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Optical soliton perturbation for complex Ginzburg–Landau equation with modified simple equation method

Anjan Biswas[a,b,c], Yakup Yildirim[d], Emrullah Yasar[d], Houria Triki[e], Ali Saleh Alshomrani[b], Malik Zaka Ullah[b], Qin Zhou[f,*], Seithuti P. Moshokoa[c], Milivoj Belic[g]

[a] *Department of Physics, Chemistry and Mathematics, Alabama A&M University, Normal, AL 35762, USA*
[b] *Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia*
[c] *Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria 0008, South Africa*
[d] *Department of Mathematics, Faculty of Arts and Sciences, Uludag University, 16059 Bursa, Turkey*
[e] *Radiation Physics Laboratory, Department of Physics, Faculty of Sciences, Badji Mokhtar University, P. O. Box 12, 23000 Annaba, Algeria*
[f] *School of Electronics and Information Engineering, Wuhan Donghu University, Wuhan 430212, People's Republic of China*
[g] *Science Program, Texas A&M University at Qatar, PO Box 23874, Doha, Qatar*

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The modified simple equation method applied to perturbed complex Ginzburg–Landau equation gives dark and singular solutions to the model. The perturbation terms appear with full nonlinearity. There are eight nonlinear forms studied in this paper. These solitons appear with constraint conditions that gurantee its existence and these are also presented.

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1. Introduction

The complex Ginzburg–Landau equation (CGLE) is one of the many models that studies the dynamics of optical soliton propagation through a wide range of waveguides such as crystals, optical fibers, optical couplers, optical metamaterials and metasurfaces as well as PCF. This model is an extended version of the usual nonlinear Schrödinger's equation that is visible all across. These models and other nonlinear evolution equations, that arise in mathematical photonics as well as other areas of mathematical physics, are all successfully addressed by the modified simple equation method [1–10]. When perturbation terms are turned on, the perturbed CGLE will be addressed in this paper by the modified simple equation method. There are eight forms of nonlinear media that will be studied. All of these nonlinear forms will lead to the retrieval of dark and singular

* Corresponding author.
E-mail address: qinzhou@whu.edu.cn (Q. Zhou).

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soliton solutions to the governing model. The existence of these solitons will be guaranteed with the constraint conditions in the parameters. The perturbation terms are all of Hamiltonian type. After a quick recapitulation of this integration scheme, soliton solutions will be derived in the subsequent subsections.

1.1. The model

The dimensions form of CGLE is [2–4,6–8]:

$$iq_t + aq_{xx} + bF\left(|q|^2\right)q = \frac{1}{|q|^2 q^*}\left[\alpha|q|^2\left(|q|^2\right)_{xx} - \beta\left\{\left(|q|^2\right)_x\right\}^2\right] + \gamma q \tag{1}$$

where x is the spatial variable that represents the non-dimensional distance along the fiber, while t is the temporal variable. Then, a, b, α, β and γ are valued constants. The coefficients a and b come from the group velocity dispersion and nonlinearity, respectively. The terms with α and β are additional nonlinear terms and γ comes from detuning effect.

In (1), F is real-valued algebraic function and it is necessary to possess the smoothness of the complex function $F\left(|q|^2\right)q$ is k times continuously differentiable, so that

$$F\left(|q|^2\right)q \in \bigcup_{m,n=1}^{\infty} C^k((-n,n) \quad \times \quad (-m,m); \quad R^2).$$

In presence of perturbation terms, CGLE gets extended to [4]:

$$iq_t + aq_{xx} + bF\left(|q|^2\right)q = \frac{1}{|q|^2 q^*}\left[\alpha|q|^2\left(|q|^2\right)_{xx} - \beta\left\{\left(|q|^2\right)_x\right\}^2\right] + \gamma q + i\left[\delta q_x + \lambda\left(|q|^{2m}q\right)_x + \mu\left(|q|^{2m}\right)_x q\right] \tag{2}$$

where δ is the inter-modal dispersion that arises in addition to chromatic dispersion, λ represents the self-steepening effect for short pulses and μ is the higher–order dispersion coefficient. The parameter m accounts for full nonlinearity.

2. Revisitation of the integration algorithm

Suppose we have a nonlinear evolution equation in the form [1,5,9,10]:

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \ldots) = 0 \tag{3}$$

where P is a polynomial in $u(x, t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method.

Step-1: We use the transformation

$$u(x,t) = U(\xi), \quad \xi = x - ct, \tag{4}$$

where c is a constant to be determined, to reduce Eq. (3) to the following ODE:

$$Q\left(U, U', U'', U''', \ldots\right) = 0 \tag{5}$$

where Q is a polynomial in $U(\xi)$ and its total derivatives, while $' = d/d\xi$.

Step-2: We suppose that Eq. (5) has the formal solution

$$U(\xi) = \sum_{l=0}^{N} a_l \left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^l, \tag{6}$$

where a_l are constants to be determined, such that $a_N \neq 0$, and $\psi(\xi)$ is an unknown function to be determined later.

Step-3: We determine the positive integer N in Eq. (6) by considering the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. (5).

Step-4: We substitute (6) into (5), then we calculate all the necessary derivatives $U', U'', \ldots$ of the unknown function $U(\xi)$ and we account the function $U(\xi)$. As a result of this substitution, we get a polynomial of $\psi'(\xi)/\psi(\xi)$ and its derivatives. In this polynomial, we gather all the terms of the same power of $\psi^{-j}(\xi)$, $j = 0, 1, 2, \ldots$ and its derivatives, and we equate with zero all the coefficients of this polynomial. This operation yields a system of equations which can be solved to find a_k and $\psi(\xi)$. Consequently, once can retrieve the exact solutions of Eq. (3).

3. Soliton solutions

In order to solve Eq. (2) by the trial equation method, we start with the following wave transformation

$$q(x,t) = U(\xi)e^{i\phi(x,t)}, \quad q^*(x,t) = U(\xi)e^{-i\phi(x,t)} \tag{7}$$

where $U(\xi)$ represents the shape of the pulse, $\xi = x - vt$ and $\phi = -\kappa x + \omega t + \theta$. The function $\phi(x, t)$ is the phase component of the soliton, κ is the soliton frequency, while ω is the wave number, θ is the phase constant and v is the velocity of the soliton.

Substituting Eq. (7) into Eq. (2) and then decomposing into real and imaginary parts yields a pair of relations. The imaginary part gives

$$v+\delta+2a\kappa+((2m+1)\lambda+2m\mu)U^{2m}=0 \tag{8}$$

while the real part gives

$$(a-2\alpha)U''-\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)U-\kappa\lambda U^{2m+1}+bF\left(U^2\right)U-2\left(\alpha-2\beta\right)\frac{(U')^2}{U}=0. \tag{9}$$

By choosing

$$\alpha=2\beta$$

the last term of Eq. (9) vanishes. Therefore Eq. (2) changes to

$$iq_t+aq_{xx}+bF\left(|q|^2\right)q=\frac{\beta}{|q|^2q^*}\left[2|q|^2\left(|q|^2\right)_{xx}-\left\{\left(|q|^2\right)_x\right\}^2\right]+\gamma q+i\left[\delta q_x+\lambda\left(|q|^{2m}q\right)_x+\mu\left(|q|^{2m}\right)_x q\right] \tag{10}$$

and Eq. (9) condenses to

$$\left(a-4\beta\right)U''-\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)U-\kappa\lambda U^{2m+1}+bF\left(U^2\right)U=0. \tag{11}$$

The imaginary part equation implies

$$v=-2a\kappa-\delta \tag{12}$$

and

$$(2m+1)\lambda+2m\mu=0. \tag{13}$$

Eq. (12) gives the velocity of the soliton and Eq. (13) gives the constraint relation between the two perturbation terms, while Eq. (11) can be integrated to determine the soliton profile. This form for the velocity remains the same for all types of nonlinearity, $F(s)$.

