Optik 143 (2017) 131–134

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Original research article

Optical soliton perturbation with anti-cubic nonlinearity by semi-inverse variational principle

Anjan Biswas[a,b], Qin Zhou[c,*], Malik Zaka Ullah[b], Houria Triki[d], Seithuti P. Moshokoa[a], Milivoj Belic[e]

[a] Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria 0008, South Africa
[b] Operator Theory and Applications Research Group, Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O. Box-80203, Jeddah 21589, Saudi Arabia
[c] School of Electronics and Information Engineering, Wuhan Donghu University, Wuhan 430212, People's Republic of China
[d] Radiation Physics Laboratory, Department of Physics, Faculty of Sciences, Badji Mokhtar University, P.O. Box 12, 23000 Annaba, Algeria
[e] Science Program, Texas A & M University at Qatar, P.O. Box 23874, Doha, Qatar

ARTICLE INFO

Article history:
Received 30 March 2017
Accepted 20 June 2017

Keywords:
Bright soliton
Semi-inverse variation
Anti-cubic nonlinearity

ABSTRACT

The perturbed nonlinear Schrödinger's equation with anti-cubic nonlinearity is studied with the aid of semi-inverse variational principle. The perturbation terms that are included are inter-modal dispersion, third and fourth order dispersion, nonlinear dispersion and self-steepening term, the last two of which appear with full nonlinearity.

1. Introduction

Optical solitons is one of the fastest growing areas of research in the field of telecommunications. There are various aspects of these solitons that are constantly investigated upon. A few such topics are polarizatin preserving fibers, birefringent fibers, DWDM systems and dispersion-flattened fibers, Bragg gratings, optical switching, magneto-optic waveguides, integrability, perturbation theory, super-continuum generation, rogue waves and several others. This paper is going to address one such aspect of optical solitons that are studied with anti-cubic nonlinearity. It is the integrability issue. There are several perturbation terms that the governing nonlinear Schrödinger's equation (NLSE) is studied with. These are inter-modal dispersion (IMD), third order dispersion (3OD), fourth order dispersion 4OD), self-steepening term and nonlinear dispersion, the last two of which are studied with full nonlinearity. There are several analytical tools available to study these type of nonlinear models [1–15]. Some of the most visible ones are G'/G-expansion method, extended trial equation method, Lie symmetry analysis, traveling wave hypothesis. This paper is going to employ the semi-inverse variational principle (SVP) to secure analytical solutions to the model. Although this will not be an exact solution, the variational principle will lead to a closed form analytical bright soliton solution. There are a few restrictions that are going to be implemented for the solitons to exist and these conditions will be listed.

* Corresponding author.
E-mail addresses: qinzhou@whu.edu.cn, qzh@whu.edu.cn (Q. Zhou).

http://dx.doi.org/10.1016/j.ijleo.2017.06.087
0030-4026/

2. Governing model

The governing NLSE with perturbation terms having anti-cubic nonlinearity is [3,5,6,11]:

$$iq_t + aq_{xx} + \left(b_1|q|^{-4} + b_2|q|^2 + b_3|q|^4\right) q = i\left[\alpha q_x - \gamma q_{xxx} - i\sigma q_{xxxx} + \lambda\left(|q|^{2m}q\right)_x + \theta\left(|q|^{2m}\right)_x q\right] \tag{1}$$

In (1), $q(x, t)$ is the dependent variable that represents the complex-valued wave profile while x and t are independent spatial and temporal variables respectively. The coefficient of a is the group velocity dispersion (GVD). The three nonlinear terms are the coefficients of b_j for $j = 1, 2, 3$. The term with b_1 acccounts for the anti-cubic nonlinearity. However, b_2 and b_3 are with cubic and quintic nonlinear terms. On the right hand side α is the coefficient of inter-modal dispersion. This happens when group velocity of light propagating in multimode fibers or other waveguides depend on optical frequency as well as the propagation mode involved. The coefficients of 3OD and 4OD are γ and σ respectively. The self-steepening term is with λ while nonlinear dispersion is given by θ. Finally, the full nonlinearity parameter is governed by the parameter m.

