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Cubic–quartic optical solitons in Kerr and power law media

Anjan Biswas[a,b], Houria Triki[c], Qin Zhou[d,*], Seithuti P. Moshokoa[a], Malik Zaka Ullah[b], Milivoj Belic[e]

[a] Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria 0008, South Africa
[b] Operator Theory and Applications Research Group, Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O. Box-80203, Jeddah 21589, Saudi Arabia
[c] Radiation Physics Laboratory, Department of Physics, Faculty of Sciences, Badji Mokhtar University, P.O. Box 12, 23000 Annaba, Algeria
[d] School of Electronics and Information Engineering, Wuhan Donghu University, Wuhan 430212, People's Republic of China
[e] Science Program, Texas A & M University at Qatar, P.O. Box 23874, Doha, Qatar

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ABSTRACT

In this paper, we present the exact bright and singular optical solitons of the nonlinear Schrödinger equation with third and fourth order dispersion terms. The method of undetermined coefficients is applied to obtain the reported solutions. The cases of Kerr law and power law nonlinearity are taken into account. We also find the conditions concerning the optical material parameters for the existence of these soliton structures. The results are useful in describing the propagation of optical solitons in highly dispersive media with Kerr and power law nonlinearity.

1. Introduction

Nonlinear Kerr effect naturally arises in optical fibers and plays a crucial role in the dynamics of light wave envelopes [1–15]. It is to be noted that self-phase modulation (SPM) is the nonlinear effect due to the lowest dominant nonlinear third-order susceptibility $\chi^{(3)}$ in silica fibers [10]. In addition to this cubic nonlinearity, the propagating envelope will also be influenced by the group velocity dispersion (GVD). The exact balancing between GVD and its counterpart SPM gives rise the so-called optical soliton pulse which propagates without change of its shape and velocity properties and stable against mutual collisions [13]. The unique property of optical solitons, either bright or dark, is their particle-like behavior in interaction [7].

Wave propagation in optical fibers with the above mentioned effects is governed by the well known nonlinear Schrödinger equation (NLSE), which can describe the soliton behavior applicable to a picosecond regime [2]. This equation is completely integrable by the inverse scattering transform (IST) [1]. We note that the IST was the first technique for obtaining the soliton solutions for the homogenous NLSE [15].

However, for the propagation of femtosecond pulses, it is necessary to take into account higher order effects. Particularly, the effect of third order dispersion (3OD) is significant for femtosecond pulses when the GVD is negligible [3]. Moreover, as the pulses become extremely short (below 10 fs), fourth order dispersion (4OD) must also be taken into account [9]. It is relevant to mention that these additional effects can significantly change the physical features and the stability of an optical soliton.

* Corresponding author.
E-mail addresses: qinzhou@whu.edu.cn, qzh@whu.edu.cn (Q. Zhou).

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In recent years, considerable attention is being paid theoretically and experimentally to analyze the dynamics of soliton pulses in optical fibers with a view to much potential application of solitons in long-distance communications, optical switching devices, and pulse shaping in laser sources [14]. In this context, higher order NLSE models have been analyzed from different points of view (e.g., Painlevé analysis, Hirota direct method, Ablowitz–Kaup–Newell–Segur method, method of undetermined coefficients, Bäcklund transform, and conservation laws) and there have been many results giving exact soliton solutions with a variety of forms and shapes [10,11]. More recently, Blanco-Redondo et al. [4] have investigated the propagation behavior of pure-quartic solitons, that is, in the presence of SPM and 4OD only. Unlike standard NLSE, higher order equations modeling most practical cases are not completely integrable in general.

In this paper, we study, for the first time, exact soliton solutions of a cubic–quartic NLSE including both 3OD and 4OD. Exact analytical bright and singular soliton solutions for the cubic–quartic (CQ) model are derived by using the method of undetermined coefficients. The cases of Kerr law and power law nonlinearity are taken into account. All the soliton parameters are determined in terms of the physical parameters of the system. The conditions of existence of kind of solitons are also presented.

2. Governing model

The NLSE in presence of 3OD and 4OD but without GVD is given by

$$iq_t + iaq_{xxx} + bq_{xxxx} + F\left(|q|^2\right)q = 0, \tag{1}$$

where the coefficients a and b are real parameters that control independently the values of 3OD and that of 4OD. Also $F(\cdot)$ is an arbitrary function that describes a more general form of the intensity dependent refractive index. There will be two types of this functional F that will be addressed in this paper. They are the Kerr law nonlinearity and power law nonlinearity. Each of these types of nonlinearity will be studied in separate subsections.