3.1. Kerr law

In this case,

$$F(s)=s$$

so that Eq. (10) reduces to

$$iq_t+aq_{xx}+b|q|^2q=\frac{\beta}{|q|^2q^*}\left[2|q|^2\left(|q|^2\right)_{xx}-\left\{\left(|q|^2\right)_x\right\}^2\right]+\gamma q+i\left[\delta q_x+\lambda\left(|q|^{2m}q\right)_x+\mu\left(|q|^{2m}\right)_x q\right] \tag{14}$$

and Eq. (11) simplifies to

$$\left(a-4\beta\right)U''-\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)U-\kappa\lambda U^{2m+1}+bU^3=0. \tag{15}$$

Balancing U'' with U^3 in Eq. (15), then we get $N=1$. Consequently, we reach

$$U\left(\xi\right)=a_0+a_1\left(\frac{\psi'\left(\xi\right)}{\psi\left(\xi\right)}\right). \tag{16}$$

Substituting Eq. (16) into Eq. (15) along with $m=1$ and setting the coefficients of $\psi^{-j}, j=0,1,2,3$ to zero, then we obtain a set of algebraic equations. Solving this system, we obtain

$$a_0=\pm\sqrt{-\frac{\omega+a\kappa^2+\gamma+\delta\kappa}{\kappa\lambda-b}},\quad a_1=\pm\sqrt{\frac{2\left(a-4\beta\right)}{\kappa\lambda-b}}, \tag{17}$$

and

$$\psi''=\pm\sqrt{-\frac{2\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)}{a-4\beta}}\psi', \tag{18}$$

$$\psi'''=-\frac{2\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)}{a-4\beta}\psi'. \tag{19}$$

From Eqs. (18) and (19), we can deduce that

$$\psi' = \pm\sqrt{-\frac{a-4\beta}{2\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)}}k_1 e^{\pm\sqrt{-\frac{2\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)}{a-4\beta}}\xi}, \tag{20}$$

and

$$\psi = -\frac{a-4\beta}{2\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)}k_1 e^{\pm\sqrt{-\frac{2\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)}{a-4\beta}}\xi} + k_2, \tag{21}$$

where k_1 and k_2 are constants of integration. Substituting Eqs. (20) and (21) into Eq. (16), we obtain following exact solution to Eq. (14)

$$q(x,t) = \left\{\pm\sqrt{-\frac{\omega+a\kappa^2+\gamma+\delta\kappa}{\kappa\lambda-b}} \pm \sqrt{\frac{2\left(a-4\beta\right)}{\kappa\lambda-b}}\left(\frac{\pm\sqrt{-\frac{a-4\beta}{2(\omega+a\kappa^2+\gamma+\delta\kappa)}}k_1 e^{\pm\sqrt{-\frac{2\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)}{a-4\beta}}(x-vt)}}{-\frac{a-4\beta}{2(\omega+a\kappa^2+\gamma+\delta\kappa)}k_1 e^{\pm\sqrt{-\frac{2\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)}{a-4\beta}}(x-vt)} + k_2}\right)\right\} e^{i(-\kappa x+\omega t+\theta)}$$

If we set

$$k_1 = -\frac{2\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)}{a-4\beta}e^{\pm\sqrt{-\frac{2\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)}{a-4\beta}}\xi_0}, \quad k_2 = \pm 1$$

we obtain:

$$q(x,t) = \pm\sqrt{-\frac{\omega+a\kappa^2+\gamma+\delta\kappa}{\kappa\lambda-b}}\tanh\left[\sqrt{-\frac{\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)}{2\left(a-4\beta\right)}}\left(x-vt+\xi_0\right)\right]e^{i(-\kappa x+\omega t+\theta)}, \tag{22}$$

$$q(x,t) = \pm\sqrt{-\frac{\omega+a\kappa^2+\gamma+\delta\kappa}{\kappa\lambda-b}}\coth\left[\sqrt{-\frac{\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)}{2\left(a-4\beta\right)}}\left(x-vt+\xi_0\right)\right]e^{i(-\kappa x+\omega t+\theta)}, \tag{23}$$

where (22) and (23) represent dark soliton and singular soliton solutions respectively. These solitons are valid for

$$\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)\left(a-4\beta\right) < 0.$$

$$q(x,t) = \pm\sqrt{\frac{\omega+a\kappa^2+\gamma+\delta\kappa}{\kappa\lambda-b}}\tan\left[\sqrt{\frac{\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)}{2\left(a-4\beta\right)}}\left(x-vt+\xi_0\right)\right]e^{i(-\kappa x+\omega t+\theta)}, \tag{24}$$

$$q(x,t) = \pm\sqrt{\frac{\omega+a\kappa^2+\gamma+\delta\kappa}{\kappa\lambda-b}}\cot\left[\sqrt{\frac{\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)}{2\left(a-4\beta\right)}}\left(x-vt+\xi_0\right)\right]e^{i(-\kappa x+\omega t+\theta)}, \tag{25}$$

where (24) and (25) represent singular periodic solutions. These solutions are valid for

$$\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)\left(a-4\beta\right) > 0.$$

3.2. Power law

In this case,

$$F(s) = s^n$$

so that Eq. (10) reduces to

$$iq_t + aq_{xx} + b|q|^{2n}q = \frac{\beta}{|q|^2 q^*}\left[2|q|^2\left(|q|^2\right)_{xx} - \left\{\left(|q|^2\right)_x\right\}^2\right] + \gamma q + i\left[\delta q_x + \lambda\left(|q|^{2m}q\right)_x + \mu\left(|q|^{2m}\right)_x q\right] \tag{26}$$

aand Eq. (11) simplifies to

$$\left(a-4\beta\right)U'' - \left(\omega+a\kappa^2+\gamma+\delta\kappa\right)U - \kappa\lambda U^{2m+1} + bU^{2n+1} = 0. \tag{27}$$

By using transformation $U = V^{\frac{1}{2n}}$, Eq. (27) becomes

$$\left(a-4\beta\right)\left((1-2n)\left(V'\right)^2+2nVV''\right)-4n^2V^2\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)-4n^2\kappa\lambda V^{\frac{m}{n}+2}+4n^2bV^3=0. \quad (28)$$

Balancing VV'' or $(V')^2$ with $V^{\frac{m}{n}+2}$ in Eq. (28), then we get $N=\frac{2n}{m}$. Setting $m=2n$, we reach

$$V\left(\xi\right)=a_0+a_1\left(\frac{\psi'\left(\xi\right)}{\psi\left(\xi\right)}\right). \quad (29)$$

Substituting Eq. (29) into Eq. (28) and setting the coefficients of ψ^{-j}, $j=0,1,2,3,4$ to zero, then we obtain a set of algebraic equations. Solving this system, we obtain

$$a_0=\frac{(2n+1)b}{2(n+1)\kappa\lambda},\quad a_1=\pm\frac{1}{2n}\sqrt{\frac{(2n+1)\left(a-4\beta\right)}{\kappa\lambda}}, \quad (30)$$

$$\omega=-\frac{1}{4\kappa\lambda(n+1)^2}\left(4\kappa^3a\lambda n^2+8\kappa^3a\lambda n+4\kappa^2\delta\lambda n^2+4\kappa^3a\lambda+8\kappa^2\delta\lambda n\right.$$
$$\left.+4\kappa\gamma\lambda n^2+4\kappa^2\delta\lambda+8\kappa\gamma\lambda n+4\kappa\gamma\lambda-2b^2n-b^2\right), \quad (31)$$

and

$$\psi''=\pm\frac{nb}{n+1}\sqrt{\frac{2n+1}{\left(a-4\beta\right)\kappa\lambda}}\psi', \quad (32)$$

$$\psi'''=\frac{n^2b^2(2n+1)}{(n+1)^2\left(a-4\beta\right)\kappa\lambda}\psi'. \quad (33)$$

From Eqs. (32) and (33), we can deduce that

$$\psi'=\pm\frac{n+1}{nb}\sqrt{\frac{\left(a-4\beta\right)\kappa\lambda}{2n+1}}k_1e^{\pm\frac{nb}{n+1}\sqrt{\frac{2n+1}{(a-4\beta)\kappa\lambda}}\xi}, \quad (34)$$

and

$$\psi=\frac{(n+1)^2\left(a-4\beta\right)\kappa\lambda}{n^2b^2(2n+1)}k_1e^{\pm\frac{nb}{n+1}\sqrt{\frac{2n+1}{(a-4\beta)\kappa\lambda}}\xi}+k_2, \quad (35)$$

where k_1 and k_2 are constants of integration. Substituting Eqs. (34) and (35) into Eq. (29), we obtain following exact solution to Eq. (26)

$$q(x,t)=\left\{\frac{(2n+1)b}{2(n+1)\kappa\lambda}\pm\frac{1}{2n}\sqrt{\frac{(2n+1)\left(a-4\beta\right)}{\kappa\lambda}}\left(\frac{\pm\frac{n+1}{nb}\sqrt{\frac{(a-4\beta)\kappa\lambda}{2n+1}}k_1e^{\pm\frac{nb}{n+1}\sqrt{\frac{2n+1}{(a-4\beta)\kappa\lambda}}(x-vt)}}{\frac{(n+1)^2(a-4\beta)\kappa\lambda}{n^2b^2(2n+1)}k_1e^{\pm\frac{nb}{n+1}\sqrt{\frac{2n+1}{(a-4\beta)\kappa\lambda}}(x-vt)}+k_2}\right)\right\}^{\frac{1}{2n}}e^{i(-\kappa x+\omega t+\theta)}$$

where ω is given by Eq. (97). If we set

$$k_1=\frac{n^2b^2(2n+1)}{(n+1)^2\left(a-4\beta\right)\kappa\lambda}e^{\pm\frac{nb}{n+1}\sqrt{\frac{2n+1}{(a-4\beta)\kappa\lambda}}\xi_0},\quad k_2=\pm1$$

we obtain:

$$q(x,t)=\left\{\frac{(2n+1)b}{4(n+1)\kappa\lambda}\left(1\pm\tanh\left[\frac{nb}{2(n+1)}\sqrt{\frac{2n+1}{\left(a-4\beta\right)\kappa\lambda}}\left(x-vt+\xi_0\right)\right]\right)\right\}^{\frac{1}{2n}}e^{i(-\kappa x+\omega t+\theta)}, \quad (36)$$

$$q(x,t)=\left\{\frac{(2n+1)b}{4(n+1)\kappa\lambda}\left(1\pm\coth\left[\frac{nb}{2(n+1)}\sqrt{\frac{2n+1}{\left(a-4\beta\right)\kappa\lambda}}\left(x-vt+\xi_0\right)\right]\right)\right\}^{\frac{1}{2n}}e^{i(-\kappa x+\omega t+\theta)}, \quad (37)$$

where Eqs. (36) and (37) represent dark soliton and singular soliton solutions respectively. These solitons are valid for

$$\left(a-4\beta\right)\kappa\lambda>0.$$

3.3. Parabolic law

In this case,

$$F(s) = b_1 s + b_2 s^2$$

where b_1 and b_2 are constants. Therefore, Eq. (10) reduces to

$$iq_t + aq_{xx} + \left(b_1|q|^2 + b_2|q|^4\right) q = \frac{\beta}{|q|^2 q^*}\left[2|q|^2\left(|q|^2\right)_{xx} - \left\{\left(|q|^2\right)_x\right\}^2\right] + \gamma q + i\left[\delta q_x + \lambda\left(|q|^{2m}q\right)_x + \mu\left(|q|^{2m}\right)_x q\right] \tag{38}$$

and Eq. (11) simplifies to

$$\left(a - 4\beta\right) U'' - \left(\omega + a\kappa^2 + \gamma + \delta\kappa\right) U - \kappa\lambda U^{2m+1} + b_1 U^3 + b_2 U^5 = 0. \tag{39}$$

By using transformation $U = V^{\frac{1}{2}}$, Eq. (39) becomes

$$\left(a - 4\beta\right)\left(-\left(V'\right)^2 + 2VV''\right) - 4V^2\left(\omega + a\kappa^2 + \gamma + \delta\kappa\right) - 4\kappa\lambda V^{m+2} + 4\left(b_1 V^3 + b_2 V^4\right) = 0. \tag{40}$$

Balancing VV'' or $(V')^2$ with V^4 in Eq. (40), then we get $N = 1$. Consequently, we reach

$$V\left(\xi\right) = a_0 + a_1\left(\frac{\psi'\left(\xi\right)}{\psi\left(\xi\right)}\right). \tag{41}$$

Case 1. Substituting Eq. (41) into Eq. (40) along with $m = 1$ and setting the coefficients of ψ^{-j}, $j = 0,1,2,3,4$ to zero, then we obtain a set of algebraic equations. Solving this system, we obtain

$$a_0 = \frac{3\left(\kappa\lambda - b_1\right)}{4b_2}, \quad a_1 = \pm\frac{1}{2}\sqrt{-\frac{3\left(a - 4\beta\right)}{b_2}}, \tag{42}$$

$$\omega = -\frac{16\kappa^2 a b_2 + 3\kappa^2\lambda^2 + 16\kappa\delta b_2 - 6\kappa\lambda b_1 + 16\gamma b_2 + 3b_1^2}{16b_2}, \tag{43}$$

and

$$\psi'' = \pm\frac{3\left(\kappa\lambda - b_1\right)}{2\sqrt{-3\left(a - 4\beta\right) b_2}}\psi', \tag{44}$$

$$\psi''' = -\frac{3(\kappa\lambda - b_1)^2}{4b_2\left(a - 4\beta\right)}\psi'. \tag{45}$$

From Eqs. (44) and (45), we can deduce that

$$\psi' = \pm\frac{2\sqrt{-3\left(a - 4\beta\right) b_2}}{3\left(\kappa\lambda - b_1\right)} k_1 e^{\pm\frac{3(\kappa\lambda - b_1)}{2\sqrt{-3(a-4\beta)b_2}}\xi}, \tag{46}$$

and

$$\psi = -\frac{4b_2\left(a - 4\beta\right)}{3(\kappa\lambda - b_1)^2} k_1 e^{\pm\frac{3(\kappa\lambda - b_1)}{2\sqrt{-3(a-4\beta)b_2}}\xi} + k_2, \tag{47}$$

where k_1 and k_2 are constants of integration. Substituting Eqs. (46) and (47) into Eq. (41), we obtain following exact solution to Eq. (38)

$$q(x,t) = \left\{\frac{3\left(\kappa\lambda - b_1\right)}{4b_2} \pm \frac{1}{2}\sqrt{-\frac{3\left(a - 4\beta\right)}{b_2}}\left(\frac{\pm\frac{2\sqrt{-3(a-4\beta)b_2}}{3(\kappa\lambda - b_1)} k_1 e^{\pm\frac{3(\kappa\lambda - b_1)}{2\sqrt{-3(a-4\beta)b_2}}(x - vt)}}{-\frac{4b_2(a-4\beta)}{3(\kappa\lambda - b_1)^2} k_1 e^{\pm\frac{3(\kappa\lambda - b_1)}{2\sqrt{-3(a-4\beta)b_2}}(x - vt)} + k_2}\right)\right\}^{\frac{1}{2}} e^{i\left(-\kappa x + \omega t + \theta\right)}$$

where ω is given by Eq. (43). If we set

$$k_1 = -\frac{3(\kappa\lambda - b_1)^2}{4b_2\left(a - 4\beta\right)} e^{\pm\frac{3(\kappa\lambda - b_1)}{2\sqrt{-3(a-4\beta)b_2}}\xi_0}, \quad k_2 = \pm 1$$

we obtain:

$$q(x,t)=\left\{\frac{3(\kappa\lambda-b_1)}{8b_2}\left(1\pm\tanh\left[\frac{3(\kappa\lambda-b_1)}{4\sqrt{-3(a-4\beta)b_2}}(x-vt+\xi_0)\right]\right)\right\}^{\frac{1}{2}}e^{i(-\kappa x+\omega t+\theta)}, \tag{48}$$

$$q(x,t)=\left\{\frac{3(\kappa\lambda-b_1)}{8b_2}\left(1\pm\coth\left[\frac{3(\kappa\lambda-b_1)}{4\sqrt{-3(a-4\beta)b_2}}(x-vt+\xi_0)\right]\right)\right\}^{\frac{1}{2}}e^{i(-\kappa x+\omega t+\theta)}, \tag{49}$$

where Eqs. (48) and (49) represent dark soliton and singular soliton solutions respectively. These solitons are valid for

$(a-4\beta)b_2<0.$

Case 2. Substituting Eq. (41) into Eq. (40) along with $m=2$ and setting the coefficients of ψ^{-j}, $j=0,1,2,3,4$ to zero, then we obtain a set of algebraic equations. Solving this system, we obtain

$$a_0=\frac{3b_1}{4(\kappa\lambda-b_2)},\quad a_1=\pm\frac{\sqrt{3}}{2}\sqrt{\frac{a-4\beta}{\kappa\lambda-b_2}}, \tag{50}$$

$$\omega=-\frac{16\kappa^3a\lambda-16\kappa^2ab_2+16\kappa^2\delta\lambda-16\kappa\delta b_2+16\kappa\gamma\lambda-16\gamma b_2-3b_1^2}{16(\kappa\lambda-b_2)}, \tag{51}$$

and

$$\psi''=\pm\frac{\sqrt{3}b_1}{2\sqrt{(a-4\beta)(\kappa\lambda-b_2)}}\psi', \tag{52}$$

$$\psi'''=\frac{3b_1^2}{4(a-4\beta)(\kappa\lambda-b_2)}\psi'. \tag{53}$$

From Eqs. (52) and (53), we can deduce that

$$\psi'=\pm\frac{2\sqrt{(a-4\beta)(\kappa\lambda-b_2)}}{\sqrt{3}b_1}k_1e^{\pm\frac{\sqrt{3}b_1}{2\sqrt{(a-4\beta)(\kappa\lambda-b_2)}}\xi}, \tag{54}$$

and

$$\psi=\frac{4(a-4\beta)(\kappa\lambda-b_2)}{3b_1^2}k_1e^{\pm\frac{\sqrt{3}b_1}{2\sqrt{(a-4\beta)(\kappa\lambda-b_2)}}\xi}+k_2, \tag{55}$$

where k_1 and k_2 are constants of integration. Substituting Eqs. (54) and (55) into Eq. (41), we obtain following exact solution to Eq. (38)

