln A\right)\right]\right)^2} \right) \\ & \times \exp\left[i\left\{-\kappa x + \left(-a\kappa^2 - \delta\kappa - \gamma\kappa^3 - \sigma\kappa^4 + P_2\ln^2 A - \sigma\ln^4 A\right)t^\alpha + \theta_0\right\}\right]. \end{aligned} \tag{23}$$

This represents a bright-singular combo soliton solution.

2.2. Liu's extended trial function scheme

This subsection will employ the extended trial solution mechanism [11–14,24,25] to handle the resonant NLSE with conformable time fractional and perturbation terms. To carry out the integration of Eq. (1), by this mechanism it is necessary to choose $\gamma = \sigma = 0$ which means equation (1) condenses to:

$$iD_t^\alpha q + aq_{xx} + \left(b_1\,|q| + b_2|q|^2\right)q = i\left\{\delta q_x + \lambda\left(|q|^2 q\right)_x + \theta\left(|q|^2\right)_x q\right\}.$$

Thus, Eq. (7) can be rewritten as

$$ag'' - \left(\omega + \delta\kappa + a\kappa^2\right)g + \left(b_1 g + (b_2 - \lambda\kappa)g^2\right)g = 0. \tag{24}$$

To start off with the scheme, the following initial assumption for the solution structure of (24) is considered:

$$g = \sum_{i=0}^{\varsigma} \tau_i \Psi^i, \tag{25}$$

where

$$(\Psi')^2 = \Lambda(\Psi) = \frac{\Phi(\Psi)}{\Upsilon(\Psi)} = \frac{\mu_\sigma \Psi^\sigma + \cdots + \mu_1 \Psi + \mu_0}{\chi_\rho \Psi^\rho + \cdots + \chi_1 \Psi + \chi_0}. \tag{26}$$

Here $\tau_0, \ldots, \tau_\varsigma$; $\mu_0, \ldots, \mu_\sigma$ and $\chi_0, \ldots, \chi_\rho$ are constants to be determined later. Terms $(g')^2$ and g'' by the aid of Eqs. (25) and (26) can be acquired as

$$(g')^2 = \frac{\Phi(\Psi)}{\Upsilon(\Psi)}\left(\sum_{i=0}^{\varsigma} i\tau_i \Psi^{i-1}\right)^2, \tag{27}$$

and

$$g'' = \frac{\Phi'(\Psi)\Upsilon(\Psi) - \Phi(\Psi)\Upsilon'(\Psi)}{2\Upsilon^2(\Psi)}\left(\sum_{i=0}^{\varsigma} i\tau_i \Psi^{i-1}\right) + \frac{\Phi(\Psi)}{\Upsilon(\Psi)}\left(\sum_{i=0}^{\varsigma} i(i-1)\tau_i \Psi^{i-2}\right), \tag{28}$$

where $\Phi(\Psi)$ and $\Upsilon(\Psi)$ are polynomials of Ψ. Eq. (26) can be reduced elementary integral form as

$$\pm(s - s_0) = \int \frac{d\Psi}{\sqrt{\Lambda(\Psi)}} = \int \sqrt{\frac{\Upsilon(\Psi)}{\Phi(\Psi)}}\,d\Psi. \tag{29}$$

Balancing the order of g'' and g^3 in Eq. (24) leads to

$$\sigma - \rho - 2\varsigma = 2. \tag{30}$$

Let us choose $\sigma=4$, $\rho=0$ and $\varsigma=1$ in Eq. (30). Then, the adopted approach allows us to use the substitution

$$g=\tau_0+\tau_1\Psi, \tag{31}$$

where τ_0 and τ_1 are constants to be determined later such that $\tau_1\neq 0$, and Ψ satisfies Eq. (26). Substituting (31) into (24), collecting the coefficients of each power of Ψ, and solving the system of coefficients, the following set of solutions is found:

