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ORIGINAL PAPER

Optical solitons in nonlinear directional couplers by sine–cosine function method and Bernoulli's equation approach

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Abstract This paper obtains soliton solutions to optical couplers by two methods. These are sine–cosine function method and Bernoulli's equation approach. There are four laws that are touched upon in this paper. These are Kerr law, power law, parabolic law and dual-power law. The first integration scheme is applicable to Kerr and power laws only where bright soliton solutions are retrievable. The second tool is applicable to parabolic and dual-power laws only that leads to dark and singular solitons for these two nonlinear media.

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1 Introduction

Theory of solitons in optical fibers is a very rich area of research in the field of nonlinear dynamics, in particular nonlinear optics [1–50]. There are several papers that describe the dynamics in optical fibers using a plethora of integration schemes. It is about time to change gears for the time being. This paper focuses on soliton solutions in nonlinear directional optical couplers. Three types of couplers will be the focus in this paper. They are twin-core couplers, coupling with nearest neighbors and coupling with all neighbors. Each type is studied with four nonlinear forms. These are Kerr law, power law, parabolic law and dual-power law.

This paper considers the governing nonlinear Schrödinger's equation with spatiotemporal dispersion (STD) in addition to the usual group velocity dispersion (GVD). STD makes the model well posed as pointed out during 2012 [17]. Therefore, it is imperative to study NLSE in couplers as well as in fibers with STD term included. It must be noted that optical couplers have been studied earlier by ansatz scheme and other methods [1,6–9,43]. There are a couple of different integration schemes that will be exploited, in this paper, to retrieve soliton solutions for these kind of couplers. There are several constraint conditions that will be listed for these soliton solutions to exist. It will be clearly noticeable that Kerr and power laws retrieve bright soliton solutions only, while parabolic and dual-power laws give dark and singular soliton solutions. Such are the limitations of the schemes adopted in this paper.

2 Integration algorithms: an overview

There are two integration algorithms that will be implemented in this paper. These are sine–cosine function method and Bernoulli's equation method. The first integration scheme is applicable to Kerr and power laws of nonlinearity for all three kinds of couplers, while the second integration scheme is applicable to parabolic and dual-power laws of nonlinearity. After a brief overview of these schemes, we will dive into the extraction of soliton solutions for all kinds of couplers from the next section.

2.1 Sine–cosine function method

A partial differential equation (PDE)

$$P(u, u_t, u_x, u_{xx}, u_{xt}, u_{tt}, \ldots) = 0, \tag{1}$$

can be converted to an ordinary differential equation (ODE)

$$Q\left(U, U', U'', \ldots\right) = 0, \tag{2}$$

upon using a traveling wave variable $u(x,t) = U(z),\ z = x - vt$. If possible, integrate Eq. (2) term by term one or more times. This will reduce the order of Eq. (2). For simplicity, the integration constants can be set to zero. The solutions of the reduced ODE can be expressed in the form

$$U(z) = \lambda \cos^{\beta}(\mu z), \quad |z| \le \frac{\pi}{2\mu}, \tag{3}$$

or in the form

$$U(z) = \lambda \sin^{\beta}(\mu z), \quad |z| \le \frac{\pi}{\mu}, \tag{4}$$

where $\lambda,\ \mu$, and β are parameters that will be determined, μ and v are the wave number and the wave speed respectively. These assumptions give

$$\begin{aligned}\left(U^n\right)'' &= -n^2\mu^2\beta^2\lambda^n \cos^{n\beta}(\mu z)\\ &\quad + n\mu^2\lambda^n\beta(n\beta-1)\cos^{n\beta-2}(\mu z),\end{aligned} \tag{5}$$

and

$$\begin{aligned}\left(U^n\right)'' &= -n^2\mu^2\beta^2\lambda^n \sin^{n\beta}(\mu z)\\ &\quad + n\mu^2\lambda^n\beta(n\beta-1)\sin^{n\beta-2}(\mu z).\end{aligned} \tag{6}$$

Using (3)–(6) in the reduced ODE gives a trigonometric equation in $\cos^K(z)$ or $\sin^K(z)$ terms. The parameters are then determined by first balancing the exponents of each pair of cosines or sines to determine K. We next collect all coefficients of the same power in $\cos^K(z)$ or $\sin^K(z)$, where these coefficients have to vanish. This gives a system of algebraic equations among the unknowns $\beta,\ \lambda,\ v$ and μ that will be determined. The solutions proposed in (3) and (4) follow immediately.

2.2 Bernoulli's equation method

Let us present the algorithm of Bernoulli's equation approach for finding exact solutions of nonlinear PDEs. We consider the nonlinear PDE in the following form:

$$P_1(u, u_t, u_x, u_{xx}, \ldots) = 0. \tag{7}$$

Using traveling wave $u(x,t) = U(z),\ z = x - vt$ carries Eq. (7) into the following ordinary differential equation (ODE):

$$P_2(U, -vU_z, U_z, U_{zz}, \ldots) = 0. \tag{8}$$

The Bernoulli's equation approach utilizes the following steps:

Step-1 We look for exact solution of Eq. (8) in the form

$$U = \sum_{l=0}^{N} A_l\,(G(z))^l \tag{9}$$

where $A_l(l = 0, 1, \ldots, N)$ are constants to be determined later, such that $A_N \neq 0$, while $G(z)$ has the form

$$G(z) = \frac{\delta}{2}\left\{1 + \tanh\left(\frac{\delta}{2}(z + z_0)\right)\right\} \tag{10}$$

a solution to the Bernoulli's equation

$$G'(z) = \delta G(z) - G^2(z) \tag{11}$$

where δ is an arbitrary constant.

Step-2 We determine the positive integer N in Eq. (9) by considering the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. (8).

Step-3 We substitute Eq. (9) into Eq. (8) and calculate all the necessary derivatives $U_z, U_{zz}, \ldots$ of the unknown function $U(z)$ as follows:

$$U_z = \sum_{l=1}^{N} A_l l(\delta - G)G^l, \tag{12}$$

$$U_{zz} = \sum_{l=1}^{N} A_l l \left\{(1+l)G^2 - \delta(2l+1)G + l\delta^2\right\} G^l, \tag{13}$$

and so on. Substituting Eqs. (9), (12) and (13) into Eq. (8), we obtain the polynomial

$$E_2[G(z)] = 0. \tag{14}$$

Step-4 Collecting all the terms of the same powers of the function $G(z)$ in the polynomial (14) and equating them to zero, we obtain a system of algebraic equations which can be solved by computer programs such as Maple and Mathematica to get the unknown parameters A_l, δ and v. Consequently, we obtain the exact solutions of Eq. (7).

3 Twin-core couplers

The governing equation for twin-core couplers is given by [6–9,43]

$$iq_t + a_1 q_{xx} + b_1 q_{xt} + c_1\, F\left(|q|^2\right) q = k_1 r, \tag{15}$$

$$ir_t + a_2 r_{xx} + b_2 r_{xt} + c_2\, F\left(|r|^2\right) r = k_2 q. \tag{16}$$

Equations (15) and (16) represent the coupled NLSE, with GVD and STD, that governs soliton propagation through twin-core optical fibers, typically for non-Kerr law media. The first term, for both equations, represents the evolution term. The coefficients of GVD are a_l, while the coefficients of STD are b_l for $l = 1, 2$. Then, c_l represents the coefficients of nonlinearity where the functional F gives the type of nonlinearity that will be studied. Here, $F\left(|q|^2\right) q : C \longmapsto C$. Considering the complex plane C as a two-dimensional linear space R_2, the function $F\left(|q|^2\right) q$ is k times continuously differentiable, so that

$$F\left(|q|^2\right) q \in \bigcup_{m,n=1}^{\infty} C^k\left((-n,n)\times(-m,m)\,;\,R^2\right). \tag{17}$$

On the right-hand sides of (15) and (16), constant k_1, k_2 represent the coupling coefficients. In order to study integrability of these equations by sine–cosine function method and Bernoulli's equation approach, the following solution structure is selected.