Generally, Eq. (1) is not integrable by the classical method of IST. It is always useful and desirable to construct exact analytical solutions (in particular soliton solutions) for the understanding of most nonlinear physical phenomena. Based on the exact solutions directly, we can accurately analyze the properties of propagating waves in nonlinear physical systems.

In order to get started, the hypothesis is taken as

$$q(x,t) = g(s)e^{i\phi(x,t)}, \tag{2}$$

where

$$s = x - vt, \tag{3}$$

where v is the speed of the soliton and the phase is taken to be

$$\phi(x,t) = -\kappa x + \omega t + \theta_0. \tag{4}$$

Substitute (2) into (1) and equate the real and imaginary parts. The imaginary part gives:

$$\left(v + 3a\kappa^2 - 4b\kappa^3\right)g' - (a - 4b\kappa)g''' = 0. \tag{5}$$

Here $g' = dg/ds$, $g'' = d^2g/ds^2$ and so on. From (5), setting the coefficients of the linearly independent functions to zero gives:

$$a = 4b\kappa, \tag{6}$$

and

$$v = -3a\kappa^2 + 4b\kappa^3. \tag{7}$$

Eq. (6) serves as a constraint relation between the coefficients of 3OD and 4OD. Then, Eq. (7) gives the speed of the soliton. Next, the real part gives:

$$bg^{(iv)} + P_2g'' + P_1g + F\left(g^2\right)g = 0, \tag{8}$$

where

$$P_1 = -\omega - a\kappa^3 + b\kappa^4, \tag{9}$$

and

$$P_2 = 3a\kappa - 6b\kappa^2. \tag{10}$$

Eq. (8) will now be analyzed in the next two sections with various nonlinear forms.

3. Kerr law

Here, $F(u) = cu$, for some real-valued constant c. This changes Eq. (1) to the form

$$iq_t + iaq_{xxx} + bq_{xxxx} + c|q|^2 q = 0. \tag{11}$$

It is interesting to note that if $a = 0$, Eq. (11) collapses to the case of pure-quartic NLSE studied in [4]. For general case (i.e., $a \neq 0$ and $b \neq 0$), we will demonstrate the existence of bright and singular soliton solutions under certain parametric conditions.

Therefore, the real part Eq. (8) changes to:

$$bg^{(iv)} + P_2 g'' + P_1 g + cg^3 = 0, \tag{12}$$

Multiplying both sides of (12) by g' and integrating while taking the integration constants to be zero, gives after simplification:

$$4bg'g''' - 2b\left(g''\right)^2 + 2P_2\left(g'\right)^2 + 2P_1 g^2 + cg^4 = 0. \tag{13}$$

3.1. Bright solitons

To solve (13) for bright solitons, the hypothesis

$$g(s) = A \quad \text{sech}^p(Bs), \tag{14}$$

is picked. Here A and B are the amplitude and the inverse width of the soliton, respectively.

Substituting this hypothesis (14) into (13) gives:

$$\begin{aligned}
&2bp^4B^4 \quad \text{sech}^{2p}\tau - 4bp^2B^4\left\{(p+1)(p+2) + p^2\right\} \quad \text{sech}^{2p+2}\tau \\
&+4bp^2(p+1)(p+2)B^4 \quad \text{sech}^{2p+4}\tau + 4bp^3(p+1)B^4 \quad \text{sech}^{2p+2}\tau \\
&-2bp^2(p+1)^2B^4 \quad \text{sech}^{2p+4}\tau + 2P_2p^2B^2 \quad \text{sech}^{2p}\tau \\
&-2P_2p^2B^2 \quad \text{sech}^{2p+2}\tau + 2P_1 \quad \text{sech}^{2p}\tau + cA^2 \quad \text{sech}^{4p}\tau = 0,
\end{aligned} \tag{15}$$

where $\tau = Bs$. By the aid of balancing principle, equating the exponents $4p$ and $2p + 4$ gives

$$4p = 2p + 4, \tag{16}$$

so that

$$p = 2. \tag{17}$$

Next setting the coefficients of the linearly independent functions $\text{sech}^{2p+j}\tau$ to zero, where $j = 0, 2, 4$ gives the width of the soliton as