3. Semi-inverse variational principle

To solve (1) by SVP, an initial hypothesis is [1–3]:

$$q(x, t) = g(s)e^{i\phi(x,t)} \tag{2}$$

where

$$s = x - vt \tag{3}$$

and the phase component ϕ is:

$$\phi(x, t) = -\kappa x + \omega t + \theta_0. \tag{4}$$

In (2) and (3), $g(x, t)$ represents the amplitude component of the wave and v is the speed of the wave. From (4), κ is the soliton frequency, ω is its wave number and θ_0 is the phase constant. Substituting (2) into (1) and splitting into real and imaginary parts yield [3,13]

$$\sigma g^{(iv)} - P_2 g'' + P_1 g + \left(b_1 g^{-4} + b_2 g^2 + b_3 g^4\right) g + \lambda\kappa g^{2m+1} = 0 \tag{5}$$

and

$$\left(v + 2a\kappa + \alpha + 3\gamma\kappa^2 + 4\sigma\kappa^3\right) g' - (\gamma + 4\sigma\kappa) g'' + \left\{(2m+1)\lambda + 2m\theta\right\} g^{2m} g' = 0, \tag{6}$$

respectively, where the notations $g' = dg/ds$ and $g'' = d^2g/ds^2$ etc are adopted. Here, in (5)

$$P_1 = \omega + a\kappa^2 + \alpha\kappa + \gamma\kappa^3 + \sigma\kappa^4, \tag{7}$$

$$P_2 = a + 3\gamma\kappa + 6\sigma\kappa^3. \tag{8}$$

From (6), setting the coefficients of linearly independent functions to zero implies:

$$v = -2a\kappa - \alpha - 3\gamma\kappa^2 - 4\sigma\kappa^3, \tag{9}$$

$$\gamma + 4\sigma\kappa = 0, \tag{10}$$

and

$$(2m + 1)\lambda + 2m\theta = 0. \tag{11}$$

Thus, Eq. (9) gives the speed of the soliton in presence of the perturbation terms while relations (10) and (11) are the constraints on the perturbation parameters.

Next, multiplying both sides of (5) by g' and integrating once yields

$$\sigma\left(g''\right)^2 - 2\sigma g' g''' + P_1 g^2 + P_2\left(g'\right)^2 - \frac{b_1}{g^2} + \frac{b_2}{2}g^4 + \frac{b_3}{3}g^6 + \frac{\lambda g^{2m+2}}{m+1} = K, \tag{12}$$

where K is the integration constant. The stationary integral is then defined as [3,4,13]:

$$J = \int_{-\infty}^{\infty} K ds = \int_{-\infty}^{\infty}\left[3\sigma\left(g''\right)^2 + P_1 g^2 + P_2\left(g'\right)^2 - \frac{b_1}{g^2} + \frac{b_2}{2}g^4 + \frac{b_3}{3}g^6 + \frac{\lambda\kappa g^{2m+2}}{m+1}\right] ds \tag{13}$$

Now choose [3,13]

$$g(s) = A\sqrt{\text{sech}(Bs)} \tag{14}$$

where A is the soliton amplitude, B is its inverse width. SVP states that the solution of the perturbed Eq. (1) will be same as the unperturbed model. However, its amplitude and width will vary according to the coupled system of equations [3,4,13]:

$$\frac{\partial J}{\partial A}=0, \tag{15}$$

and

$$\frac{\partial J}{\partial B}=0. \tag{16}$$

Substituting (14) into (13) and carrying out the integrations gives

$$J=\frac{57\sigma\pi A^2B^3}{64}+\frac{P_2\pi A^2B}{8}+\frac{P_1A^2\pi}{B}-\frac{b_1I}{A^2B}+\frac{b_2A^4}{B}+\frac{b_3A^6\pi}{6B}+\frac{\lambda\kappa A^{2m+2}}{(m+1)B}\frac{\Gamma\left(1/2\right)\Gamma\left((m+1)/2\right)}{\Gamma\left((m/2)+1\right)}, \tag{17}$$

where $\Gamma(x)$ is Euler's gamma function and

$$I=\int_{-\infty}^{\infty}\cosh u \quad du. \tag{18}$$

This is a divergent integral and can be approximated numerically between finite limits of integration.

