Optical solitons with fractional temporal evolution having anti-cubic nonlinearity

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This paper obtains dark optical soliton solutions, with fractional temporal evolution, for anti-cubic nonlinear medium. The definition of Khalil's conformable fractional derivative, coupled with Bernoulli's equation approach, is utilized. The soliton solution appears with constraint conditions, for its existence.

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1. Introduction

Optical solitons is one of the most fascinating areas of research in photonic sciences [1-20]. While integrability aspect of the governing equations for the solitons is a major focus on most of the papers, it is the fractional temporal evolution that is drawing a lot of attention these days. The focus on this issue leads to a lot of advantages. One of the advantages is that it can control the Internet bottleneck that is a growing problem across the globe. The consideration of fractional evolution of pulses will slow down its evolution in one direction, thus Internet traffic can flow at a normal pace in the other direction and vice versa. Therefore, it is very important to divert the focus of research towards fractional evolution of pulses.

This paper will focus on the fractional temporal evolution of solitons in optical fibers that maintain anti-cubic nonlinearity [4, 5, 8]. Khalil's conformable fractional derivative will be revisited [6]. Subsequently, Bernoulli's method will be implemented to carry out the integration of the governing nonlinear Schrödinger's equation (NLSE) with anti-cubic nonlinearity. The resulting soliton solution will appear with a number of constraints for its existence.

2. Khalil's conformable fractional derivative

The conformable derivative of order α is defined as [1]:

$$T_\alpha(f)(t) = \frac{\partial^\alpha}{\partial t^\alpha} f(t) = \lim_{\varepsilon \to 0} \frac{f(t+\varepsilon t^{1-\alpha}) - f(t)}{\varepsilon} \quad (1)$$

for all $t > 0, \alpha \in (0,1]$ and a function $f:[0,\infty) \to R$.

The conformable derivative satisfies the properties as described by the following theorems:

Theorem-I: Assume that the order of the derivative $\alpha \in (0,1]$, and suppose that f and g are α-differentiable for all positive t. Then,

$$\begin{gathered} T_\alpha(af+bg) = aT_\alpha(f) + bT_\alpha(g), \forall\ a,b \in R \\ T_\alpha(t^p) = pt^{p-\alpha}, \forall\ p \in R, \\ T_\alpha(fg) = fT_\alpha(g) + gT_\alpha(f), \\ T_\alpha\left(\frac{f}{g}\right) = \frac{gT_\alpha(f) - fT_\alpha(g)}{g^2}. \end{gathered} \quad (2)$$

If, in addition, f is differentiable, then $T_\alpha(f)(t) = t^{1-\alpha}\frac{df}{dt}$.

Some more properties covering the chain rule, Gronwall's inequality, a few integration techniques, Laplace transform, Taylor series expansion and exponential function with respect to the conformable derivative are expressed in the work [2].

Theorem-II: If $f:(0,\infty) \to R$, a function such that f is differentiable and also α-differentiable. Let g be a function defined in the range of f and also differentiable; then, one has the following rule:

$$T_\alpha(f \circ g)(t) = t^{1-\alpha} g'(t) f'(g(t)). \quad (3)$$

3. Mathematical model

We investigate the conformable fractional cubic-quintic NLSE that with an additional anti-cubic nonlinear term, first introduced during 2003, and is of the form [3]

$$iq_t^\alpha + aq_{xx} + (b_1|q|^{-4} + b_2|q|^2 + b_3|q|^4)q = 0. \quad (4)$$

where a, b_1, b_2 and b_3 are all real-valued constants and $\alpha \in (0,1]$. The independent variables are x and t that represents spatial and temporal co-ordinates. Again the dependent variable is $q(x,t)$ that is a complex-valued function. In (1), if $b_1 = 0$ it reduces to NLSE with parabolic law or cubic-quintic law of nonlinearity that has been extensively studied. It is this b_1 that introduces the anti-cubic nonlinear term. Next, introduce the transformations

$$q(x,t) = e^{i\theta(x,t)}u(\xi), \theta = -\kappa x + \left(\frac{\omega}{\alpha}\right)t^\alpha + \varepsilon_0, \xi = x - \left(\frac{\lambda}{\alpha}\right)t^\alpha + x_0 \quad (5)$$

where κ, ω, λ, ε_0 and x_0 are real constants.