$$q(x,t)=\left\{\frac{3b_1}{4(\kappa\lambda-b_2)}\pm\frac{\sqrt{3}}{2}\sqrt{\frac{a-4\beta}{\kappa\lambda-b_2}}\left(\frac{\pm\frac{2\sqrt{(a-4\beta)(\kappa\lambda-b_2)}}{\sqrt{3}b_1}k_1e^{\pm\frac{\sqrt{3}b_1}{2\sqrt{(a-4\beta)(\kappa\lambda-b_2)}}(x-vt)}}{\frac{4(a-4\beta)(\kappa\lambda-b_2)}{3b_1^2}k_1e^{\pm\frac{\sqrt{3}b_1}{2\sqrt{(a-4\beta)(\kappa\lambda-b_2)}}(x-vt)}+k_2}\right)\right\}^{\frac{1}{2}}e^{i(-\kappa x+\omega t+\theta)}$$

where ω is given by Eq. (51). If we set

$$k_1=\frac{3b_1^2}{4(a-4\beta)(\kappa\lambda-b_2)}e^{\pm\frac{\sqrt{3}b_1}{2\sqrt{(a-4\beta)(\kappa\lambda-b_2)}}\xi_0},\quad k_2=\pm1$$

we obtain:

$$q(x,t)=\left\{\frac{3b_1}{8(\kappa\lambda-b_2)}\left(1\pm\tanh\left[\frac{\sqrt{3}b_1}{4\sqrt{(a-4\beta)(\kappa\lambda-b_2)}}(x-vt+\xi_0)\right]\right)\right\}^{\frac{1}{2}}e^{i(-\kappa x+\omega t+\theta)}, \tag{56}$$

$$q(x,t)=\left\{\frac{3b_1}{8(\kappa\lambda-b_2)}\left(1\pm\coth\left[\frac{\sqrt{3}b_1}{4\sqrt{(a-4\beta)(\kappa\lambda-b_2)}}\left(x-vt+\xi_0\right)\right]\right)\right\}^{\frac{1}{2}}e^{i(-\kappa x+\omega t+\theta)}, \quad (57)$$

where Eqs. (56) and (57) represent dark soliton and singular soliton solutions respectively. These solitons are valid for

$$(a-4\beta)(\kappa\lambda-b_2)>0.$$

3.4. Dual-power law

In this case,

$$F(s)=b_1s^n+b_2s^{2n}$$

where b_1 and b_2 are constants. Therefore, Eq. (10) reduces to

$$iq_t+aq_{xx}+\left(b_1|q|^{2n}+b_2|q|^{4n}\right)q=\frac{\beta}{|q|^2q^*}\left[2|q|^2\left(|q|^2\right)_{xx}-\left\{\left(|q|^2\right)_x\right\}^2\right]+\gamma q+i\left[\delta q_x+\lambda\left(|q|^{2m}q\right)_x+\mu\left(|q|^{2m}\right)_xq\right] \quad (58)$$

and Eq. (11) simplifies to

$$(a-4\beta)U''-\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)U-\kappa\lambda U^{2m+1}+b_1U^{2n+1}+b_2U^{4n+1}=0. \quad (59)$$

By using transformation $U=V^{\frac{1}{2n}}$, Eq. (59) becomes

$$(a-4\beta)\left((1-2n)\left(V'\right)^2+2nVV''\right)-4n^2V^2\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)-4n^2\kappa\lambda V^{\frac{m}{n}+2}+4n^2\left(b_1V^3+b_2V^4\right)=0. \quad (60)$$

Balancing VV'' or $(V')^2$ with V^4 in Eq. (60), then we get $N=1$. Consequently, we reach

$$V(\xi)=a_0+a_1\left(\frac{\psi'(\xi)}{\psi(\xi)}\right). \quad (61)$$

Case 1. Substituting Eq. (61) into Eq. (60) along with $m=n$ and setting the coefficients of ψ^{-j}, $j=0,1,2,3,4$ to zero, then we obtain a set of algebraic equations. Solving this system, we obtain

$$a_0=\frac{(2n+1)(\kappa\lambda-b_1)}{2(n+1)b_2},\quad a_1=\pm\frac{1}{2n}\sqrt{-\frac{(2n+1)(a-4\beta)}{b_2}}, \quad (62)$$

$$\begin{aligned}\omega\ =&-\frac{1}{4b_2(n+1)^2}\left(4\kappa^2an^2b_2+8\kappa^2anb_2+2\kappa^2\lambda^2n+4\kappa\delta n^2b_2+4\kappa^2ab_2+\kappa^2\lambda^2\right.\\&\left.+8\kappa\delta nb_2-4\kappa\lambda nb_1+4\gamma n^2b_2+4\kappa\delta b_2-2\kappa\lambda b_1+8\gamma nb_2+2nb_1^2+4\gamma b_2+b_1^2\right),\end{aligned} \quad (63)$$

and

$$\psi''=\pm\frac{n(\kappa\lambda-b_1)}{(n+1)}\sqrt{-\frac{2n+1}{(a-4\beta)b_2}}\psi', \quad (64)$$

$$\psi'''=-\frac{n^2(\kappa\lambda-b_1)^2(2n+1)}{(n+1)^2(a-4\beta)b_2}\psi'. \quad (65)$$

From Eqs. (64) and (65), we can deduce that

$$\psi'=\pm\frac{(n+1)}{n(\kappa\lambda-b_1)}\sqrt{-\frac{(a-4\beta)b_2}{2n+1}}k_1e^{\pm\frac{n(\kappa\lambda-b_1)}{(n+1)}\sqrt{-\frac{2n+1}{(a-4\beta)b_2}}\xi}, \quad (66)$$

and

$$\psi=-\frac{(n+1)^2(a-4\beta)b_2}{n^2(\kappa\lambda-b_1)^2(2n+1)}k_1e^{\pm\frac{n(\kappa\lambda-b_1)}{(n+1)}\sqrt{-\frac{2n+1}{(a-4\beta)b_2}}\xi}+k_2, \quad (67)$$

where k_1 and k_2 are constants of integration. Substituting Eqs. (66) and (67) into Eq. (61), we obtain following exact solution to Eq. (58)

$$q(x,t)=\left\{\frac{(2n+1)(\kappa\lambda-b_1)}{2(n+1)b_2}\pm\frac{1}{2n}\sqrt{-\frac{(2n+1)\left(a-4\beta\right)}{b_2}}\left(\frac{\pm\frac{(n+1)}{n(\kappa\lambda-b_1)}\sqrt{-\frac{(a-4\beta)b_2}{2n+1}}k_1e^{\pm\frac{n(\kappa\lambda-b_1)}{(n+1)}\sqrt{-\frac{2n+1}{(a-4\beta)b_2}}(x-vt)}}{-\frac{(n+1)^2(a-4\beta)b_2}{n^2(\kappa\lambda-b_1)^2(2n+1)}k_1e^{\pm\frac{n(\kappa\lambda-b_1)}{(n+1)}\sqrt{-\frac{2n+1}{(a-4\beta)b_2}}(x-vt)}+k_2}\right)\right\}^{\frac{1}{2n}}$$

$$\times e^{i(-\kappa x+\omega t+\theta)}$$

where ω is given by Eq. (63). If we set

$$k_1=-\frac{n^2(\kappa\lambda-b_1)^2(2n+1)}{(n+1)^2\left(a-4\beta\right)b_2}e^{\pm\frac{n(\kappa\lambda-b_1)}{(n+1)}\sqrt{-\frac{2n+1}{(a-4\beta)b_2}}\xi_0},\quad k_2=\pm1$$

we obtain:

$$q(x,t)=\left\{\frac{(2n+1)(\kappa\lambda-b_1)}{4(n+1)b_2}\left(1\pm\tanh\left[\frac{n(\kappa\lambda-b_1)}{2(n+1)}\sqrt{-\frac{2n+1}{\left(a-4\beta\right)b_2}}\left(x-vt+\xi_0\right)\right]\right)\right\}^{\frac{1}{2n}}e^{i(-\kappa x+\omega t+\theta)}, \tag{68}$$

$$q(x,t)=\left\{\frac{(2n+1)(\kappa\lambda-b_1)}{4(n+1)b_2}\left(1\pm\coth\left[\frac{n(\kappa\lambda-b_1)}{2(n+1)}\sqrt{-\frac{2n+1}{\left(a-4\beta\right)b_2}}\left(x-vt+\xi_0\right)\right]\right)\right\}^{\frac{1}{2n}}e^{i(-\kappa x+\omega t+\theta)}, \tag{69}$$

where Eqs. (68) and (69) represent dark soliton and singular soliton solutions respectively. These solitons are valid for

$$\left(a-4\beta\right)b_2<0.$$

Case 2. Substituting Eq. (61) into Eq. (60) along with $m=2n$ and setting the coefficients of $\psi^{-j}, j=0,1,2,3,4$ to zero, then we obtain a set of algebraic equations. Solving this system, we obtain

$$a_0=\frac{(2n+1)b_1}{2(n+1)(\kappa\lambda-b_2)},\quad a_1=\pm\frac{1}{2n}\sqrt{\frac{(2n+1)\left(a-4\beta\right)}{\kappa\lambda-b_2}}, \tag{70}$$

$$\begin{aligned}\omega&=-\frac{1}{4(n+1)^2(\kappa\lambda-b_2)}\left(4\kappa^3a\lambda n^2+8\kappa^3a\lambda n-4\kappa^2an^2b_2+4\kappa^2\delta\lambda n^2\right.\\&+4\kappa^3a\lambda-8\kappa^2anb_2+8\kappa^2\delta\lambda n-4\kappa\delta n^2b_2+4\kappa\gamma\lambda n^2-4\kappa^2ab_2+4\kappa^2\delta\lambda\\&\left.-8\kappa\delta nb_2+8\kappa\gamma\lambda n-4\gamma n^2b_2-4\kappa\delta b_2+4\kappa\gamma\lambda-8\gamma nb_2-2nb_1^2-4\gamma b_2-b_1^2\right),\end{aligned} \tag{71}$$

and

$$\psi''=\pm\frac{b_1n\sqrt{2n+1}}{(n+1)\sqrt{\left(a-4\beta\right)(\kappa\lambda-b_2)}}\psi', \tag{72}$$

$$\psi'''=\frac{b_1^2n^2(2n+1)}{(n+1)^2\left(a-4\beta\right)(\kappa\lambda-b_2)}\psi'. \tag{73}$$