$$q(x,t) = P_1(x,t)\mathrm{e}^{i\phi(x,t)}, \tag{18}$$

$$r(x,t) = P_2(x,t)\mathrm{e}^{i\phi(x,t)} \tag{19}$$

where $P_l(x,t)$ $(l = 1, 2)$ represents the amplitude component of the soliton solution, while the phase component $\phi(x,t)$ is defined as

$$\phi(x,t) = -\kappa x + \omega t + \theta. \tag{20}$$

Here, κ is the frequency of the solitons while ω represents the wave number and θ is the phase constant. Substituting (18) and (19) into (15) and (16) and then decomposing into real and imaginary parts gives

$$a_l \frac{\partial^2 P_l}{\partial x^2} + b_l \frac{\partial^2 P_l}{\partial x \partial t} + P_l\left(b_l\omega\kappa - \omega - a_l\kappa^2\right) + c_l\, F(P_l^2)P_l - k_l P_{\bar{l}} = 0, \tag{21}$$

and

$$(1 - b_l\kappa)\frac{\partial P_l}{\partial t} + (b_l\omega - 2a_l\kappa)\frac{\partial P_l}{\partial x} = 0, \tag{22}$$

respectively. Here, $l = 1, 2$ and $\bar{l} = 3 - l$. Under the traveling wave transformation

$$P_1(x,t) = U_1(\tau),\; P_2(x,t) = U_2(\tau),\quad \tau = B(x - vt) \tag{23}$$

we have

$$(a_l - b_l v)B^2\frac{\mathrm{d}^2 U_l}{\mathrm{d}\tau^2} + U_l\left(b_l\omega\kappa - \omega - a_l\kappa^2\right) + c_l\, F(U_l^2)U_l - k_l U_{\bar{l}} = 0, \tag{24}$$

and

$$\{-v(1 - b_l\kappa) + b_l\omega - 2a_l\kappa\}\, B\frac{\mathrm{d}U_l}{\mathrm{d}\tau} = 0. \tag{25}$$

Now, from Eq. (25), we get

$$v = \frac{b_l\omega - 2a_l\kappa}{1 - b_l\kappa}. \tag{26}$$

Now, equating the two values of the soliton speed leads to

$$a_1 = a_2 \tag{27}$$

and

$$b_1 = b_2. \tag{28}$$

The speed of the soliton therefore reduces to

$$v = \frac{b\omega - 2a\kappa}{1 - b\kappa}. \tag{29}$$

The coupled NLSE for twin-core couplers given by (15) and (16) modifies to

$$iq_t + aq_{xx} + bq_{xt} + c_1\ F\left(|q|^2\right) q = k_1 r, \tag{30}$$

$$ir_t + ar_{xx} + br_{xt} + c_2\ F\left(|r|^2\right) r = k_2 q. \tag{31}$$

where $a_1 = a_2 = a$ and $b_1 = b_2 = b$. Consequently, the Eq. (24) changes to

$$(a - bv)B^2 \frac{d^2 U_l}{d\tau^2} + U_l\left(b\omega\kappa - \omega - a\kappa^2\right) + c_l\ F(U_l^2)U_l - k_l U_{\bar{l}} = 0. \tag{32}$$

3.1 Kerr law nonlinearity

For Kerr law nonlinearity, $F(s) = s$. The model equations (30) and (31), for twin-core couplers with Kerr law nonlinearity, reduce to [6–9,43]

$$iq_t + aq_{xx} + bq_{xt} + c_1|q|^2 q = k_1 r, \tag{33}$$

$$ir_t + ar_{xx} + br_{xt} + c_2|r|^2 r = k_2 q. \tag{34}$$

Therefore, real part equation (32) is

$$(a - bv)B^2 \frac{d^2 U_l}{d\tau^2} + U_l\left(b\omega\kappa - \omega - a\kappa^2\right) + c_l U_l^3 - k_l U_{\bar{l}} = 0. \tag{35}$$

Using the assumption

$$U_l(\tau) = \lambda_l \cos^{\beta}(\mu\tau), \tag{36}$$

in Eq. (35), we obtain

$$(U_l)_{\tau\tau} = -\mu^2\beta^2\lambda_l \cos^{\beta}(\mu\tau) + \mu^2\lambda_l\beta(\beta - 1)\cos^{\beta-2}(\mu\tau). \tag{37}$$

Substituting Eqs. (36) and (37) into Eq. (35), we have

$$\lambda_l\left\{-(a - bv)B^2\mu^2\beta^2 + b\omega\kappa - \omega - a\kappa^2\right\}\cos^{\beta}(\mu\tau) + \lambda_l(a - bv)B^2\mu^2\beta(\beta - 1)\cos^{\beta-2}(\mu\tau) + c_l\lambda_l^3\cos^{3\beta}(\mu\tau) - k_l\lambda_{\bar{l}}\cos^{\beta}(\mu\tau) = 0. \tag{38}$$

Using the balance method, by equating the exponents and the coefficients of $\cos^K(\cdot)$, we get

$$\beta(\beta - 1) \neq 0, \tag{39}$$

$$3\beta = \beta - 2, \tag{40}$$

$$\lambda_l(a - bv)B^2\mu^2\beta(\beta - 1) + c_l\lambda_l^3 = 0, \tag{41}$$

$$\lambda_l\left\{-(a - bv)B^2\mu^2\beta^2 + b\omega\kappa - \omega - a\kappa^2\right\} - k_l\lambda_{\bar{l}} = 0. \tag{42}$$

Solving the system (Eqs (39)–(42)) simultaneously, we get the solution set

$$\beta = -1, \tag{43}$$

$$v = \frac{c_l\lambda_l^2 + 2aB^2\mu^2}{2bB^2\mu^2}, \tag{44}$$

$$\omega = \frac{2a\kappa^2\lambda_l - c_l\lambda_l^3 + 2k_l\lambda_{\bar{l}}}{2\lambda_l(b\kappa - 1)}. \tag{45}$$

Next, equating the two expressions for the soliton wave number from (45) for $l = 1, 2$ gives

$$\lambda_1\lambda_2\left(c_2\lambda_2^2 - c_1\lambda_1^2\right) = 2\left(k_2\lambda_1^2 - k_1\lambda_2^2\right). \tag{46}$$

Equating the two expressions for the soliton speed v from (44) implies

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{c_2}{c_1}}, \tag{47}$$

which immediately prompts the constraint

$$c_1 c_2 > 0. \tag{48}$$

The amplitudes of the solitons are given by the nonlinear coupled system (46) and (47). This shows that nonlinear terms from two components of NLSE must bear the same sign for bright solitons to exist.

Finally, equating the two expressions for the soliton speed v from (29) and (44) implies

$$B = \pm\sqrt{\frac{(1 - b\kappa)c_l}{2\mu^2\left(b^2\omega - a\kappa b - a\right)}}\lambda_l, \tag{49}$$

which introduces the constraint

$$(1 - b\kappa)c_l\left(b^2\omega - a\kappa b - a\right) > 0. \tag{50}$$

Consequently, we obtain singular periodic solutions with periodic blow-ups:

$$q(x,t)=\lambda_1 \sec\left[\sqrt{\frac{(1-b\kappa)c_1}{2\left(b^2\omega-a\kappa b-a\right)}}\lambda_1 \times\left\{x-\left(\frac{b\omega-2a\kappa}{1-b\kappa}\right)t\right\}\right]\mathrm{e}^{i(-\kappa x+\omega t+\theta)}, \tag{51}$$

$$r(x,t)=\lambda_2 \sec\left[\sqrt{\frac{(1-b\kappa)c_2}{2\left(b^2\omega-a\kappa b-a\right)}}\lambda_2 \times\left\{x-\left(\frac{b\omega-2a\kappa}{1-b\kappa}\right)t\right\}\right]\mathrm{e}^{i(-\kappa x+\omega t+\theta)} \tag{52}$$

where ω is given by Eq. (45). These periodic solutions will exist provided the constraint condition given by (48). Additional constraints that must also remain valid are

$$b\omega \neq 1, \tag{53}$$

$$b\kappa \neq 1, \tag{54}$$

$$b\omega \neq 0, \tag{55}$$

which follows from (26), (44) and (45).

It is easy to see that solutions (51) and (52) can reduce to bright soliton solutions given by

$$q(x,t)=\lambda_1 \mathrm{sech}\left[\sqrt{\frac{(b\kappa-1)c_1}{2\left(b^2\omega-a\kappa b-a\right)}}\lambda_1 \times\left\{x-\left(\frac{b\omega-2a\kappa}{1-b\kappa}\right)t\right\}\right]\mathrm{e}^{i(-\kappa x+\omega t+\theta)}, \tag{56}$$

$$r(x,t)=\lambda_2 \mathrm{sech}\left[\sqrt{\frac{(b\kappa-1)c_2}{2\left(b^2\omega-a\kappa b-a\right)}}\lambda_2 \times\left\{x-\left(\frac{b\omega-2a\kappa}{1-b\kappa}\right)t\right\}\right]\mathrm{e}^{i(-\kappa x+\omega t+\theta)} \tag{57}$$

where ω is given by Eq. (45). The bright solitons will exist provided (48) and (53)–(55) hold.