Substituting (5) into (4) and then splitting into real and imaginary parts yields a pair of relations. The imaginary part gives

$$\lambda = -2\kappa a \quad (6)$$

while the real part gives

$$u''(\xi) - \frac{(\omega + a\kappa^2)}{a}u(\xi) + \frac{b_1}{a}u^{-3}(\xi) + \frac{b_2}{a}u^3(\xi) + \frac{b_3}{a}u^5(\xi) = 0. \quad (7)$$

Multiplying both sides of (8) by u' and integrating with respect to ξ, we get

$$\frac{(u')^2}{2} - \frac{(\omega + a\kappa^2)}{2a}u^2 - \frac{b_1}{2a}u^{-2}(\xi) + \frac{b_2}{4a}u^4 + \frac{b_3}{6a}u^6 + b_4 = 0, \quad (8)$$

where b_4 is an integration constant. Thus, we have

$$(u')^2 - c_0u^2 - c_1u^{-2} + c_2\frac{u^4}{2} + c_3\frac{u^6}{3} + c_4 = 0, \quad (9)$$

where

$$c_0 = \frac{(\omega + a\kappa^2)}{a}, c_1 = \frac{b_1}{a}, c_2 = \frac{b_2}{a}, c_3 = \frac{b_3}{a}, c_4 = 2b_4. \quad (10)$$

Let $u^2 = v$, then

$$u' = \frac{1}{2u}v'. \quad (11)$$

Substitute Eq. (11) in Eq. (9) gives:

$$(v')^2 - 4c_0v^2 - 4c_1 + 2c_2v^3 + \frac{4}{3}c_3v^4 + 4c_4v = 0. \quad (12)$$

4. Bernoulli's equation method

By employing the Bernoulli's equation method to Eq. (12), the traveling wave solutions can be written [4,5] as

$$v(\xi) = a_0 + a_1G(\xi) \quad (13)$$

where

$$G(\xi) = \frac{\delta}{2}\left\{1 + tanh\left(\frac{\delta}{2}\xi\right)\right\} \quad (14)$$

is the solution to the Bernoulli's equation $G'(\xi) = \delta G(\xi) - G^2(\xi)$ and δ, a_0, and a_1 are constants to be evaluated later. Then, Substituting (13) into Eq. (12) and equating the coefficient of each power of $G(\xi)$ to zero, we obtain a system of nonlinear algebraic equations and by solving it, we get

$$a_0 = \pm\frac{2\left(b_4^2a^2 + \sqrt{b_4^4a^4 - b_1^2b_2b_4a}\right)}{b_1b_2}, \quad (15)$$

$$a_1 = \pm\frac{2\sqrt{b_1a}ab_4}{b_1b_2}, \quad (16)$$

$$\delta = \mp\frac{2\sqrt{b_4^4a^4 - b_1^2b_2b_4a}}{\sqrt{b_1a}ab_4}, \quad (17)$$

$$\omega = \frac{2b_4^3a^3 + b_1^2b_2 - 2b_4a^2\kappa^2b_1}{2b_4ab_1}, \quad (18)$$

$$b_3 = -\frac{3b_1b_2^2}{16a^2b_4^2}, \quad (19)$$

which immediately prompt the constraints:

$$(b_4^4a^4 - b_1^2b_2b_4a) > 0, b_1a > 0, b_4 \neq 0, b_1b_2 > 0. \quad (20)$$

Using the values of parameters (15)-(19) we have the following solution of Eq. (12)

$$v(\xi) = \frac{2a_0 + a_1\delta}{2} + \frac{a_1\delta}{2}tanh\left(\frac{\delta}{2}\xi\right). \quad (21)$$