From Eqs. (72) and (73), we can deduce that

$$\psi'=\pm\frac{(n+1)\sqrt{\left(a-4\beta\right)(\kappa\lambda-b_2)}}{b_1n\sqrt{2n+1}}k_1e^{\pm\frac{b_1n\sqrt{2n+1}}{(n+1)\sqrt{(a-4\beta)(\kappa\lambda-b_2)}}\xi}, \tag{74}$$

and

$$\psi=\frac{(n+1)^2\left(a-4\beta\right)(\kappa\lambda-b_2)}{b_1^2n^2(2n+1)}k_1e^{\pm\frac{b_1n\sqrt{2n+1}}{(n+1)\sqrt{(a-4\beta)(\kappa\lambda-b_2)}}\xi}+k_2, \tag{75}$$

where k_1 and k_2 are constants of integration. Substituting Eqs. (74) and (75) into Eq. (61), we obtain following exact solution to Eq. (58)

$$q(x,t) = \left\{ \frac{(2n+1)b_1}{2(n+1)(\kappa\lambda - b_2)} \pm \frac{1}{2n}\sqrt{\frac{(2n+1)\left(a-4\beta\right)}{\kappa\lambda - b_2}} \left(\frac{\pm \frac{(n+1)\sqrt{\left(a-4\beta\right)(\kappa\lambda-b_2)}}{b_1 n\sqrt{2n+1}} k_1 e^{\pm \frac{b_1 n\sqrt{2n+1}}{(n+1)\sqrt{\left(a-4\beta\right)(\kappa\lambda-b_2)}}(x-vt)}}{\frac{(n+1)^2\left(a-4\beta\right)(\kappa\lambda-b_2)}{b_1^2 n^2(2n+1)} k_1 e^{\pm \frac{b_1 n\sqrt{2n+1}}{(n+1)\sqrt{\left(a-4\beta\right)(\kappa\lambda-b_2)}}(x-vt)} + k_2} \right) \right\}^{\frac{1}{2n}} \times e^{i\left(-\kappa x+\omega t+\theta\right)}$$

where ω is given by Eq. (71). If we set

$$k_1 = \frac{b_1^2 n^2 (2n+1)}{(n+1)^2\left(a-4\beta\right)(\kappa\lambda-b_2)} e^{\pm \frac{b_1 n\sqrt{2n+1}}{(n+1)\sqrt{(a-4\beta)(\kappa\lambda-b_2)}}\xi_0}, \quad k_2 = \pm 1$$

we obtain:

$$q(x,t) = \left\{ \frac{(2n+1)b_1}{4(n+1)(\kappa\lambda-b_2)} \left(1 \pm \tanh\left[\frac{b_1 n\sqrt{2n+1}}{2(n+1)\sqrt{\left(a-4\beta\right)(\kappa\lambda-b_2)}} \left(x-vt+\xi_0\right) \right] \right) \right\}^{\frac{1}{2n}} e^{i\left(-\kappa x+\omega t+\theta\right)}, \quad (76)$$

$$q(x,t) = \left\{ \frac{(2n+1)b_1}{4(n+1)(\kappa\lambda-b_2)} \left(1 \pm \coth\left[\frac{b_1 n\sqrt{2n+1}}{2(n+1)\sqrt{\left(a-4\beta\right)(\kappa\lambda-b_2)}} \left(x-vt+\xi_0\right) \right] \right) \right\}^{\frac{1}{2n}} e^{i\left(-\kappa x+\omega t+\theta\right)}, \quad (77)$$

where Eqs. (76) and (77) represent dark soliton and singular soliton solutions respectively. These solitons are valid for

$$\left(a-4\beta\right)(\kappa\lambda-b_2) > 0.$$

3.5. Quadratic-cubic law

In this case,

$$F(s) = b_1\sqrt{s} + b_2 s$$

where b_1 and b_2 are constants. Therefore, Eq. (10) reduces to

$$iq_t + aq_{xx} + \left(b_1|q| + b_2|q|^2\right)q = \frac{\beta}{|q|^2 q^*}\left[2|q|^2\left(|q|^2\right)_{xx} - \left\{\left(|q|^2\right)_x\right\}^2\right] + \gamma q + i\left[\delta q_x + \lambda\left(|q|^{2m}q\right)_x + \mu\left(|q|^{2m}\right)_x q\right] \quad (78)$$

and Eq. (11) simplifies to

$$\left(a-4\beta\right)U'' - \left(\omega + a\kappa^2 + \gamma + \delta\kappa\right)U - \kappa\lambda U^{2m+1} + b_1 U^2 + b_2 U^3 = 0. \quad (79)$$

Balancing U'' with U^3 in Eq. (79), then we get $N = 1$. Consequently, we reach

$$U\left(\xi\right) = a_0 + a_1\left(\frac{\psi'\left(\xi\right)}{\psi\left(\xi\right)}\right). \quad (80)$$

Substituting Eq. (80) into Eq. (79) along with $m = 1$ and setting the coefficients of $\psi^{-j}, j = 0,1,2,3$ to zero, then we obtain a set of algebraic equations. Solving this system, we obtain

$$a_0 = \frac{2b_1}{3(\kappa\lambda-b_2)}, \quad a_1 = \pm\sqrt{\frac{2\left(a-4\beta\right)}{\kappa\lambda-b_2}}, \quad (81)$$

$$\omega = -\frac{9\kappa^3 a\lambda - 9\kappa^2 ab_2 + 9\kappa^2\delta\lambda - 9\kappa\delta b_2 + 9\kappa\gamma\lambda - 9\gamma b_2 - 2b_1^2}{9(\kappa\lambda-b_2)}, \quad (82)$$

and

$$\psi'' = \pm \frac{\sqrt{2}b_1}{3\sqrt{(\kappa\lambda - b_2)\left(a - 4\beta\right)}}\psi', \tag{83}$$

$$\psi''' = \frac{2b_1^2}{9(\kappa\lambda - b_2)\left(a - 4\beta\right)}\psi'. \tag{84}$$

From Eqs. (83) and (84), we can deduce that

$$\psi' = \pm \frac{3\sqrt{(\kappa\lambda - b_2)\left(a - 4\beta\right)}}{\sqrt{2}b_1} k_1 e^{\pm \frac{\sqrt{2}b_1}{3\sqrt{(\kappa\lambda - b_2)(a-4\beta)}}\xi}, \tag{85}$$

and

$$\psi = \frac{9(\kappa\lambda - b_2)\left(a - 4\beta\right)}{2b_1^2} k_1 e^{\pm \frac{\sqrt{2}b_1}{3\sqrt{(\kappa\lambda - b_2)(a-4\beta)}}\xi} + k_2, \tag{86}$$

where k_1 and k_2 are constants of integration. Substituting Eqs. (85) and (86) into Eq. (80), we obtain following exact solution to Eq. (78)

$$q(x,t) = \left\{ \frac{2b_1}{3(\kappa\lambda - b_2)} \pm \sqrt{\frac{2\left(a - 4\beta\right)}{\kappa\lambda - b_2}} \left(\frac{\pm \frac{3\sqrt{(\kappa\lambda - b_2)(a-4\beta)}}{\sqrt{2}b_1} k_1 e^{\pm \frac{\sqrt{2}b_1}{3\sqrt{(\kappa\lambda - b_2)(a-4\beta)}}(x - vt)}}{\frac{9(\kappa\lambda - b_2)(a-4\beta)}{2b_1^2} k_1 e^{\pm \frac{\sqrt{2}b_1}{3\sqrt{(\kappa\lambda - b_2)(a-4\beta)}}(x - vt)} + k_2} \right) \right\} e^{i(-\kappa x + \omega t + \theta)}$$