Next, for J given by (17), Eqs. (15) and (16) reduce to

$$\frac{57\sigma\pi B^4}{32}+\frac{P_2\pi B^2}{4}+2P_1\pi+\frac{2b_1I}{A^4}+4b_2A^2+b_3\pi A^4+2\lambda\kappa A^{2m}\frac{\Gamma\left(1/2\right)\Gamma\left((m+1)/2\right)}{\Gamma\left((m/2)+1\right)}=0 \tag{19}$$

and

$$\frac{171\sigma\pi B^4}{64}+\frac{P_2\pi B^2}{8}-P_1\pi+\frac{b_1I}{A^4}-b_2A^2+\frac{b_3\pi A^4}{6}-\frac{\lambda\kappa A^{2m}}{m+1}\frac{\Gamma\left(1/2\right)\Gamma\left((m+1)/2\right)}{\Gamma\left((m/2)+1\right)}=0 \tag{20}$$

Upon uncoupling (19) and (20) leads to a biquadratic equation for the width B in terms of the soliton amplitude A that is given by

$$\frac{57\sigma\pi B^4}{16}+\frac{P_2\pi B^2}{4}+2\frac{b_1I}{A^4}+b_2A^2+\frac{b_3\pi A^4}{3}+\frac{m\lambda\kappa A^{2m}}{m+1}\frac{\Gamma\left(1/2\right)\Gamma\left((m+1)/2\right)}{\Gamma\left((m/2)+1\right)}=0 \tag{21}$$

which solves to

$$B=\sqrt{\frac{8}{57\sigma\pi}\left[-\frac{P_2\pi}{4}+\left\{\frac{P_2^2\pi^2}{16}-\frac{57\sigma\pi}{4}\left(\frac{2b_1I}{A^4}+b_2A^2+\frac{b_3\pi A^4}{3}+\frac{m\lambda\kappa A^{2m}}{m+1}\frac{\Gamma\left(1/2\right)\Gamma\left((m+1)/2\right)}{\Gamma\left((m/2)+1\right)}\right)\right\}^{1/2}\right]} \tag{22}$$

This relation between the soliton amplitude and width will remain valid provided

$$\sigma\left[-\frac{P_2\pi}{4}+\left\{\frac{P_2^2\pi^2}{16}-\frac{57\sigma\pi}{4}\left(\frac{2b_1I}{A^4}+b_2A^2+\frac{b_3\pi A^4}{3}+\frac{m\lambda\kappa A^{2m}}{m+1}\frac{\Gamma\left(1/2\right)\Gamma\left((m+1)/2\right)}{\Gamma\left((m/2)+1\right)}\right)\right\}^{1/2}\right]>0 \tag{23}$$

and

$$\left|57\sigma\left\{\frac{2b_1I}{A^4}+b_2A^2+\frac{b_3\pi A^4}{3}+\frac{m\lambda\kappa A^{2m}}{m+1}\frac{\Gamma\left(1/2\right)\Gamma\left((m+1)/2\right)}{\Gamma\left((m/2)+1\right)}\right\}\right|<\frac{P_2\pi}{4} \tag{24}$$

Thus, finally, the 1-soliton solution to (1) is given by

$$q(x,t)=A\sqrt{\mathrm{sech}[B(x-vt)]}e^{i\left(-\kappa x+\omega t+\theta_0\right)} \tag{25}$$

where the parameter relations and restrictions are all described above.

4. Conclusions

This paper obtained an analytical soliton solution to the pertured NLSE with anti-cubic nonlinearity. The perturbation terms are of Hamiltonian type and two of these perturbations appear with full nonlinearity. SVP yielded the analytical solution although this is not an exact solution. It is not possible to secure an exact soliton solution to the model studied in this paper since the perturbation terms appear with full nonlinearity. The restrictions, or constraint conditions, to the parameters are also listed for existence of these bright solitons.

The results of this paper carry a lot of future prospects. This integration scheme can be applied to other situations as well, whenever an exact soliton solution is not available. Some of these situations are in birefringent fibers, DWDM systems, optical switching, as well as other models such as complex Ginzburg-Landau equation, Gerdjikov–Ivanov equation and others. Those results will be visible fairly soon.

Conflict of interest

The authors also declare that there is no conflict of interest.