3.2 Power law nonlinearity

For power law nonlinear media, $F(s) = s^n$ where n represents power law nonlinearity factor. Thus, for twin-core couplers, the NLSE [6–9,43]

$$iq_t + aq_{xx} + bq_{xt} + c_1|q|^{2n}q = k_1 r, \tag{58}$$

$$ir_t + ar_{xx} + br_{xt} + c_2|r|^{2n}r = k_2 q. \tag{59}$$

It must be noted that $0 < n < 2$ for stability of solitons. Additionally, $n \neq 2$ to avoid self-focusing singularity. Therefore, real part equation (32) is

$$(a-bv)B^2\frac{\mathrm{d}^2U_l}{\mathrm{d}\tau^2}+U_l\left(b\omega\kappa-\omega-a\kappa^2\right) + c_l\, U_l^{2n+1}-k_l U_{\bar{l}}=0. \tag{60}$$

Using the assumption

$$U_l(\tau)=\lambda_l\cos^{\beta}(\mu\tau), \tag{61}$$

in Eq. (60), we obtain

$$(U_l)_{\tau\tau}=-\mu^2\beta^2\lambda_l\cos^{\beta}(\mu\tau) + \mu^2\lambda_l\beta(\beta-1)\cos^{\beta-2}(\mu\tau). \tag{62}$$

Substituting Eqs. (61) and (62) into Eq. (60), we have

$$\lambda_l\left\{-(a-bv)B^2\mu^2\beta^2+b\omega\kappa-\omega-a\kappa^2\right\}\cos^{\beta}(\mu\tau) + \lambda_l(a-bv)B^2\mu^2\beta(\beta-1)\cos^{\beta-2}(\mu\tau) + c_l\lambda_l^{2n+1}\cos^{(2n+1)\beta}(\mu\tau)-k_l\lambda_{\bar{l}}\cos^{\beta}(\mu\tau)=0. \tag{63}$$

Using the balance method, by equating the exponents and the coefficients of $\cos^K(\cdot)$, we get

$$\beta(\beta-1)\neq 0, \tag{64}$$

$$(2n+1)\beta=\beta-2, \tag{65}$$

$$\lambda_l(a-bv)B^2\mu^2\beta(\beta-1)+c_l\lambda_l^{2n+1}=0, \tag{66}$$

$$\lambda_l\left\{-(a-bv)B^2\mu^2\beta^2+b\omega\kappa-\omega-a\kappa^2\right\}-k_l\lambda_{\bar{l}}=0. \tag{67}$$

Solving the system (Eqs. (64)–(67)) simultaneously, we get the solution set

$$\beta=-\frac{1}{n}, \tag{68}$$

$$v=\frac{n^2c_l\lambda_l^{2n}+(1+n)aB^2\mu^2}{(1+n)bB^2\mu^2}, \tag{69}$$

$$\omega=\frac{(1+n)a\kappa^2\lambda_l-c_l\lambda_l^{1+2n}+(1+n)k_l\lambda_{\bar{l}}}{(1+n)\lambda_l(b\kappa-1)}. \tag{70}$$

Equating the speed of the solitons from the two components implies

$$\frac{\lambda_1}{\lambda_2}=\left(\frac{c_2}{c_1}\right)^{\frac{1}{2n}}, \tag{71}$$

which poses the constraint (48). Next, after equating the two expressions for the soliton wave number the relation

$$\lambda_1\lambda_2\left(c_2\lambda_2^{2n}-c_1\lambda_1^{2n}\right)=(1+n)\left(k_2\lambda_1^2-k_1\lambda_2^2\right). \tag{72}$$

Finally, equating the two expressions for the soliton speed v from (29) and (69) implies

$$B=\pm\sqrt{\frac{(1-b\kappa)c_l}{(1+n)\mu^2\left(b^2\omega-a\kappa b-a\right)}n\lambda_l^n}, \tag{73}$$

which implies

$$(1-b\kappa)c_l\left(b^2\omega-a\kappa b-a\right)>0. \tag{74}$$

Consequently, we obtain the following singular periodic solutions:

$$q(x,t)=\lambda_1\sec^{\frac{1}{n}}\left[\sqrt{\frac{(1-b\kappa)c_1}{(1+n)\left(b^2\omega-a\kappa b-a\right)}n\lambda_1^n}\times\left\{x-\left(\frac{b\omega-2a\kappa}{1-b\kappa}\right)t\right\}\right]e^{i(-\kappa x+\omega t+\theta)}, \tag{75}$$

$$r(x,t)=\lambda_2\sec^{\frac{1}{n}}\left[\sqrt{\frac{(1-b\kappa)c_2}{(1+n)\left(b^2\omega-a\kappa b-a\right)}n\lambda_2^n}\times\left\{x-\left(\frac{b\omega-2a\kappa}{1-b\kappa}\right)t\right\}\right]e^{i(-\kappa x+\omega t+\theta)} \tag{76}$$

where ω is given by Eq. (70). This periodic solution will exist provided the relations for (48) and (53)–(55).

It is easy to see that solutions (75) and (76) can reduce to bright 1-soliton solutions

$$q(x,t)=\lambda_1\operatorname{sech}^{\frac{1}{n}}\left[\sqrt{\frac{(b\kappa-1)c_1}{(1+n)\left(b^2\omega-a\kappa b-a\right)}n\lambda_1^n}\times\left\{x-\left(\frac{b\omega-2a\kappa}{1-b\kappa}\right)t\right\}\right]e^{i(-\kappa x+\omega t+\theta)}, \tag{77}$$

$$r(x,t)=\lambda_2\operatorname{sech}^{\frac{1}{n}}\left[\sqrt{\frac{(b\kappa-1)c_2}{(1+n)\left(b^2\omega-a\kappa b-a\right)}n\lambda_2^n}\times\left\{x-\left(\frac{b\omega-2a\kappa}{1-b\kappa}\right)t\right\}\right]e^{i(-\kappa x+\omega t+\theta)} \tag{78}$$

where ω is given by Eq. (70). The bright solitons will exist provided (48) and (53)–(55) hold.

3.3 Parabolic law nonlinearity

For parabolic law nonlinear media, the governing NLSE is given by [6–9,43]

$$iq_t+aq_{xx}+bq_{xt}+\left(\xi_1|q|^2+\eta_1|q|^4\right)q=k_1r, \tag{79}$$

$$ir_t+ar_{xx}+br_{xt}+\left(\xi_2|r|^2+\eta_2|r|^4\right)r=k_2q. \tag{80}$$

The parameters ξ_l and η_l for $l=1,2$ represent the coefficients of cubic and quintic nonlinear terms for the two components. In this case, real part equation (32) reduces to

$$(a-bv)B^2\frac{d^2U_l}{d\tau^2}+U_l\left(b\omega\kappa-\omega-a\kappa^2\right)+\xi_lU_l^3+\eta_lU_l^5-k_lU_{\bar{l}}=0. \tag{81}$$

Balancing U_l'' with U_l^5 in Eq. (81), we have

$$N+2=5N\Leftrightarrow 2=4N\Leftrightarrow N=\frac{1}{2}.$$

We then assume that Eq. (81) has the following formal solution:

$$U_l(\tau)=A_l\left(G(\tau)\right)^{\frac{1}{2}},\quad A_l\neq 0 \tag{82}$$

where A_l are constants to be determined later and G satisfies

$$G'(\tau)=\delta G(\tau)-G^2(\tau). \tag{83}$$

Thus, we obtain

$$(a-bv)B^2\left(\frac{3}{4}A_lG^{\frac{3}{2}}-A_l\delta G^{\frac{1}{2}}+\frac{\delta^2}{4}A_l\right)+A_l\left(b\omega\kappa-\omega-a\kappa^2\right)+\xi_lA_l^3G^{\frac{1}{2}}+\eta_lA_l^5G^{\frac{3}{2}}-k_lA_{\bar{l}}=0. \tag{84}$$

Then, equating the coefficient of each power of G to zero, we obtain a system of nonlinear algebraic equations and by solving it, we get

$$v=\frac{4\eta_lA_l^4+3B^2a}{3B^2b}, \tag{85}$$

$$\delta=-\frac{3\xi_l}{4A_l^2\eta_l}, \tag{86}$$

$$\omega=\frac{3\xi_l^2A_l+16A_la\kappa^2\eta_l+16k_l\eta_lA_{\bar{l}}}{16A_l\eta_l\left(b\kappa-1\right)} \tag{87}$$

where $B,\ \kappa,\ A_l,\ k_l$ are arbitrary constants.