Combining (21) with (5), we obtain the exact solution to Eq. (7) and the exact solution to the conformable fractional cubic-quintic NLSE can be written as

$$q(x,t) = \left\{\pm\frac{2}{b_1b_2}\left[b_4^2a^2 - \sqrt{b_4^4a^4 - b_1^2b_2b_4a}\, tanh\left(\mp\frac{\sqrt{b_4^4a^4 - b_1^2b_2b_4a}}{\sqrt{b_1a}\,ab_4}\left(x + \left(\frac{2a\kappa}{\alpha}\right)t^\alpha + x_0\right)\right)\right]\right\}^{\frac{1}{2}} \times e^{i\left\{-\kappa x + \left(\frac{2b_4^3a^3 + b_1^2b_2 - 2b_4a^2\kappa^2b_1}{2\alpha b_4ab_1}\right)t^\alpha + \epsilon_0\right\}}, \quad (22)$$

which exist only when conditions (19) and (20) are satisfied.

3. Conclusions

This paper obtained dark optical soliton solutions to the governing NLSE with anti-cubic nonlinearity having fractional temporal evolution. The definition of Khalil's conformable fractional derivative is utilized. Finally, Bernoulli's equation approach leads to the dark soliton solution. This appears with a few constraint relations that are also listed. These relations guarantee the existence of dark solitons. The results of this paper are encouraging to study further in this avenue. Later, research results with the inclusion of fractional spatio-temporal dispersion will be considered. The results of that research will be available shortly.

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References

[1] T. Abdeljawad. J. Computational Appl. Math. **279**, 57 (2015).
[2] A. Biswas, S. Konar, CRC Press. Boca Raton FL. (2006).
[3] M. G. Hafez, M. R. Talukder, M. H. Ali, Physics of Plasmas **23**, 012902 (2016).
[4] R. Fedele, H. Schamel, V. I. Karpman, P. K. Shukla, Journal of Physics **A36**(4), 1169 (2003).
[5] A. J. M. Jawad, M. Mirzazadeh, Q. Zhou, A. Biswas, Submitted.
[6] R. Khalil, M. Al Horani, A. Yousef, M. Sababheh, Journal Computational and Applied Mathematics **264**, 65 (2014).
[7] M. Mirzazadeh, M. Eslami, E. Zerrad, M. F. Mahmood, A. Biswas, M. Belic, Nonlinear Dynamics **81**, 1933 (2015).
[8] H. Triki, A. H. Kara, A. Biswas, S. P. Moshokoa, M. Belic, Optik **127**, 12056 (2016).
[9] B. Kilic, M. Inc, D. Baleanu, Open Physics **14**(1), 65 (2016).
[10] B. Kilic, M. Inc, Journal of Electromagnetic Waves and Applications **30**(7), 871 (2016).
[11] M. Inc, B. Kilic, Waves in Random and Complex Media **27**(1), 28 (2017).
[12] F. Tchier, E. C. Aslan, M. Inc, Nonlinear Dynamics **85**(4), 2577 (2016).
[13] F. Tchier, M. Inc, B. Kilic, A. Akgül, Optik **128**, 218 (2017).
[14] F. Tciher, E. C. Aslan, M. Inc, Journal of Nanoelectronics and Optoelectronics **12**(5), 526 (2017).
[15] B. Kilic, M. Inc, Optik **138**, 64 (2017).
[16] M. M. A. Qurashi, A. Yusuf, A. Aliyu, M. Inc, Superlattices and Microstructures **105**, 183 (2017).
[17] M. Inc, A. Aliyu, A. Yusuf, Modern Physics Letters B **31**(14), 1750163 (2017).
[18] D. Baleanu, M. Inc, Optik **140**, 114 (2017).
[19] M. M. A. Qurashi, Z. S. Korpinar, M. Inc, Optik **140**, 45 (2017).
[20] E. C. Aslan, F. Tciher, M. Inc, Superlattices and Microstructures **105**, 48 (2017).

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