where ω is given by Eq. (82). If we set

$$k_1 = \frac{2b_1^2}{9(\kappa\lambda - b_2)\left(a - 4\beta\right)} e^{\pm \frac{\sqrt{2}b_1}{3\sqrt{(\kappa\lambda - b_2)(a-4\beta)}}\xi_0}, \qquad k_2 = \pm 1$$

we obtain:

$$q(x,t) = \pm \frac{2b_1}{3(\kappa\lambda - b_2)} \tanh\left[\frac{\sqrt{2}b_1}{6\sqrt{(\kappa\lambda - b_2)\left(a - 4\beta\right)}} \left(x - vt + \xi_0\right) \right] e^{i(-\kappa x + \omega t + \theta)}, \tag{87}$$

$$q(x,t) = \pm \frac{2b_1}{3(\kappa\lambda - b_2)} \coth\left[\frac{\sqrt{2}b_1}{6\sqrt{(\kappa\lambda - b_2)\left(a - 4\beta\right)}} \left(x - vt + \xi_0\right) \right] e^{i(-\kappa x + \omega t + \theta)}, \tag{88}$$

where (87) and (88) represent dark soliton and singular soliton solutions respectively. These solitons are valid for

$$(\kappa\lambda - b_2)\left(a - 4\beta\right) > 0.$$

$$q(x,t) = \pm \frac{2b_1 i}{3(\kappa\lambda - b_2)} \tan\left[\frac{\sqrt{2}b_1}{6\sqrt{(b_2 - \kappa\lambda)\left(a - 4\beta\right)}} \left(x - vt + \xi_0\right) \right] e^{i(-\kappa x + \omega t + \theta)}, \tag{89}$$

$$q(x,t) = \pm \frac{2b_1 i}{3(\kappa\lambda - b_2)} \cot\left[\frac{\sqrt{2}b_1}{6\sqrt{(b_2 - \kappa\lambda)\left(a - 4\beta\right)}} \left(x - vt + \xi_0\right) \right] e^{i(-\kappa x + \omega t + \theta)}, \tag{90}$$

where (89) and (90) represent singular periodic solutions. These solutions are valid for

$$(b_2 - \kappa\lambda)\left(a - 4\beta\right) > 0.$$

3.6. Anti-cubic law

In this case,

$$F(s) = \frac{b_1}{s^2} + b_2 s + b_3 s^2$$

where b_1, b_2 and b_3 are all constants. Therefore, Eq. (10) reduces to

$$iq_t + aq_{xx} + \left(b_1|q|^{-4} + b_2|q|^2 + b_3|q|^4\right) q = \frac{\beta}{|q|^2 q^*}\left[2|q|^2\left(|q|^2\right)_{xx} - \left\{\left(|q|^2\right)_x\right\}^2\right] + \gamma q + i\left[\delta q_x + \lambda\left(|q|^{2m} q\right)_x + \mu\left(|q|^{2m}\right)_x q\right] \tag{91}$$

and Eq. (11) simplifies to

$$\left(a - 4\beta\right) U'' - \left(\omega + a\kappa^2 + \gamma + \delta\kappa\right) U - \kappa\lambda U^{2m+1} + b_1 U^{-3} + b_2 U^3 + b_3 U^5 = 0. \tag{92}$$

By using transformation $U = V^{\frac{1}{2}}$, Eq. (92) becomes

$$\left(a - 4\beta\right)\left(-\left(V'\right)^2 + 2VV''\right) - 4V^2\left(\omega + a\kappa^2 + \gamma + \delta\kappa\right) - 4\lambda\kappa V^{m+2} + 4\left(b_1 + b_2 V^3 + b_3 V^4\right) = 0. \tag{93}$$

Balancing VV'' or $(V')^2$ with V^4 in Eq. (93), then we get $N = 1$. Consequently, we reach

$$V\left(\xi\right) = a_0 + a_1\left(\frac{\psi'\left(\xi\right)}{\psi\left(\xi\right)}\right). \tag{94}$$

Case 1. Substituting Eq. (94) into Eq. (93) along with $m = 1$ and setting the coefficients of ψ^{-j}, $j = 0,1,2,3,4$ to zero, then we obtain a set of algebraic equations. Solving this system, we obtain

$$a_1 = \pm\frac{\sqrt{3}}{2}\sqrt{\frac{4\beta - a}{b_3}}, \tag{95}$$

$$16b_3^2 a_0^4 + (-24\kappa\lambda b_3 + 24b_2 b_3)a_0^3 + (9\kappa^2\lambda^2 - 18\kappa\lambda b_2 + 9b_2^2)a_0^2 + 48b_1 b_3 = 0, \tag{96}$$

$$\omega = -\frac{48\kappa^2 ab_3 + 9\kappa^2\lambda^2 + 24\kappa\lambda a_0 b_3 - 32a_0^2 b_3^2 + 48\kappa\delta b_3 - 18\kappa\lambda b_2 - 24a_0 b_2 b_3 + 48\gamma b_3 + 9b_2^2}{48b_3}, \tag{97}$$

and

$$\psi'' = \pm\frac{(3\kappa\lambda - 8a_0 b_3 - 3b_2)\sqrt{3}}{6\sqrt{-\left(a - 4\beta\right)b_3}}\psi', \tag{98}$$

$$\psi''' = -\frac{(3\kappa\lambda - 8a_0 b_3 - 3b_2)^2}{12\left(a - 4\beta\right)b_3}\psi'. \tag{99}$$

From Eqs. (98) and (99), we can deduce that

$$\psi' = \pm\frac{6\sqrt{-\left(a - 4\beta\right)b_3}}{(3\kappa\lambda - 8a_0 b_3 - 3b_2)\sqrt{3}}k_1 e^{\pm\frac{(3\kappa\lambda - 8a_0 b_3 - 3b_2)\sqrt{3}}{6\sqrt{-(a-4\beta)b_3}}\xi}, \tag{100}$$

and

$$\psi = -\frac{12\left(a - 4\beta\right)b_3}{(3\kappa\lambda - 8a_0 b_3 - 3b_2)^2}k_1 e^{\pm\frac{(3\kappa\lambda - 8a_0 b_3 - 3b_2)\sqrt{3}}{6\sqrt{-(a-4\beta)b_3}}\xi} + k_2, \tag{101}$$

where k_1 and k_2 are constants of integration. Substituting Eqs. (100) and (101) into Eq. (94), we obtain following exact solution to Eq. (91)

$$q(x,t) = \left\{a_0 \pm \frac{\sqrt{3}}{2}\sqrt{\frac{4\beta - a}{b_3}}\left(\frac{\pm\frac{6\sqrt{-(a-4\beta)b_3}}{(3\kappa\lambda - 8a_0 b_3 - 3b_2)\sqrt{3}}k_1 e^{\pm\frac{(3\kappa\lambda - 8a_0 b_3 - 3b_2)\sqrt{3}}{6\sqrt{-(a-4\beta)b_3}}(x - vt)}}{-\frac{12(a-4\beta)b_3}{(3\kappa\lambda - 8a_0 b_3 - 3b_2)^2}k_1 e^{\pm\frac{(3\kappa\lambda - 8a_0 b_3 - 3b_2)\sqrt{3}}{6\sqrt{-(a-4\beta)b_3}}(x - vt)} + k_2}\right)\right\}^{\frac{1}{2}} e^{i(-\kappa x + \omega t + \theta)}$$

where a_0 and ω is given by Eqs. (96) and (97) respectively. If we set

$$k_1 = -\frac{(3\kappa\lambda - 8a_0 b_3 - 3b_2)^2}{12\left(a - 4\beta\right)b_3}e^{\pm\frac{(3\kappa\lambda - 8a_0 b_3 - 3b_2)\sqrt{3}}{6\sqrt{-(a-4\beta)b_3}}\xi_0}, \quad k_2 = \pm 1$$

we obtain:

$$q(x,t)=\left\{\frac{a_0}{2}\left(1\pm\tanh\left[\frac{(3\kappa\lambda-8a_0b_3-3b_2)\sqrt{3}}{12\sqrt{-\left(a-4\beta\right)b_3}}\left(x-vt+\xi_0\right)\right]\right)\right\}^{\frac{1}{2}}e^{i\left(-\kappa x+\omega t+\theta\right)}, \tag{102}$$

$$q(x,t)=\left\{\frac{a_0}{2}\left(1\pm\coth\left[\frac{(3\kappa\lambda-8a_0b_3-3b_2)\sqrt{3}}{12\sqrt{-\left(a-4\beta\right)b_3}}\left(x-vt+\xi_0\right)\right]\right)\right\}^{\frac{1}{2}}e^{i\left(-\kappa x+\omega t+\theta\right)}, \tag{103}$$

where Eqs. (102) and (103) represent dark soliton and singular soliton solutions respectively. These solitons are valid for