$$\begin{aligned}
&\mu_0=\mu_0,\quad \mu_1=\mu_1,\quad \mu_3=\mu_3,\quad \tau_0=\tau_0,\quad \tau_1=\tau_1,\\
&\chi_0=-\frac{3a\mu_3}{2\tau_1\left(b_1+3\tau_0\left(b_2-\kappa\lambda\right)\right)},\\
&\mu_2=\frac{\mu_1\tau_1}{2\tau_0}+\frac{3\mu_3\tau_0\left(b_1+2\tau_0\left(b_2-\kappa\lambda\right)\right)}{2\tau_1\left(b_1+3\tau_0\left(b_2-\kappa\lambda\right)\right)},\\
&\mu_4=\frac{3\mu_3\tau_1\left(b_2-\kappa\lambda\right)}{4\left(b_1+3\tau_0\left(b_2-\kappa\lambda\right)\right)},\\
&\omega=-\frac{3\mu_3\tau_0\left(\kappa(\delta+a\kappa)-b_1\tau_0+\tau_0^2\left(\kappa\lambda-b_2\right)\right)+\mu_1\tau_1^2\left(b_1+3\tau_0\left(b_2-\kappa\lambda\right)\right)}{3\mu_3\tau_0}.
\end{aligned} \tag{32}$$

Substituting this set of solutions into (26) and (29) leads to

$$\pm(s-s_0)=\Omega\int\frac{d\Psi}{\sqrt{\Theta(\Psi)}}, \tag{33}$$

where

$$\Theta(\Psi)=\Psi^4+\frac{\mu_3}{\mu_4}\Psi^3+\frac{\mu_2}{\mu_4}\Psi^2+\frac{\mu_1}{\mu_4}\Psi+\frac{\mu_0}{\mu_4},\qquad \Omega=\sqrt{\frac{\chi_0}{\mu_4}}. \tag{34}$$

As a consequence, traveling wave solutions for the governing model given by (1) are secured in the forms:

For $\Theta(\Psi)=(\Psi-\ell_1)^4$,

$$\begin{aligned}
q(x,t)=&\left\{\tau_0+\tau_1\ell_1\pm\frac{\tau_1\Omega}{x+\left\{\dfrac{\delta+2a\kappa}{\alpha}\right\}t^\alpha-s_0}\right\}\\
&\times\exp\left[i\left\{-\kappa x-\left(\frac{3\mu_3\tau_0\left(\kappa(\delta+a\kappa)-b_1\tau_0+\tau_0^2\left(\kappa\lambda-b_2\right)\right)+\mu_1\tau_1^2\left(b_1+3\tau_0\left(b_2-\kappa\lambda\right)\right)}{3\alpha\mu_3\tau_0}\right)t^\alpha+\theta_0\right\}\right].
\end{aligned} \tag{35}$$

If $\Theta(\Psi)=(\Psi-\ell_1)^3(\Psi-\ell_2)$ and $\ell_2>\ell_1$,

$$\begin{aligned}
q(x,t)=&\left\{\tau_0+\tau_1\ell_1+\frac{4\tau_1\Omega^2(\ell_2-\ell_1)}{4\Omega^2-\left[(\ell_1-\ell_2)\left(x+\left\{\dfrac{\delta+2a\kappa}{\alpha}\right\}t^\alpha-s_0\right)\right]^2}\right\}\\
&\times\exp\left[i\left\{-\kappa x-\left(\frac{3\mu_3\tau_0\left(\kappa(\delta+a\kappa)-b_1\tau_0+\tau_0^2\left(\kappa\lambda-b_2\right)\right)+\mu_1\tau_1^2\left(b_1+3\tau_0\left(b_2-\kappa\lambda\right)\right)}{3\alpha\mu_3\tau_0}\right)t^\alpha+\theta_0\right\}\right].
\end{aligned} \tag{36}$$