Next, equating the two expressions for the soliton wave number from (87) for $l = 1, 2$ gives

$$3A_1A_2\left(\xi_1^2\eta_2 - \xi_2^2\eta_1\right) = 16\left(k_2A_1^2 - k_1A_2^2\right)\eta_1\eta_2. \tag{88}$$

Equating the two expressions for the soliton speed v from (85) implies

$$\frac{A_1}{A_2} = \left(\frac{\eta_2}{\eta_1}\right)^{\frac{1}{4}}, \tag{89}$$

which leads to

$$\eta_1\eta_2 > 0. \tag{90}$$

The amplitudes of the solitons are given by the nonlinear coupled system (88) and (89).

Finally, equating the two expressions for the soliton speed v from (29) and (85) implies

$$B = \pm\frac{2A_l^2\sqrt{\eta_l\,(1-b\kappa)}}{\sqrt{3\left(b^2\omega - ab\kappa - a\right)}} \tag{91}$$

that compels

$$\eta_l\,(1-b\kappa)\left(b^2\omega - ab\kappa - a\right) > 0. \tag{92}$$

Thus, we obtain the exact traveling wave solution of Eqs. (79) and (80) as

$$q(x,t) = \left\{-\frac{3\xi_1}{8\eta_1}\left[1 \pm \tanh\left(\frac{\xi_1}{4}\sqrt{\frac{3(1-b\kappa)}{\eta_1(b^2\omega - ab\kappa - a)}}\right.\right.\right.$$
$$\left.\left.\left.\times\left(x - \left(\frac{b\omega - 2a\kappa}{1-b\kappa}\right)t\right)\right)\right]\right\}^{\frac{1}{2}} e^{i(-\kappa x + \omega t + \theta)}, \tag{93}$$

$$r(x,t) = \left\{-\frac{3\xi_2}{8\eta_2}\left[1 \pm \tanh\left(\frac{\xi_2}{4}\sqrt{\frac{3(1-b\kappa)}{\eta_2(b^2\omega - ab\kappa - a)}}\right.\right.\right.$$
$$\left.\left.\left.\times\left(x - \left(\frac{b\omega - 2a\kappa}{1-b\kappa}\right)t\right)\right)\right]\right\}^{\frac{1}{2}} e^{i(-\kappa x + \omega t + \theta)}, \tag{94}$$

which are dark 1-soliton solutions and

$$q(x,t) = \left\{-\frac{3\xi_1}{8\eta_1}\left[1 \pm \coth\left(\frac{\xi_1}{4}\sqrt{\frac{3(1-b\kappa)}{\eta_1(b^2\omega - ab\kappa - a)}}\right.\right.\right.$$
$$\left.\left.\left.\times\left(x - \left(\frac{b\omega - 2a\kappa}{1-b\kappa}\right)t\right)\right)\right]\right\}^{\frac{1}{2}} e^{i(-\kappa x + \omega t + \theta)}, \tag{95}$$

$$r(x,t) = \left\{-\frac{3\xi_2}{8\eta_2}\left[1 \pm \coth\left(\frac{\xi_2}{4}\sqrt{\frac{3(1-b\kappa)}{\eta_2(b^2\omega - ab\kappa - a)}}\right.\right.\right.$$
$$\left.\left.\left.\times\left(x - \left(\frac{b\omega - 2a\kappa}{1-b\kappa}\right)t\right)\right)\right]\right\}^{\frac{1}{2}} e^{i(-\kappa x + \omega t + \theta)}, \tag{96}$$

which are singular 1-soliton solutions, where ω is given by Eq. (87).

3.4 Dual-power law nonlinearity

For dual-power law nonlinearity, the governing coupled NLSE is [6–9,43]

$$iq_t + aq_{xx} + bq_{xt} + \left(\xi_1|q|^{2n} + \eta_1|q|^{4n}\right)q = k_1 r, \tag{97}$$

$$ir_t + ar_{xx} + br_{xt} + \left(\xi_2|r|^{2n} + \eta_2|r|^{4n}\right)r = k_2 q. \tag{98}$$

The special case, for $n = 1$, is parabolic law nonlinearity, discussed in the previous subsection. In this case, real part Eq. (32) reduces to

$$(a - bv)B^2\frac{d^2U_l}{d\tau^2} + U_l\left(b\omega\kappa - \omega - a\kappa^2\right)$$
$$+ \xi_l U_l^{2n+1} + \eta_l U_l^{4n+1} - k_l U_{\bar{l}} = 0. \tag{99}$$

Balancing U_l'' with U_l^{4n+1} in Eq. (99), we have

$$N + 2 = (4n+1)N \Leftrightarrow 2 = 4nN \Leftrightarrow N = \frac{1}{2n}.$$

We then assume that Eq. (99) has the following formal solution:

$$U_l(\tau) = A_l G^{\frac{1}{2n}}(\tau), \quad A_l \neq 0 \tag{100}$$

where A_l are constants to be determined later and G satisfies Eq. (83). Thus, we obtain

$$(a-bv)B^2\left\{\frac{1+2n}{4n^2}A_lG^2 - \frac{A_l\delta(1+n)}{2n^2}G + \frac{\delta^2}{4n^2}A_l\right\}$$
$$+ A_l\left(b\omega\kappa - \omega - a\kappa^2\right) + \xi_l A_l^{2n+1}G$$
$$+ \eta_l A_l^{4n+1}G^2 - k_l \quad A_{\bar{l}} = 0. \tag{101}$$

Then, equating the coefficient of each power of G to zero, we obtain a system of nonlinear algebraic equations and by solving it, we get

$$v=\frac{4n^2\eta_l A_l^{4n}+(1+2n)B^2a}{(1+2n)B^2b}, \tag{102}$$

$$\delta=-\frac{(1+2n)\xi_l}{2(1+n)A_l^{2n}\eta_l}, \tag{103}$$

$$\omega=\frac{(1+2n)\xi_l^2A_l+4(1+n)^2A_la\kappa^2\eta_l+4(1+n)^2k_l\eta_lA_{\bar{l}}}{4(1+n)^2A_l\eta_l\,(b\kappa-1)} \tag{104}$$

where $B,\ \kappa,\ A_l,\ k_l$ are arbitrary constants. Next, equating the two expressions for the soliton wave number from (104) for $l=1,2$ gives

$$(1+2n)A_1A_2\left(\xi_1^2\eta_2-\xi_2^2\eta_1\right) = 4(1+n)^2\left(k_2A_1^2-k_1A_2^2\right)\eta_1\eta_2. \tag{105}$$

Equating the two expressions for the soliton speed v from (102) implies

$$\frac{A_1}{A_2}=\left(\frac{\eta_2}{\eta_1}\right)^{\frac{1}{4n}}, \tag{106}$$

which remains valid provided

$$\eta_1\eta_2>0. \tag{107}$$

The amplitudes of the solitons are given by the nonlinear coupled system (105) and (106).