$\left(a-4\beta\right)b_3<0.$

Case 2. Substituting Eq. (94) into Eq. (93) along with $m=2$ and setting the coefficients of $\psi^{-j}, j=0,1,2,3,4$ to zero, then we obtain a set of algebraic equations. Solving this system, we obtain

$$a_1=\pm\frac{\sqrt{3}}{2}\sqrt{\frac{a-4\beta}{\kappa\lambda-b_3}}, \tag{104}$$

$$(16\kappa^2\lambda^2-32\kappa\lambda b_3+16b_3^2)a_0^4+(-24\kappa\lambda b_2+24b_2b_3)a_0^3+9b_2^2a_0^2-48\kappa\lambda b_1+48b_1b_3=0, \tag{105}$$

$$\begin{aligned}\omega\ &=-\frac{1}{48\left(\kappa\lambda-b_3\right)}\left(32\kappa^2\lambda^2a_0^2+48\kappa^3a\lambda-64\kappa\lambda a_0^2b_3-48\kappa^2ab_3+48\kappa^2\delta\lambda\right.\\&\left.-24\kappa\lambda a_0b_2+32a_0^2b_3^2-48\kappa\delta b_3+48\kappa\gamma\lambda+24a_0b_2b_3-48\gamma b_3-9b_2^2\right),\end{aligned} \tag{106}$$

and

$$\psi''=\pm\frac{(8\kappa\lambda a_0-8a_0b_3-3b_2)\sqrt{3}}{6\sqrt{\left(a-4\beta\right)(\kappa\lambda-b_3)}}\psi', \tag{107}$$

$$\psi'''=\frac{(8\kappa\lambda a_0-8a_0b_3-3b_2)^2}{12\left(a-4\beta\right)(\kappa\lambda-b_3)}\psi'. \tag{108}$$

From Eqs. (107) and (108), we can deduce that

$$\psi'=\pm\frac{6\sqrt{\left(a-4\beta\right)(\kappa\lambda-b_3)}}{(8\kappa\lambda a_0-8a_0b_3-3b_2)\sqrt{3}}k_1e^{\pm\frac{(8\kappa\lambda a_0-8a_0b_3-3b_2)\sqrt{3}}{6\sqrt{(a-4\beta)(\kappa\lambda-b_3)}}\xi}, \tag{109}$$

and

$$\psi=\frac{12\left(a-4\beta\right)(\kappa\lambda-b_3)}{(8\kappa\lambda a_0-8a_0b_3-3b_2)^2}k_1e^{\pm\frac{(8\kappa\lambda a_0-8a_0b_3-3b_2)\sqrt{3}}{6\sqrt{(a-4\beta)(\kappa\lambda-b_3)}}\xi}+k_2, \tag{110}$$

where k_1 and k_2 are constants of integration. Substituting Eqs. (109) and (110) into Eq. (94), we obtain following exact solution to Eq. (91)

$$q(x,t)=\left\{a_0\pm\frac{\sqrt{3}}{2}\sqrt{\frac{a-4\beta}{\kappa\lambda-b_3}}\left(\frac{\pm\frac{6\sqrt{(a-4\beta)(\kappa\lambda-b_3)}}{(8\kappa\lambda a_0-8a_0b_3-3b_2)\sqrt{3}}k_1e^{\pm\frac{(8\kappa\lambda a_0-8a_0b_3-3b_2)\sqrt{3}}{6\sqrt{(a-4\beta)(\kappa\lambda-b_3)}}(x-vt)}}{\frac{12(a-4\beta)(\kappa\lambda-b_3)}{(8\kappa\lambda a_0-8a_0b_3-3b_2)^2}k_1e^{\pm\frac{(8\kappa\lambda a_0-8a_0b_3-3b_2)\sqrt{3}}{6\sqrt{(a-4\beta)(\kappa\lambda-b_3)}}(x-vt)}+k_2}\right)\right\}^{\frac{1}{2}}e^{i\left(-\kappa x+\omega t+\theta\right)}$$

where a_0 and ω are given by Eqs. (105) and (106) respectively. If we set

$$k_1=\frac{(8\kappa\lambda a_0-8a_0b_3-3b_2)^2}{12\left(a-4\beta\right)(\kappa\lambda-b_3)}e^{\pm\frac{(8\kappa\lambda a_0-8a_0b_3-3b_2)\sqrt{3}}{6\sqrt{(a-4\beta)(\kappa\lambda-b_3)}}\xi_0},\qquad k_2=\pm1$$

we obtain:

$$q(x,t)=\left\{\frac{a_0}{2}\left(1\pm\tanh\left[\frac{(8\kappa\lambda a_0-8a_0b_3-3b_2)\sqrt{3}}{12\sqrt{\left(a-4\beta\right)(\kappa\lambda-b_3)}}\left(x-vt+\xi_0\right)\right]\right)\right\}^{\frac{1}{2}}e^{i\left(-\kappa x+\omega t+\theta\right)}, \tag{111}$$

$$q(x,t)=\left\{\frac{a_0}{2}\left(1\pm\coth\left[\frac{(8\kappa\lambda a_0-8a_0b_3-3b_2)\sqrt{3}}{12\sqrt{\left(a-4\beta\right)(\kappa\lambda-b_3)}}\left(x-vt+\xi_0\right)\right]\right)\right\}^{\frac{1}{2}}e^{i(-\kappa x+\omega t+\theta)}, \tag{112}$$

where Eqs. (111) and (112) represent dark soliton and singular soliton solutions respectively. These solitons are valid for

$$\left(a-4\beta\right)(\kappa\lambda-b_3)>0.$$

3.7. Cubic-quintic-septic law

In this case,

$$F(s)=b_1s+b_2s^2+b_3s^3$$

where b_1, b_2 and b_3 are all constants. Therefore, Eq. (10) reduces to

$$\begin{aligned} iq_t+aq_{xx}+\left(b_1|q|^2+b_2|q|^4+b_3|q|^6\right)q &= \frac{\beta}{|q|^2q^*}\left[2|q|^2\left(|q|^2\right)_{xx}-\left\{\left(|q|^2\right)_x\right\}^2\right] \\ &\quad +\gamma q+i\left[\delta q_x+\lambda\left(|q|^{2m}q\right)_x+\mu\left(|q|^{2m}\right)_xq\right] \end{aligned} \tag{113}$$

and Eq. (11) simplifies to

$$\left(a-4\beta\right)U''-\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)U-\kappa\lambda U^{2m+1}+b_1U^3+b_2U^5+b_3U^7=0. \tag{114}$$

By using transformation $U=V^{\frac{1}{2}}$, Eq. (114) becomes

$$\left(a-4\beta\right)\left(-\left(V'\right)^2+2VV''\right)-4V^2\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)-4\lambda\kappa V^{m+2}+4b_1V^3+4b_2V^4+4b_3V^5=0. \tag{115}$$

Balancing VV'' or $(V')^2$ with V^4 in Eq. (115), then we get $N=1$. Consequently, we reach

$$V\left(\xi\right)=a_0+a_1\left(\frac{\psi'\left(\xi\right)}{\psi\left(\xi\right)}\right). \tag{116}$$

Substituting Eq. (116) into Eq. (115) along with $m=3$ and then setting the coefficients of ψ^{-j}, $j=0,1,2,3,4,5$ to zero, then we obtain a set of algebraic equations. Solving this system, we obtain

$$b_3=\kappa\lambda,\quad a_1=\pm\frac{\sqrt{3}}{2}\sqrt{-\frac{a-4\beta}{b_2}}, \tag{117}$$

$$(48\kappa\lambda b_2-48b_2b_3)a_0^3-16b_2^2a_0^2-24b_1b_2a_0-9b_1^2=0 \tag{118}$$

$$\omega=-\frac{48\kappa^2ab_2-32a_0^2b_2^2+48\kappa\delta b_2-24a_0b_1b_2+48\gamma b_2+9b_1^2}{48b_2}, \tag{119}$$

and

$$\psi''=\pm\frac{(8a_0b_2+3b_1)\sqrt{3}}{6\sqrt{-\left(a-4\beta\right)b_2}}\psi', \tag{120}$$

$$\psi'''=-\frac{(8a_0b_2+3b_1)^2}{12\left(a-4\beta\right)b_2}\psi'. \tag{121}$$

From Eqs. (120) and (121), we can deduce that

$$\psi'=\pm\frac{6\sqrt{-\left(a-4\beta\right)b_2}}{(8a_0b_2+3b_1)\sqrt{3}}k_1e^{\pm\frac{(8a_0b_2+3b_1)\sqrt{3}}{6\sqrt{-(a-4\beta)b_2}}\xi}, \tag{122}$$

and

$$\psi=-\frac{12\left(a-4\beta\right)b_2}{(8a_0b_2+3b_1)^2}k_1e^{\pm\frac{(8a_0b_2+3b_1)\sqrt{3}}{6\sqrt{-(a-4\beta)b_2}}\xi}+k_2, \tag{123}$$

where k_1 and k_2 are constants of integration. Substituting Eqs. (122) and (123) into Eq. (116), we obtain following exact solution to Eq. (113)