However, when $\Theta(\Psi)=(\Psi-\ell_1)^2(\Psi-\ell_2)^2$,

$$\begin{aligned}
q(x,t)=&\left\{\tau_0+\tau_1\ell_2+\frac{\tau_1(\ell_2-\ell_1)}{\exp\left[\dfrac{\ell_1-\ell_2}{\Omega}\left(x+\left\{\dfrac{\delta+2a\kappa}{\alpha}\right\}t^\alpha-s_0\right)\right]-1}\right\}\\
&\times\exp\left[i\left\{-\kappa x-\left(\frac{3\mu_3\tau_0\left(\kappa(\delta+a\kappa)-b_1\tau_0+\tau_0^2\left(\kappa\lambda-b_2\right)\right)+\mu_1\tau_1^2\left(b_1+3\tau_0\left(b_2-\kappa\lambda\right)\right)}{3\alpha\mu_3\tau_0}\right)t^\alpha+\theta_0\right\}\right],
\end{aligned} \tag{37}$$

and

$$q(x,t)=\left\{\tau_0+\tau_1\ell_1+\frac{\tau_1(\ell_1-\ell_2)}{\exp\left[\frac{\ell_1-\ell_2}{\Omega}\left(x+\left\{\frac{\delta+2a\kappa}{\alpha}\right\}t^{\alpha}-s_0\right)\right]-1}\right\}\times\exp\left[i\left\{-\kappa x-\left(\frac{3\mu_3\tau_0\left(\kappa(\delta+a\kappa)-b_1\tau_0+\tau_0^2(\kappa\lambda-b_2)\right)+\mu_1\tau_1^2(b_1+3\tau_0(b_2-\kappa\lambda))}{3\alpha\mu_3\tau_0}\right)t^{\alpha}+\theta_0\right\}\right]. \tag{38}$$

Whenever $\Theta(\Psi)=(\Psi-\ell_1)^2(\Psi-\ell_2)(\Psi-\ell_3)$ and $\ell_1>\ell_2>\ell_3$,

$$q(x,t)=\left\{\tau_0+\tau_1\ell_1-\frac{2\tau_1(\ell_1-\ell_2)(\ell_1-\ell_3)}{2\ell_1-\ell_2-\ell_3+(\ell_3-\ell_2)\cosh\left[\frac{\sqrt{(\ell_1-\ell_2)(\ell_1-\ell_3)}}{\Omega}\left(x+\left\{\frac{\delta+2a\kappa}{\alpha}\right\}t^{\alpha}\right)\right]}\right\}\times\exp\left[i\left\{-\kappa x-\left(\frac{3\mu_3\tau_0\left(\kappa(\delta+a\kappa)-b_1\tau_0+\tau_0^2(\kappa\lambda-b_2)\right)+\mu_1\tau_1^2(b_1+3\tau_0(b_2-\kappa\lambda))}{3\alpha\mu_3\tau_0}\right)t^{\alpha}+\theta_0\right\}\right]. \tag{39}$$

On the other hand, if $\Theta(\Psi)=(\Psi-\ell_1)(\Psi-\ell_2)(\Psi-\ell_3)(\Psi-\ell_4)$ and $\ell_1>\ell_2>\ell_3>\ell_4$,

$$q(x,t)=\left\{\tau_0+\tau_1\ell_2+\frac{\tau_1(\ell_1-\ell_2)(\ell_4-\ell_2)}{\ell_4-\ell_2+(\ell_1-\ell_4)\,\mathrm{sn}^2\left[\pm\frac{\sqrt{(\ell_1-\ell_3)(\ell_2-\ell_4)}}{2\Omega}\left(x+\left\{\frac{\delta+2a\kappa}{\alpha}\right\}t^{\alpha}-s_0\right),m\right]}\right\}\times\exp\left[i\left\{-\kappa x-\left(\frac{3\mu_3\tau_0\left(\kappa(\delta+a\kappa)-b_1\tau_0+\tau_0^2(\kappa\lambda-b_2)\right)+\mu_1\tau_1^2(b_1+3\tau_0(b_2-\kappa\lambda))}{3\alpha\mu_3\tau_0}\right)t^{\alpha}+\theta_0\right\}\right], \tag{40}$$

where modulus m is given by

$$m^2=\frac{(\ell_2-\ell_3)(\ell_1-\ell_4)}{(\ell_1-\ell_3)(\ell_2-\ell_4)}. \tag{41}$$