Finally, equating the two expressions for the soliton speed v from (29) and (102) implies

$$B=\pm\frac{2nA_l^{2n}\sqrt{\eta_l\,(1-b\kappa)}}{\sqrt{(1+2n)\left(b^2\omega-ab\kappa-a\right)}} \tag{108}$$

for

$$\eta_l\,(1-b\kappa)\left(b^2\omega-ab\kappa-a\right)>0. \tag{109}$$

Thus, we obtain the exact traveling wave solution of Eqs. (97) and (98) as

$$q(x,t)=\left\{-\frac{(1+2n)\xi_1}{4(1+n)\eta_1}\left[1\pm\tanh\left(\frac{n\xi_1}{2(1+n)}\sqrt{\frac{(1+2n)(1-b\kappa)}{\eta_1(b^2\omega-ab\kappa-a)}}\times\left(x-\left(\frac{b\omega-2a\kappa}{1-b\kappa}\right)t\right)\right)\right]\right\}^{\frac{1}{2n}}e^{i(-\kappa x+\omega t+\theta)}, \tag{110}$$

$$r(x,t)=\left\{-\frac{(1+2n)\xi_2}{4(1+n)\eta_2}\left[1\pm\tanh\left(\frac{n\xi_2}{2(1+n)}\sqrt{\frac{(1+2n)(1-b\kappa)}{\eta_2(b^2\omega-ab\kappa-a)}}\times\left(x-\left(\frac{b\omega-2a\kappa}{1-b\kappa}\right)t\right)\right)\right]\right\}^{\frac{1}{2n}}e^{i(-\kappa x+\omega t+\theta)}, \tag{111}$$

which are dark 1-soliton solutions and

$$q(x,t)=\left\{-\frac{(1+2n)\xi_1}{4(1+n)\eta_1}\left[1\pm\coth\left(\frac{n\xi_1}{2(1+n)}\sqrt{\frac{(1+2n)(1-b\kappa)}{\eta_1(b^2\omega-ab\kappa-a)}}\times\left(x-\left(\frac{b\omega-2a\kappa}{1-b\kappa}\right)t\right)\right)\right]\right\}^{\frac{1}{2n}}e^{i(-\kappa x+\omega t+\theta)}, \tag{112}$$

$$r(x,t)=\left\{-\frac{(1+2n)\xi_2}{4(1+n)\eta_2}\left[1\pm\coth\left(\frac{n\xi_2}{2(1+n)}\sqrt{\frac{(1+2n)(1-b\kappa)}{\eta_2(b^2\omega-ab\kappa-a)}}\times\left(x-\left(\frac{b\omega-2a\kappa}{1-b\kappa}\right)t\right)\right)\right]\right\}^{\frac{1}{2n}}e^{i(-\kappa x+\omega t+\theta)}, \tag{113}$$

which are singular 1-soliton solutions, where ω is given by Eq. (104).

4 Multiple-core couplers (coupling with nearest neighbors)

The governing equation for twin- core couplers is given by [6–9,43]

$$iq_t^{(l)}+a_lq_{xx}^{(l)}+b_lq_{xt}^{(l)}+c_l\,F\left(|q^{(l)}|^2\right)q^{(l)} = k_l\left[q^{(l-1)}-2q^{(l)}+q^{(l+1)}\right]. \tag{114}$$

where $1\le l\le N$. Equation (114) represents the general model for optical couplers where coupling with nearest neighbors is considered. Here, k_l are, as before, the coupling coefficients. In order to address this model for the four forms of nonlinear media, the initial hypothesis is taken to be

$$q^{(l)}(x,t)=P_l(x,t)e^{i\phi(x,t)} \tag{115}$$

where the amplitude component of soliton is $P_l(x,t)$ while the amplitude component carries the same definition as in (18) or (19). After substituting this hypothesis (115) into (114), the resulting expression is split into real and imaginary components. The imaginary part gives the speed of the soliton as

$$v = \frac{b_l\omega - 2a_l\kappa}{1 - b_l\kappa}. \tag{116}$$

The speed of the soliton stays the same irrespective of the type of nonlinearity and type of solitons that is going to be addressed. Next, the real part implies

$$a_l \frac{\partial^2 P_l}{\partial x^2} + b_l \frac{\partial^2 P_l}{\partial x \partial t} + P_l \left(b_l\omega\kappa - \omega - a_l\kappa^2\right) + c_l\, F(P_l^2)P_l - k_l\left[P_{l-1} - 2P_l + P_{l+1}\right] = 0. \tag{117}$$

Under the traveling wave transformation

$$P_l(x,t) = U_l(\tau), \quad \tau = B(x - vt) \tag{118}$$

we have

$$(a_l - b_l v)B^2 \frac{\mathrm{d}^2 U_l}{\mathrm{d}\tau^2} + U_l\left(b_l\omega\kappa - \omega - a_l\kappa^2\right) + c_l\, F(U_l^2)U_l - k_l\left[U_{l-1} - 2U_l + U_{l+1}\right] = 0. \tag{119}$$

4.1 Kerr law nonlinearity

For Kerr law, the coupled NLSE modifies to [6–9,43]

$$iq_t^{(l)} + a_l q_{xx}^{(l)} + b_l q_{xt}^{(l)} + c_l\, |q^{(l)}|^2 q^{(l)} = k_l\left[q^{(l-1)} - 2q^{(l)} + q^{(l+1)}\right]. \tag{120}$$

For hypothesis given by (115) and (118), Eq. (120) reduces to

$$(a_l - b_l v)B^2 \frac{\mathrm{d}^2 U_l}{d\tau^2} + U_l\left(b_l\omega\kappa - \omega - a_l\kappa^2\right) + c_l\, U_l^3 - k_l\left[U_{l-1} - 2U_l + U_{l+1}\right] = 0. \tag{121}$$

Using the assumption

$$U_l(\tau) = \lambda_l \cos^\beta(\mu\tau), \tag{122}$$

in Eq. (121), we obtain

$$(U_l)_{\tau\tau} = -\mu^2\beta^2\lambda_l\cos^\beta(\mu\tau) + \mu^2\lambda_l\beta(\beta-1)\cos^{\beta-2}(\mu\tau). \tag{123}$$

Substituting Eqs. (122) and (123) into Eq. (121), we have

$$\lambda_l\left\{-(a_l - b_l v)B^2\mu^2\beta^2 + b_l\omega\kappa - \omega - a_l\kappa^2\right\}\cos^\beta(\mu\tau) + \lambda_l(a_l - b_l v)B^2\mu^2\beta(\beta-1)\cos^{\beta-2}(\mu\tau) + c_l\lambda_l^3\cos^{3\beta}(\mu\tau) - k_l\left(\lambda_{l-1} - 2\lambda_l + \lambda_{l+1}\right)\cos^\beta(\mu\tau) = 0. \tag{124}$$

Using the balance method, by equating the exponents and the coefficients of $\cos^K(\cdot)$, we get

$$\beta(\beta-1) \neq 0, \tag{125}$$

$$3\beta = \beta - 2, \tag{126}$$

$$\lambda_l(a_l - b_l v)B^2\mu^2\beta(\beta-1) + c_l\lambda_l^3 = 0, \tag{127}$$

$$\lambda_l\left\{-(a_l - b_l v)B^2\mu^2\beta^2 + b_l\omega\kappa - \omega - a_l\kappa^2\right\} - k_l\left(\lambda_{l-1} - 2\lambda_l + \lambda_{l+1}\right) = 0. \tag{128}$$

Solving the system (Eqs. (125)–(128)) simultaneously, we get the solution set

$$\beta = -1, \tag{129}$$

$$v = \frac{c_l\lambda_l^2 + 2a_l B^2\mu^2}{2b_l B^2\mu^2}, \tag{130}$$

$$\omega = \frac{2a_l\kappa^2\lambda_l - c_l\lambda_l^3 + 2k_l\left(\lambda_{l-1} - 2\lambda_l + \lambda_{l+1}\right)}{2\lambda_l(b_l\kappa - 1)}. \tag{131}$$

equating the two expressions for the soliton speed v from (116) and (130) implies

$$B = \pm\sqrt{\frac{(1 - b_l\kappa)c_l}{2\mu^2\left(b_l^2\omega - a_l\kappa b_l - a_l\right)}}\lambda_l, \tag{132}$$

which kicks in the constraint

$$(1 - b_l\kappa)c_l\left(b_l^2\omega - a_l\kappa b_l - a_l\right) > 0. \tag{133}$$

Therefore, we obtain the following singular periodic solution:

$$q^{(l)}(x,t) = \lambda_l \sec\left[\sqrt{\frac{(1 - b_l\kappa)c_l}{2\left(b_l^2\omega - a_l\kappa b_l - a_l\right)}}\lambda_l \times\left\{x - \left(\frac{b_l\omega - 2a_l\kappa}{1 - b_l\kappa}\right)t\right\}\right]\mathrm{e}^{i(-\kappa x + \omega t + \theta)}, \tag{134}$$

where ω is given by Eq. (131).