$$B = \left[\frac{-P_2 + \sqrt{P_2^2 - 4bP_1}}{8b}\right]^{1/2}, \tag{18}$$

or

$$B = \frac{1}{2}\sqrt{-\frac{P_2}{5b}}, \tag{19}$$

and the amplitude of the soliton is

$$A = P_2\sqrt{-\frac{3}{10bc}}. \tag{20}$$

The expressions for the width introduces the following constraints:

$$b\left\{-P_2 + \sqrt{P_2^2 - 4bP_1}\right\} > 0, \tag{21}$$

$$|P_2| > 2\sqrt{bP_1}, \tag{22}$$

$$bP_2 < 0. \tag{23}$$

The amplitude gives the condition

$$bc < 0. \tag{24}$$

Finally, equating the two expressions for the soliton width (B) gives the wave number of the soliton as

$$\omega = -\frac{36\kappa^2(a-2b\kappa)^2 + 25b\kappa^3(a-b\kappa)}{25b}. \tag{25}$$

Hence, bright CQ 1-soliton solution is:

$$q(x,t) = A \quad \mathrm{sech}^2[B(x-vt)]e^{i(-\kappa x+\omega t+\theta_0)}. \tag{26}$$

3.2. Singular solitons

For (13), the hypothesis is:

$$g(s) = A \quad \mathrm{csch}^p(Bs), \tag{27}$$

where $p > 0$ for solitons to exist. Also, A and B are the unknown parameters representing the amplitude and pulse width, respectively, to be determined. Substituting this hypothesis (27) into (13) gives:

$$\begin{aligned}
&4bB^4p^4\mathrm{csch}^{2p}\tau + 4bB^4p^2\left\{p^2 \quad + (p+1)(p+2)\right\}\mathrm{csch}^{2p+2}\tau \\
&+4bB^4p^2(p+1)(p+2)\,\mathrm{csch}^{2p+4}\tau - 2bB^4p^4\mathrm{csch}^{2p}\tau - 2bB^4p^2(p+1)^2\mathrm{csch}^{2p+4}\tau \\
&-4bB^4p^3(p+1)\mathrm{csch}^{2p+2}\tau + \quad 2P_2p^2B^2 \quad \mathrm{csch}^{2p}\tau + 2P_2p^2 \quad B^2 \quad \mathrm{csch}^{2p+2}\tau \\
&+2P_1\mathrm{csch}^{2p}\tau + cA^2\mathrm{csch}^{4p}\tau = 0.
\end{aligned} \tag{28}$$

where $\tau = Bs$. Now, from (28), equating the exponents $4p$ and $2p+4$ leads to the same value of p as in (17). The parameter definitions and their corresponding restrictions also stay the same as given from (18) to (25). Hence, singular CQ 1-soliton solution is:

$$q(x,t) = A \quad \mathrm{csch}^2[B(x-vt)]e^{i(-\kappa x+\omega t+\theta_0)}. \tag{29}$$

4. Power law

Now, we consider a nonlinear optical medium that is characterized by a nonlinear refractive index having a power law-type dependence on the electric field amplitude so that $F(u) = cu^n$, for some real-valued constant c. This changes Eq. (1) to

$$iq_t + iaq_{xxx} + bq_{xxxx} + c|q|^{2n}q = 0. \tag{30}$$

where the parameter n is the exponent in the power law which can take integer values.

Therefore, the real part Eq. (8) changes to:

$$bg^{(iv)} + P_2g'' + P_1g + cg^{2n+1} = 0, \tag{31}$$

Multiplying both sides of (31) by g' and integrating while taking the integration constants to be zero, gives after simplification:

$$4bg'g''' - 2b\left(g''\right)^2 + 2P_2\left(g'\right)^2 + 2P_1g^2 + cg^{2n+2} = 0. \tag{32}$$