It is important to note that ℓ_j for $j=1,\ldots,4$ are the roots of

$$\Theta(\Psi)=0. \tag{42}$$

Let $\tau_0=-\tau_1\ell_1$ and $s_0=0$. Then, the solutions (35)–(39) can be reduced to plane wave solutions

$$q(x,t)=\left\{\pm\frac{\tau_1\Omega}{x+\left\{\frac{\delta+2a\kappa}{\alpha}\right\}t^{\alpha}}\right\}\times\exp\left[i\left\{-\kappa x-\left(\frac{3\mu_3\tau_0\left(\kappa(\delta+a\kappa)-b_1\tau_0+\tau_0^2(\kappa\lambda-b_2)\right)+\mu_1\tau_1^2(b_1+3\tau_0(b_2-\kappa\lambda))}{3\alpha\mu_3\tau_0}\right)t^{\alpha}+\theta_0\right\}\right], \tag{43}$$

$$q(x,t)=\left\{\frac{4\tau_1\Omega^2(\ell_2-\ell_1)}{4\Omega^2-\left[(\ell_1-\ell_2)\left(x+\left\{\frac{\delta+2a\kappa}{\alpha}\right\}t^{\alpha}\right)\right]^2}\right\}\times\exp\left[i\left\{-\kappa x-\left(\frac{3\mu_3\tau_0\left(\kappa(\delta+a\kappa)-b_1\tau_0+\tau_0^2(\kappa\lambda-b_2)\right)+\mu_1\tau_1^2(b_1+3\tau_0(b_2-\kappa\lambda))}{3\alpha\mu_3\tau_0}\right)t^{\alpha}+\theta_0\right\}\right], \tag{44}$$

singular soliton solutions

$$q(x,t)=\left\{\frac{\tau_1(\ell_2-\ell_1)}{2}\left(1\mp\coth\left[\frac{\ell_1-\ell_2}{2\Omega}\left(x+\left\{\frac{\delta+2a\kappa}{\alpha}\right\}t^{\alpha}\right)\right]\right)\right\}\times\exp\left[i\left\{-\kappa x-\left(\frac{3\mu_3\tau_0\left(\kappa(\delta+a\kappa)-b_1\tau_0+\tau_0^2(\kappa\lambda-b_2)\right)+\mu_1\tau_1^2(b_1+3\tau_0(b_2-\kappa\lambda))}{3\alpha\mu_3\tau_0}\right)t^{\alpha}+\theta_0\right\}\right], \tag{45}$$

and bright soliton solution

$$q(x,t)=\left\{\frac{M}{R+\cosh\left[N\left(x+\left\{\frac{\delta+2a\kappa}{\alpha}\right\}t^{\alpha}\right)\right]}\right\}\times\exp\left[i\left\{-\kappa x-\left(\frac{3\mu_3\tau_0\left(\kappa(\delta+a\kappa)-b_1\tau_0+\tau_0^2(\kappa\lambda-b_2)\right)+\mu_1\tau_1^2(b_1+3\tau_0(b_2-\kappa\lambda))}{3\alpha\mu_3\tau_0}\right)t^{\alpha}+\theta_0\right\}\right], \tag{46}$$