It is easy to see that solution (134) can reduce to bright 1-soliton solution as

$$q^{(l)}(x,t) = \lambda_l \,\mathrm{sech}\left[\sqrt{\frac{(b_l\kappa - 1)c_l}{2\left(b_l^2\omega - a_l\kappa b_l - a_l\right)}}\lambda_l \times\left\{x - \left(\frac{b_l\omega - 2a_l\kappa}{1 - b_l\kappa}\right)t\right\}\right]\mathrm{e}^{i(-\kappa x + \omega t + \theta)}, \tag{135}$$

where ω is given by Eq. (131).

4.2 Power law nonlinearity

For power law, the coupled NLSE modifies to [6–9,43]

$$iq_t^{(l)} + a_l q_{xx}^{(l)} + b_l q_{xt}^{(l)} + c_l \, |q^{(l)}|^{2n} q^{(l)} = k_l \left[q^{(l-1)} - 2q^{(l)} + q^{(l+1)} \right]. \tag{136}$$

In this case, Eq. (119) gives

$$(a_l - b_l v)B^2 \frac{\mathrm{d}^2 U_l}{\mathrm{d}\tau^2} + U_l \left(b_l \omega\kappa - \omega - a_l \kappa^2 \right) + c_l \, U_l^{2n+1} - k_l \left[U_{l-1} - 2U_l + U_{l+1} \right] = 0. \tag{137}$$

Using the assumption

$$U_l(\tau) = \lambda_l \cos^{\beta}(\mu\tau), \tag{138}$$

in Eq. (137), we obtain

$$(U_l)_{\tau\tau} = -\mu^2\beta^2\lambda_l \cos^{\beta}(\mu\tau) + \mu^2\lambda_l\beta(\beta-1)\cos^{\beta-2}(\mu\tau). \tag{139}$$

Substituting Eqs. (138) and (139) into Eq. (137), we have

$$\lambda_l \left\{ -(a-bv)B^2\mu^2\beta^2 + b\omega\kappa - \omega - a\kappa^2 \right\} \cos^{\beta}(\mu\tau) + \lambda_l (a-bv)B^2\mu^2\beta(\beta-1)\cos^{\beta-2}(\mu\tau) + c_l \lambda_l^{2n+1} \cos^{(2n+1)\beta}(\mu\tau) - k_l \left(\lambda_{l-1} - 2\lambda_l + \lambda_{l+1} \right) \cos^{\beta}(\mu\tau) = 0. \tag{140}$$

Using the balance method, by equating the exponents and the coefficients of $\cos^{K}(\cdot)$, we get

$$\beta(\beta-1) \neq 0, \tag{141}$$

$$(2n+1)\beta = \beta - 2, \tag{142}$$

$$\lambda_l (a_l - b_l v)B^2\mu^2\beta(\beta-1) + c_l \lambda_l^{2n+1} = 0, \tag{143}$$

$$\lambda_l \left\{ -(a_l - b_l v)B^2\mu^2\beta^2 + b_l\omega\kappa - \omega - a_l\kappa^2 \right\} - k_l \left(\lambda_{l-1} - 2\lambda_l + \lambda_{l+1} \right) = 0. \tag{144}$$

Solving the system (Eqs. (141)–(144)) simultaneously, we get the solution set

$$\beta = -\frac{1}{n}, \tag{145}$$

$$v = \frac{n^2 c_l \lambda_l^{2n} + (1+n)a_l B^2\mu^2}{(1+n)b_l B^2\mu^2}, \tag{146}$$

$$\omega = \frac{(1+n)a_l\kappa^2\lambda_l - c_l\lambda_l^{1+2n} + (1+n)k_l \left(\lambda_{l-1} - 2\lambda_l + \lambda_{l+1} \right)}{(1+n)\lambda_l (b_l\kappa - 1)}. \tag{147}$$

Equating the two expressions for the soliton speed v from (116) and (146) implies

$$B = \pm \sqrt{\frac{(1-b_l\kappa)c_l}{(1+n)\mu^2 \left(b_l^2\omega - a_l\kappa b_l - a_l \right)}} \, n\lambda_l^n, \tag{148}$$

which forces

$$(1-b_l\kappa)c_l \left(b_l^2\omega - a_l\kappa b_l - a_l \right) > 0. \tag{149}$$

Hence, one recovers singular periodic solution:

$$q^{(l)}(x,t) = \lambda_l \sec^{\frac{1}{n}} \left[\sqrt{\frac{(1-b_l\kappa)c_l}{(1+n)\left(b_l^2\omega - a_l\kappa b_l - a_l \right)}} \, n\lambda_l^n \times \left(x - \left\{ \frac{b_l\omega - 2a_l\kappa}{1-b_l\kappa} \right) t \right\} \right] \mathrm{e}^{i(-\kappa x + \omega t + \theta)}, \tag{150}$$

where ω is given by Eq. (147).

It is easy to see that solution (150) leads to bright 1-soliton solution:

$$q^{(l)}(x,t) = \lambda_l \operatorname{sech}^{\frac{1}{n}} \left[\sqrt{\frac{(b_l\kappa - 1)c_l}{(1+n)\left(b_l^2\omega - a_l\kappa b_l - a_l \right)}} \, n\lambda_l^n \times \left\{ x - \left(\frac{b_l\omega - 2a_l\kappa}{1-b_l\kappa} \right) t \right\} \right] \mathrm{e}^{i(-\kappa x + \omega t + \theta)}, \tag{151}$$

where ω is given by Eq. (147).

4.3 Parabolic law nonlinearity

In this case, the governing equation reduces to [6–9,43]

$$iq_t^{(l)} + a_l q_{xx}^{(l)} + b_l q_{xt}^{(l)} + \left(\xi_l \, |q^{(l)}|^2 + \eta_l \, |q^{(l)}|^4 \right) q^{(l)} = k_l \left[q^{(l-1)} - 2q^{(l)} + q^{(l+1)} \right]. \tag{152}$$

where $1 \le l \le N$. The real part equation therefore is

$$(a_l - b_l v)B^2 \frac{\mathrm{d}^2 U_l}{\mathrm{d}\tau^2} + U_l \left(b_l\omega\kappa - \omega - a_l\kappa^2 \right) + \xi_l \, U_l^3 + \eta_l \, U_l^5 - k_l \left(U_{l-1} - 2U_l + U_{l+1} \right) = 0. \tag{153}$$

We then assume that Eq. (153) has the following formal solution:

$$U_l(\tau) = A_l \left(G(\tau) \right)^{\frac{1}{2}}, \quad A_l \neq 0 \tag{154}$$

where A_l are constants to be determined later and G satisfies Eq. (83).

Thus, we obtain

$$(a_l - b_l v)B^2\left(\frac{3}{4}A_l G^{\frac{3}{2}} - A_l\delta G^{\frac{1}{2}} + \frac{\delta^2}{4}A_l\right) + A_l\left(b\omega\kappa - \omega - a\kappa^2\right) + \xi_l A_l^3 G^{\frac{1}{2}} + \eta_l A_l^5 G^{\frac{3}{2}} - k_l\left(A_{l-1} - 2A_l + A_{l+1}\right) = 0. \quad (155)$$

Then, equating the coefficient of each power of G to zero, we obtain a system of nonlinear algebraic equations and by solving it, we get

$$v = \frac{4\eta_l A_l^4 + 3B^2 a_l}{3B^2 b_l}, \quad (156)$$

$$\delta = -\frac{3\xi_l}{4A_l^2\eta_l}, \quad (157)$$

$$\omega = \frac{3\xi_l^2 A_l + 16A_l a_l\kappa^2\eta_l + 16k_l\eta_l\left(A_{l-1} - 2A_l + A_{l+1}\right)}{16A_l\eta_l\left(b_l\kappa - 1\right)} \quad (158)$$

where $B,\ \kappa,\ A_l,\ k_l$ are arbitrary constants.