4.1. Bright solitons

The hypothesis for bright solitons is the same as in (14). Now substituting (14) into (32) gives

$$\begin{aligned}
&2bp^4B^4 \quad \mathrm{sech}^{2p}\tau - 4bp^2B^4\left\{(p+1)(p+2) + p^2\right\} \quad \mathrm{sech}^{2p+2}\tau \\
&+4bp^2(p+1)(p+2)B^4 \quad \mathrm{sech}^{2p+4}\tau + 4bp^3(p+1)B^4 \quad \mathrm{sech}^{2p+2}\tau \\
&-2bp^2(p+1)^2B^4 \quad \mathrm{sech}^{2p+4}\tau + 2P_2p^2B^2 \quad \mathrm{sech}^{2p}\tau \\
&-2P_2p^2B^2 \quad \mathrm{sech}^{2p+2}\tau + 2P_1 \quad \mathrm{sech}^{2p}\tau + cA^2 \quad \mathrm{sech}^{(2n+2)p}\tau = 0,
\end{aligned} \tag{33}$$

where $\tau = Bs$. From (33), equating the exponents $(2n+2)p$ and $2p+4$ gives

$$(2n+2)p = 2p+4, \tag{34}$$

so that

$$p = \frac{2}{n}. \tag{35}$$

Next, setting the coefficients of the linearly independent functions $\operatorname{sech}^{2p+j}\tau$ to zero, where $j = 0, 2, 4$ gives the width of the soliton as

$$B = n\left[\frac{-P_2+\sqrt{P_2^2-4bP_1}}{8b}\right]^{1/2}, \tag{36}$$

or

$$B = \frac{n}{2}\sqrt{-\frac{P_2}{b(n^2+2n+2)}}, \tag{37}$$

and the amplitude of the soliton is

$$A = \left[-\frac{(n+2)(3n+2)P_2^2}{2bc(n^2+2n+2)^2}\right]^{1/2n}. \tag{38}$$

Also, from (36) to (38), the same restrictions as in (21)–(24) are recovered. From (36) to (37), the wave number of the power-law soliton emerges:

$$\omega = -\frac{9(n+1)^2\kappa^2(a-2b\kappa)^2+\left(n^2+2n+2\right)^2 b\kappa^3(a-b\kappa)}{\left(n^2+2n+2\right)^2 b}. \tag{39}$$

Hence, bright CQ 1-soliton solution is:

$$q(x,t) = A\ \operatorname{sech}^{2/n}[B(x-vt)]e^{i(-\kappa x+\omega t+\theta_0)}. \tag{40}$$

4.2. Singular solitons

The hypothesis for singular solitons is the same as in (27). The substitution of (27) into (33) gives:

$$\begin{aligned}&4bA^2B^4p^4\operatorname{csch}^{2p}\tau+4bA^2B^4p^2\left\{p^2\ +(p+1)(p+2)\right\}\operatorname{csch}^{2p+2}\tau\\&+4bA^2B^4p^2(p+1)(p+2)\operatorname{csch}^{2p+4}\tau-2bA^2B^4p^4\operatorname{csch}^{2p}\tau-2bA^2B^4p^2(p+1)^2\operatorname{csch}^{2p+4}\tau\\&-4bA^2B^4p^3(p+1)\operatorname{csch}^{2p+2}\tau+\ 2P_2A^2p^2B^2\ \operatorname{csch}^{2p}\tau+2P_2A^2p^2\ B^2\ \operatorname{csch}^{2p+2}\tau\\&+2P_1A^2\operatorname{csch}^{2p}\tau+cA^{2n+2}\operatorname{csch}^{(2n+2)p}\tau=0,\end{aligned} \tag{41}$$

where $\tau = Bs$. By the aid of balancing principle, the same value of p as in (35) is obtained. Next, all parameters and constraints stay the same as (36)–(39) along with (21)–(24). Hence, finally, the singular CQ 1-soliton solution of Eq. (31) is:

$$q(x,t) = A\ \operatorname{csch}^{2/n}[B(x-vt)]e^{i(-\kappa x+\omega t+\theta_0)}. \tag{42}$$

From the above results, we can see that the presence of higher-order dispersion terms, in general, changes all the parameters of both bright and singular solitons (velocity, amplitude, and width).

5. Conclusions

We have obtained exact analytic bright and singular soliton solutions of the CQ-NLSE by means of the method of undetermined coefficients. The parameters of the solitons as well as the conditions of their existence are determined for the two cases of Kerr law and power law nonlinearity. The results show clearly how the higher-order dispersion terms influence these solutions. These propagating envelopes exist due to the exact balance among the third order dispersion, the fourth order dispersion, and the self-phase modulation effect. A drawback of this technique, applied to the CQ-NLSE, is that it cannot retrieve dark optical solitons. Another disadvantage is that it is not applicable to parabolic and dual-power laws.

Conflict of interest

The authors also declare that there is no conflict of interest.

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