where

$$M=\frac{2\tau_1(\ell_1-\ell_2)(\ell_1-\ell_3)}{\ell_3-\ell_2}, \tag{47}$$

$$N=\frac{\sqrt{(\ell_1-\ell_2)(\ell_1-\ell_3)}}{\Omega}, \tag{48}$$

$$R=\frac{2\ell_1-\ell_2-\ell_3}{\ell_3-\ell_2}. \tag{49}$$

It is worth mentioning here that the amplitude of the soliton is given by (47), while the inverse width of the soliton is given by (48). These solitons exist for $\tau_1<0$. Moreover, for $\tau_0=-\tau_1\ell_2$ and $s_0=0$, Jacobi elliptic function solutions (40) are reduced to

$$q(x,t)=\left\{\frac{M_1}{R_1+\operatorname{sn}^2\left[N_j\left(x+\left\{\frac{\delta+2a\kappa}{\alpha}\right\}t^{\alpha}\right),\frac{(\ell_2-\ell_3)(\ell_1-\ell_4)}{(\ell_1-\ell_3)(\ell_2-\ell_4)}\right]}\right\}\times\exp\left[i\left\{-\kappa x-\left(\frac{3\mu_3\tau_0\left(\kappa(\delta+a\kappa)-b_1\tau_0+\tau_0^2(\kappa\lambda-b_2)\right)+\mu_1\tau_1^2(b_1+3\tau_0(b_2-\kappa\lambda))}{3\alpha\mu_3\tau_0}\right)t^{\alpha}+\theta_0\right\}\right], \tag{50}$$

where

$$M_1=\frac{\tau_1(\ell_1-\ell_2)(\ell_4-\ell_2)}{\ell_1-\ell_4}, \tag{51}$$

$$R_1=\frac{\ell_4-\ell_2}{\ell_1-\ell_4}, \tag{52}$$

$$N_j=\frac{(-1)^j\sqrt{(\ell_1-\ell_3)(\ell_2-\ell_4)}}{2\Omega}\quad \text{for}\quad j=1,2. \tag{53}$$

Remark 1. When the modulus $m\to 1$, singular optical soliton solutions are emerge

$$q(x,t)=\left\{\frac{M_1}{R_1+\tanh^2\left[N_j\left(x+\left\{\frac{\delta+2a\kappa}{\alpha}\right\}t^{\alpha}\right)\right]}\right\}\times\exp\left[i\left\{-\kappa x-\left(\frac{3\mu_3\tau_0\left(\kappa(\delta+a\kappa)-b_1\tau_0+\tau_0^2(\kappa\lambda-b_2)\right)+\mu_1\tau_1^2(b_1+3\tau_0(b_2-\kappa\lambda))}{3\alpha\mu_3\tau_0}\right)t^{\alpha}+\theta_0\right\}\right], \tag{54}$$

for $\ell_3=\ell_4$.

Remark 2. However, if $m\to 0$, the following periodic singular solutions are procured:

$$q(x,t)=\left\{\frac{M_1}{R_1+\sin^2\left[N_j\left(x+\left\{\frac{\delta+2a\kappa}{\alpha}\right\}t^{\alpha}\right)\right]}\right\}\times\exp\left[i\left\{-\kappa x-\left(\frac{3\mu_3\tau_0\left(\kappa(\delta+a\kappa)-b_1\tau_0+\tau_0^2(\kappa\lambda-b_2)\right)+\mu_1\tau_1^2(b_1+3\tau_0(b_2-\kappa\lambda))}{3\alpha\mu_3\tau_0}\right)t^{\alpha}+\theta_0\right\}\right],\tag{55}$$

for $\ell_2=\ell_3$.

3. Conclusions

This paper derived several forms of optical soliton solutions to the NLSE that is studied with QC nonlinearity in presence of perturbation terms including 3OD and 4OD dispersions. These are bright-singular combo solitons, bright and singular solitons. Two integration approaches revealed these soliton solutions. These are Kudryashov's method and Liu's extended trial equation method. The soliton solutions contain the parameter α that makes it move slowly, as we desire, to control the Internet traffic flow in any direction of propagation. The results of this paper thus accounts for engineering marvel that will be a great asset in telecommunications industry.

Conflict of interest

The authors also declare that there is no conflict of interest.

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