Equating the two expressions for the soliton speed v from (116) and (156) implies

$$B = \pm\frac{2A_l^2\sqrt{\eta_l\left(1 - b_l\kappa\right)}}{\sqrt{3\left(b_l^2\omega - a_l b_l\kappa - a_l\right)}} \quad (159)$$

as long as

$$\eta_l\left(1 - b_l\kappa\right)\left(b_l^2\omega - a_l b_l\kappa - a_l\right) > 0. \quad (160)$$

Thus, we obtain the exact traveling wave solution of Eq. (152) as

$$q^{(l)}(x,t) = \left\{-\frac{3\xi_l}{8\eta_l}\left[1 \pm \tanh\left(\frac{\xi_l}{4}\sqrt{\frac{3(1-b_l\kappa)}{\eta_l(b_l^2\omega - a_l b_l\kappa - a_l)}} \times \left(x - \left(\frac{b_l\omega - 2a_l\kappa}{1 - b_l\kappa}\right)t\right)\right)\right]\right\}^{\frac{1}{2}} e^{i(-\kappa x + \omega t + \theta)}, \quad (161)$$

which is a dark 1-soliton solution and a singular 1-soliton solution given by

$$q^{(l)}(x,t) = \left\{-\frac{3\xi_l}{8\eta_l}\left[1 \pm \coth\left(\frac{\xi_l}{4}\sqrt{\frac{3(1-b_l\kappa)}{\eta_l(b_l^2\omega - a_l b_l\kappa - a_l)}} \times \left(x - \left(\frac{b_l\omega - 2a_l\kappa}{1 - b_l\kappa}\right)t\right)\right)\right]\right\}^{\frac{1}{2}} e^{i(-\kappa x + \omega t + \theta)}, \quad (162)$$

where ω is given by Eq. (158).

4.4 Dual-power law nonlinearity

For dual-power law nonlinearity, the governing equation is [1–6]

$$iq_t^{(l)} + a_l q_{xx}^{(l)} + b_l q_{xt}^{(l)} + \left(\xi_l\,|q^{(l)}|^{2n} + \eta_l\,|q^{(l)}|^{4n}\right)q^{(l)} = k_l\left[q^{(l-1)} - 2q^{(l)} + q^{(l+1)}\right]. \quad (163)$$

where $1 \leq l \leq N$. The real part equation therefore is

$$(a_l - b_l v)B^2\frac{d^2U_l}{d\tau^2} + U_l\left(b_l\omega\kappa - \omega - a_l\kappa^2\right) + \xi_l\,U_l^{2n+1} + \eta_l\,U_l^{4n+1} - k_l\left(U_{l-1} - 2U_l + U_{l+1}\right) = 0. \quad (164)$$

We then assume that Eq. (164) has the following formal solution:

$$U_l(\tau) = A_l G^{\frac{1}{2n}}(\tau), \quad A_l \neq 0 \quad (165)$$

where A_l are constants to be determined later and G satisfies Eq. (83). Thus, we obtain

$$(a_l - b_l v)B^2\left\{\frac{1+2n}{4n^2}A_l G^2 - \frac{A_l\delta(1+n)}{2n^2}G + \frac{\delta^2}{4n^2}A_l\right\} + A_l\left(b_l\omega\kappa - \omega - a_l\kappa^2\right) + \xi_l A_l^{2n+1}G + \eta_l A_l^{4n+1}G^2 - k_l\left(A_{l-1} - 2A_l + A_{l+1}\right) = 0. \quad (166)$$

Then, equating the coefficient of each power of G to zero, we obtain a system of nonlinear algebraic equations and by solving it, we get

$$v = \frac{4n^2\eta_l A_l^{4n} + (1+2n)B^2 a_l}{(1+2n)B^2 b_l}, \quad (167)$$

$$\delta = -\frac{(1+2n)\xi_l}{2(1+n)A_l^{2n}\eta_l}, \quad (168)$$

$$\omega = \frac{(1+2n)\xi_l^2 A_l + 4(1+n)^2 A_l a_l \kappa^2 \eta_l + 4(1+n)^2 k_l \eta_l \left(A_{l-1} - 2A_l + A_{l+1}\right)}{4(1+n)^2 A_l \eta_l \left(b_l \kappa - 1\right)} \tag{169}$$

where $B,\ \kappa,\ A_l,\ k_l$ are arbitrary constants.

Equating the two values of the speed v from the imaginary part equation (116) and real part equation (167) gives the free parameter

$$B = \pm \frac{2nA_l^{2n}\sqrt{\eta_l\left(1-b_l\kappa\right)}}{\sqrt{(1+2n)\left(b_l^2\omega - a_l b_l \kappa - a_l\right)}} \tag{170}$$

which therefore induces the constraint

$$\eta_l\left(1-b_l\kappa\right)\left(b_l^2\omega - a_l b_l\kappa - a_l\right) > 0. \tag{171}$$

Thus, we obtain the exact traveling wave solution of Eq. (163) as

$$q^{(l)}(x,t) = \left\{ -\frac{(1+2n)\xi_l}{4(1+n)\eta_l}\left[1 \pm \tanh \left(\frac{n\xi_l}{2(1+n)}\sqrt{\frac{(1+2n)(1-b_l\kappa)}{\eta_l(b_l^2\omega - a_l b_l\kappa - a_l)}} \times \left(x - \left(\frac{b_l\omega - 2a_l\kappa}{1-b_l\kappa}\right)t\right)\right)\right]\right\}^{\frac{1}{2n}} e^{i(-\kappa x+\omega t+\theta)}, \tag{172}$$

and

$$q^{(l)}(x,t) = \left\{ -\frac{(1+2n)\xi_l}{4(1+n)\eta_l}\left[1 \pm \coth \left(\frac{n\xi_l}{2(1+n)}\sqrt{\frac{(1+2n)(1-b_l\kappa)}{\eta_l(b_l^2\omega - a_l b_l\kappa - a_l)}} \times \left(x - \left(\frac{b_l\omega - 2a_l\kappa}{1-b_l\kappa}\right)t\right)\right)\right]\right\}^{\frac{1}{2n}} e^{i(-\kappa x+\omega t+\theta)}, \tag{173}$$

which are dark and singular 1-soliton solutions respectively, where ω is given by Eq. (169).

5 Multiple-core couplers (coupling with all neighbors)

The governing equation for multiple-core couplers, where coupling is with all neighbors is [6–9,43]

$$iq_t^{(l)} + a_l q_{xx}^{(l)} + b_l q_{xt}^{(l)} + c_l\, F\left(|q^{(l)}|^2\right) q^{(l)} = \sum_{m=1}^{N} k_{lm} q^{(m)}. \tag{174}$$

where $1 \le l \le N$ and k_{lm} represents the coupling coefficient with all neighbors. The solution hypothesis is taken to be the same as given by (115). Substituting this hypothesis into (174) and again splitting into real and imaginary parts, one obtains the same speed of solitons, as in (116), that is valid for all types of solitons in all nonlinear media considered in this paper. The real part equation now is

$$a_l \frac{\partial^2 P_l}{\partial x^2} + b_l \frac{\partial^2 P_l}{\partial x \partial t} + P_l\left(b_l\omega\kappa - \omega - a_l\kappa^2\right) + c_l\, F(P_l^2)P_l - \sum_{m=1}^{N} k_{lm} P_m = 0. \tag{175}$$

Under the traveling wave transformation

$$P_l(x,t) = U_l(\tau), \quad \tau = B(x - vt) \tag{176}$$

we have

$$(a_l - b_l v)B^2 \frac{d^2 U_l}{d\tau^2} + U_l\left(b_l\omega\kappa - \omega - a_l\kappa^2\right) + c_l\, F(U_l^2)U_l - \sum_{m=1}^{N} k_{lm} U_m = 0. \tag{177}$$

5.1 Kerr law nonlinearity

For Kerr law, governing equation is [6–9,43]

$$iq_t^{(l)} + a_l q_{xx}^{(l)} + b_l q_{xt}^{(l)} + c_l\, |q^{(l)}|^2 q^{(l)} = \sum_{m=1}^{N} k_{lm} q^{(m)}. \tag{178}$$

The real part equation (177) therefore reduces to

$$(a_l - b_l v)B^2 \frac{d^2 U_l}{d\tau^2} + U_l\left(b_l\omega\kappa - \omega - a_l\kappa^2\right) + c_l\, U_l^3 - \sum_{m=1}^{N} k_{lm} U_m = 0. \tag{179}$$

Using the assumption

$$U_l(\tau) = \lambda_l \cos^{\beta}(\mu\tau), \tag{180}$$

in Eq. (179), we obtain

$$(U_l)_{\tau\tau} = -\mu^2\beta^2\lambda_l \cos^{\beta}(\mu\tau) + \mu^2\lambda_l\beta(\beta-1)\cos^{\beta-2}(\mu\tau). \tag{181}$$