Acknowledgements

The second author (QZ) was funded by the Young Foundation of Wuhan Donghu University under the grant number WDU201701 and the National Science Foundation of Hubei Province in China under the grant number 2015CFC891. The fifth author (SPM) would like to thank the research support provided by the Department of Mathematics and Statistics at Tshwane University of Technology and the support from the South African National Foundation under Grant Number 92052 IRF1202210126. The research work of sixth author (MB) was supported by Qatar National Research Fund (QNRF) under the grant number NPRP-8-028-1-001.

References

[1] A.H. Bhrawy, M.A. Abdelkawy, A. Biswas, Optical solitons in (1+1) and (2+1) dimensions, Optik 125 (4) (2014) 1537–1549.
[2] A.H. Bhrawy, A.A. Alshaery, E.M. Hilal, K.R. Khan, M.F. Mahmood, A. Biswas, Optical soliton perturbation with spatio-temporal dispersion in parabolic and dual-power law media by semi-inverse variational principle, Optik 125 (17) (2014) 4945–4950.
[3] A. Biswas, M.Z. Ullah, M. Asma, Q. Zhou, S.P. Moshokoa, M. Belic, Optical solitons with quadratic-cubic nonlinearity by semi-inverse variational principle, Optik (2017), http://dx.doi.org/10.1016/j.ijleo.2017.03.111.
[4] A. Biswas, D. Milovic, M. Savescu, M.F. Mahmood, K.R. Khan, R. Kohl, Optical soliton perturbation in nanofibers with improved nonlinear Schrödinger's equation by semi-inverse variational principle, J. Nonlinear Opt. Phys. Mater. 12 (4) (2012) 1250054.
[5] M. Ekici, M. Mirzazadeh, A. Sonmezoglu, M.Z. Ullah, Q. Zhou, H. Triki, S.P. Moshokoa, A. Biswas, Optical solitons with anti-cubic nonlinearity by extended trial equation method, Optik 136 (2017) 368–373.
[6] A.J.M. Jawad, M. Mirzazadeh, Q. Zhou, A. Biswas, Optical solitons with anti-cubic nonlinearity using three integration schemes, Superlattices Microstruct. 105 (2017) 1–10.
[7] R. Kohl, D. Milovic, E. Zerrad, A. Biswas, Optical solitons by He's variational principle in a non-Kerr law media, J. Infrared Millim. Terahertz Waves 30 (5) (2009) 526–537.
[8] X.-W. Li, Y. Li, J.-H. He, On the semi-inverse method and variational principle, Thermal Sci. 17 (5) (2013) 1565–1568.
[9] M. Najafi, S. Arbabi, Dark soliton and periodic wave solutions of the Biswas–Milovic equation, Optik 127 (5) (2016) 2679–2682.
[10] T. Ozis, A. Yildirim, Application of He's semi-inverse method to the nonlinear Schrödinger equation, Comput. Math. Appl. 54 (7–8) (2007) 1039–1042.
[11] H. Triki, A.H. Kara, A. Biswas, S.P. Moshokoa, M. Belic, Optical solitons and conservation laws with anti-cubic nonlinearity, Optik 127 (24) (2016) 12056–12062.
[12] Y. Wu, Variational approach to the generalized Zakharov equations, Int. J. Nonlinear Sci. Numer. Simul. 10 (9) (2009) 1245–1247.
[13] Y. Xu, J. Vega-Guzman, D. Milovic, M. Mirzazadeh, M. Eslami, M.F. Mahmood, A. Biswas, M. Belic, Bright and exotic solitons in optical metamaterials by semi-inverse variational principle, J. Nonlinear Opt. Phys. Mater. 24 (4) (2015) 155042.
[14] A. Zerarka, K. Libarir, A semi-inverse variational method for generating the bound state energy eigenvalues in a quantum system: the Schrödinger equation, Commun. Nonlinear Sci. Numer. Simul. 14 (7) (2009) 3195–3199.
[15] C.B. Zheng, B. Liu, Z.-J. Wang, S.-K. Zheng, Generalized variational principle for electromagnetic field with magnetic monopoles by He's semi-inverse method, Int. J. Nonlinear Sci. Numer. Simul. 10 (11–12) (2009) 1369–1372.