$$q(x,t)=\left\{a_0\pm\frac{\sqrt{3}}{2}\sqrt{-\frac{a-4\beta}{b_2}}\left(\frac{\pm\frac{6\sqrt{-(a-4\beta)b_2}}{(8a_0b_2+3b_1)\sqrt{3}}k_1e^{\pm\frac{(8a_0b_2+3b_1)\sqrt{3}}{6\sqrt{-(a-4\beta)b_2}}(x-vt)}}{-\frac{12(a-4\beta)b_2}{(8a_0b_2+3b_1)^2}k_1e^{\pm\frac{(8a_0b_2+3b_1)\sqrt{3}}{6\sqrt{-(a-4\beta)b_2}}(x-vt)}+k_2}\right)\right\}^{\frac{1}{2}}e^{i(-\kappa x+\omega t+\theta)}$$

where a_0 and ω are given by Eqs. (118) and (119) respectively. If we set

$$k_1=-\frac{(8a_0b_2+3b_1)^2}{12\left(a-4\beta\right)b_2}e^{\pm\frac{(8a_0b_2+3b_1)\sqrt{3}}{6\sqrt{-(a-4\beta)b_2}}\xi_0},\quad k_2=\pm1$$

we obtain:

$$q(x,t)=\left\{\frac{a_0}{2}\left(1\pm\tanh\left[\frac{(8a_0b_2+3b_1)\sqrt{3}}{12\sqrt{-\left(a-4\beta\right)b_2}}\left(x-vt+\xi_0\right)\right]\right)\right\}^{\frac{1}{2}}e^{i(-\kappa x+\omega t+\theta)},\tag{124}$$

$$q(x,t)=\left\{\frac{a_0}{2}\left(1\pm\coth\left[\frac{(8a_0b_2+3b_1)\sqrt{3}}{12\sqrt{-\left(a-4\beta\right)b_2}}\left(x-vt+\xi_0\right)\right]\right)\right\}^{\frac{1}{2}}e^{i(-\kappa x+\omega t+\theta)},\tag{125}$$

where Eqs. (124) and (125) represent dark soliton and singular soliton solutions respectively. These solitons are valid for

$$\left(a-4\beta\right)b_2<0.$$

3.8. Polynomial law

In this case,

$$F(s)=b_1s^n+b_2s^{2n}+b_3s^{3n}$$

where b_1, b_2 and b_3 are all constants. Therefore, Eq. (10) reduces to

$$\begin{aligned}iq_t+aq_{xx}+\left(b_1|q|^{2n}+b_2|q|^{4n}+b_3|q|^{6n}\right)q &=\frac{\beta}{|q|^2q^*}\left[2|q|^2\left(|q|^2\right)_{xx}-\left\{\left(|q|^2\right)_x\right\}^2\right]\\&\quad+\gamma q+i\left[\delta q_x+\lambda\left(|q|^{2m}q\right)_x+\mu\left(|q|^{2m}\right)_xq\right]\end{aligned}\tag{126}$$

and Eq. (11) simplifies to

$$\left(a-4\beta\right)U''-\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)U-\kappa\lambda U^{2m+1}+b_1U^{2n+1}+b_2U^{4n+1}+b_3U^{6n+1}=0.\tag{127}$$

By using transformation $U=V^{\frac{1}{2n}}$, Eq. (127) becomes

$$\begin{aligned}&\left(a-4\beta\right)\left((1-2n)(V')^2+2nVV''\right)-4n^2V^2\left(\omega+a\kappa^2+\gamma+\delta\kappa\right)\\&-4n^2\lambda\kappa V^{\frac{m}{n}+2}+4n^2b_1V^3+4n^2b_2V^4+4n^2b_3V^5=0.\end{aligned}\tag{128}$$

Balancing VV'' or $(V')^2$ with V^4 in Eq. (128), then we get $N=1$. Consequently, we reach

$$V\left(\xi\right)=a_0+a_1\left(\frac{\psi'\left(\xi\right)}{\psi\left(\xi\right)}\right).\tag{129}$$

Substituting Eq. (129) into Eq. (128) along with $m=3n$ and setting the coefficients of ψ^{-j}, $j=0,1,2,3,4,5$ to zero, then we obtain a set of algebraic equations. Solving this system, we obtain

$$b_3=\kappa\lambda,\quad a_0=-\frac{(2n+1)b_1}{2(n+1)b_2},\quad a_1=\pm\frac{1}{2n}\sqrt{-\frac{(2n+1)\left(a-4\beta\right)}{b_2}},\tag{130}$$

$$\omega=-\frac{4\kappa^2an^2b_2+8\kappa^2anb_2+4\kappa\delta n^2b_2+4\kappa^2ab_2+8\kappa\delta nb_2+4\gamma n^2b_2+4\kappa\delta b_2+8\gamma nb_2+2nb_1^2+4\gamma b_2+b_1^2}{4b_2(n+1)^2}, \tag{131}$$

and

$$\psi''=\pm\frac{\sqrt{2n+1}nb_1}{(n+1)\sqrt{-\left(a-4\beta\right)b_2}}\psi', \tag{132}$$

$$\psi'''=-\frac{(2n+1)n^2b_1^2}{(n+1)^2\left(a-4\beta\right)b_2}\psi'. \tag{133}$$

From Eqs. (132) and (133), we can deduce that

$$\psi'=\pm\frac{(n+1)\sqrt{-\left(a-4\beta\right)b_2}}{\sqrt{2n+1}nb_1}k_1e^{\pm\frac{\sqrt{2n+1}nb_1}{(n+1)\sqrt{-(a-4\beta)b_2}}\xi}, \tag{134}$$

and

$$\psi=-\frac{(n+1)^2\left(a-4\beta\right)b_2}{(2n+1)n^2b_1^2}k_1e^{\pm\frac{\sqrt{2n+1}nb_1}{(n+1)\sqrt{-(a-4\beta)b_2}}\xi}+k_2, \tag{135}$$

where k_1 and k_2 are constants of integration. Substituting Eqs. (134) and (135) into Eq. (129), we obtain following exact solution to Eq. (126)

$$q(x,t)=\left\{-\frac{(2n+1)b_1}{2(n+1)b_2}\pm\frac{1}{2n}\sqrt{-\frac{(2n+1)\left(a-4\beta\right)}{b_2}}\left(\frac{\pm\frac{(n+1)\sqrt{-(a-4\beta)b_2}}{\sqrt{2n+1}nb_1}k_1e^{\pm\frac{\sqrt{2n+1}nb_1}{(n+1)\sqrt{-(a-4\beta)b_2}}(x-vt)}}{-\frac{(n+1)^2(a-4\beta)b_2}{(2n+1)n^2b_1^2}k_1e^{\pm\frac{\sqrt{2n+1}nb_1}{(n+1)\sqrt{-(a-4\beta)b_2}}(x-vt)}+k_2}\right)\right\}^{\frac{1}{2n}}e^{i(-\kappa x+\omega t+\theta)}$$

where ω is given by Eq. (131). If we set

$$k_1=-\frac{(2n+1)n^2b_1^2}{(n+1)^2\left(a-4\beta\right)b_2}e^{\pm\frac{\sqrt{2n+1}nb_1}{(n+1)\sqrt{-(a-4\beta)b_2}}\xi_0},\quad k_2=\pm1$$

we obtain:

$$q(x,t)=\left\{-\frac{(2n+1)b_1}{4(n+1)b_2}\left(1\pm\tanh\left[\frac{\sqrt{2n+1}nb_1}{2(n+1)\sqrt{-\left(a-4\beta\right)b_2}}\left(x-vt+\xi_0\right)\right]\right)\right\}^{\frac{1}{2n}}e^{i(-\kappa x+\omega t+\theta)}, \tag{136}$$

$$q(x,t)=\left\{-\frac{(2n+1)b_1}{4(n+1)b_2}\left(1\pm\coth\left[\frac{\sqrt{2n+1}nb_1}{2(n+1)\sqrt{-\left(a-4\beta\right)b_2}}\left(x-vt+\xi_0\right)\right]\right)\right\}^{\frac{1}{2n}}e^{i(-\kappa x+\omega t+\theta)}, \tag{137}$$

where Eqs. (136) and (137) represent dark soliton and singular soliton solutions respectively. These solitons are valid for

$$\left(a-4\beta\right)b_2<0.$$

4. Conclusions

This paper secured dark and singular optical soliton solutions to the perturbed CGGLE where the perturbation terms, which are all of Hamiltonian type appeared with full nonlinearity. There are eight types of nonlinear media that was studied in this paper. The modified simple equation method was the integration scheme applied here and it has its limitations. It can only retrieve dark and singular solitons. This method fails to obtain bright soliton solution to the model. Nevertheless, this method has advantages in studying soliton dynamics. Later, this scheme will be applied to several other models to obtain soliton solutions in optics and other areas of mathematical physics. Those results are awaited currently and will be reported with time.

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