Substituting Eqs. (180) and (181) into Eq. (179), we have

$$\lambda_l \left\{-(a_l - b_l v)B^2\mu^2\beta^2 + b_l\omega\kappa - \omega - a_l\kappa^2\right\}\cos^{\beta}(\mu\tau) + \lambda_l(a_l - b_l v)B^2\mu^2\beta(\beta-1)\cos^{\beta-2}(\mu\tau) + c_l\lambda_l^3\cos^{3\beta}(\mu\tau) - \sum_{m=1}^{N} k_{lm}\lambda_m\cos^{\beta}(\mu\tau) = 0. \quad (182)$$

Using the balance method, by equating the exponents and the coefficients of $\cos^K(\cdot)$, we arrive at

$$\beta(\beta-1) \neq 0, \quad (183)$$

$$3\beta = \beta - 2, \quad (184)$$

$$\lambda_l(a_l - b_l v)B^2\mu^2\beta(\beta-1) + c_l\lambda_l^3 = 0, \quad (185)$$

$$\lambda_l \left\{-(a_l - b_l v)B^2\mu^2\beta^2 + b_l\omega\kappa - \omega - a_l\kappa^2\right\} - \sum_{m=1}^{N} k_{lm}\lambda_m = 0. \quad (186)$$

Solving the system (Eqs. (183)–(186)) simultaneously, we get the solution set

$$\beta = -1, \quad (187)$$

$$v = \frac{c_l\lambda_l^2 + 2a_l B^2\mu^2}{2b_l B^2\mu^2}, \quad (188)$$

$$\omega = \frac{2a_l\kappa^2\lambda_l - c_l\lambda_l^3 + 2\sum_{m=1}^{N} k_{lm}\lambda_m}{2\lambda_l(b_l\kappa - 1)}. \quad (189)$$

equating the two expressions for the soliton speed v from (116) and (188) implies

$$B = \pm\sqrt{\frac{(1 - b_l\kappa)c_l}{2\mu^2\left(b_l^2\omega - a_l\kappa b_l - a_l\right)}\lambda_l}, \quad (190)$$

which leads to the integrability criteria

$$(1 - b_l\kappa)c_l\left(b_l^2\omega - a_l\kappa b_l - a_l\right) > 0. \quad (191)$$

Consequently, a singular periodic solution is recovered:

$$q^{(l)}(x,t) = \lambda_l \sec\left[\sqrt{\frac{(1 - b_l\kappa)c_l}{2\left(b_l^2\omega - a_l\kappa b_l - a_l\right)}\lambda_l} \times \left\{x - \left(\frac{b_l\omega - 2a_l\kappa}{1 - b_l\kappa}\right\} t\right)\right] e^{i(-\kappa x + \omega t + \theta)}, \quad (192)$$

where ω is given by Eq. (189).

It is easy to see that solution (192) reduces to bright 1-soliton solution:

$$q^{(l)}(x,t) = \lambda_l \,\mathrm{sech}\left[\sqrt{\frac{(b_l\kappa - 1)c_l}{2\left(b_l^2\omega - a_l\kappa b_l - a_l\right)}\lambda_l} \times \left\{x - \left(\frac{b_l\omega - 2a_l\kappa}{1 - b_l\kappa}\right) t\right\}\right] e^{i(-\kappa x + \omega t + \theta)}, \quad (193)$$

where ω is given by Eq. (189).

5.2 Power law nonlinearity

For power law, the coupled NLSE modifies to [6–9,43]

$$iq_t^{(l)} + a_l q_{xx}^{(l)} + b_l q_{xt}^{(l)} + c_l \left|q^{(l)}\right|^{2n} q^{(l)} = \sum_{m=1}^{N} k_{lm} q^{(m)}. \quad (194)$$

Therefore, the real part equation reduces to

$$(a_l - b_l v)B^2\frac{d^2U_l}{d\tau^2} + U_l\left(b_l\omega\kappa - \omega - a_l\kappa^2\right) + c_l\, U_l^{2n+1} - \sum_{m=1}^{N} k_{lm}U_m = 0. \quad (195)$$

Using the assumption

$$U_l(\tau) = \lambda_l\cos^{\beta}(\mu\tau), \quad (196)$$

in Eq. (196), we obtain

$$(U_l)_{\tau\tau} = -\mu^2\beta^2\lambda_l\cos^{\beta}(\mu\tau) + \mu^2\lambda_l\beta(\beta-1)\cos^{\beta-2}(\mu\tau). \quad (197)$$

Substituting Eqs. (196) and (197) into Eq. (195), we have

$$\lambda_l \left\{-(a_l - b_l v)B^2\mu^2\beta^2 + b_l\omega\kappa - \omega - a_l\kappa^2\right\}\cos^{\beta}(\mu\tau) + \lambda_l(a_l - b_l v)B^2\mu^2\beta(\beta-1)\cos^{\beta-2}(\mu\tau) + c_l\lambda_l^{2n+1}\cos^{(2n+1)\beta}(\mu\tau) - \sum_{m=1}^{N} k_{lm}\lambda_m\cos^{\beta}(\mu\tau) = 0. \quad (198)$$

Using the balance method, by equating the exponents and the coefficients of $\cos^K(\cdot)$, we get

$$\beta(\beta-1) \neq 0, \quad (199)$$

$$(2n+1)\beta = \beta - 2, \tag{200}$$
$$\lambda_l(a_l - b_l v)B^2\mu^2\beta(\beta-1) + c_l\lambda_l^{2n+1} = 0, \tag{201}$$
$$\lambda_l\left\{-(a_l - b_l v)B^2\mu^2\beta^2 + b_l\omega\kappa - \omega - a_l\kappa^2\right\} - \sum_{m=1}^{N} k_{lm}\lambda_m = 0. \tag{202}$$

Solving the system (Eqs. (199)–(202)) simultaneously, we get the solution set

$$\beta = -\frac{1}{n}, \tag{203}$$
$$v = \frac{n^2 c_l\lambda_l^{2n} + (1+n)a_l B^2\mu^2}{(1+n)b_l B^2\mu^2}, \tag{204}$$
$$\omega = \frac{(1+n)a_l\kappa^2\lambda_l - c_l\lambda_l^{1+2n} + (1+n)\sum_{m=1}^{N} k_{lm}\lambda_m}{(1+n)\lambda_l(b_l\kappa - 1)}. \tag{205}$$

Equating the two expressions for the soliton speed v from (116) and (204) implies

$$B = \pm\sqrt{\frac{(1-b_l\kappa)c_l}{(1+n)\mu^2\left(b_l^2\omega - a_l\kappa b_l - a_l\right)}}\, n\lambda_l^n, \tag{206}$$

whenever

$$(1-b_l\kappa)c_l\left(b_l^2\omega - a_l\kappa b_l - a_l\right) > 0. \tag{207}$$

Thus, the singular periodic solution is given by

$$q^{(l)}(x,t) = \lambda_l \sec^{\frac{1}{n}}\left[\sqrt{\frac{(1-b_l\kappa)c_l}{(1+n)\left(b_l^2\omega - a_l\kappa b_l - a_l\right)}}\, n\lambda_l^n \times\left\{x - \left(\frac{b_l\omega - 2a_l\kappa}{1-b_l\kappa}\right)t\right\}\right] \mathrm{e}^{i(-\kappa x+\omega t+\theta)}, \tag{208}$$

where ω is given by Eq. (205).

Additionally, bright 1-soliton solution retrievable is:

$$q^{(l)}(x,t) = \lambda_l \operatorname{sech}^{\frac{1}{n}}\left[\sqrt{\frac{(b_l\kappa - 1)c_l}{(1+n)\left(b_l^2\omega - a_l\kappa b_l - a_l\right)}}\, n\lambda_l^n \times\left\{x - \left(\frac{b_l\omega - 2a_l\kappa}{1-b_l\kappa}\right)t\right\}\right] \mathrm{e}^{i(-\kappa x+\omega t+\theta)}, \tag{209}$$

where ω is given by Eq. (205).

5.3 Parabolic law nonlinearity

In this case, the governing equation reduces to [6–9,43]

$$iq_t^{(l)} + a_l q_{xx}^{(l)} + b_l q_{xt}^{(l)} + \left(\xi_l\,|q^{(l)}|^2 + \eta_l\,|q^{(l)}|^4\right)q^{(l)} = \sum_{m=1}^{N} k_{lm} q^{(m)}, \tag{210}$$

where $1 \le l \le N$. The real part equation therefore is

$$(a_l - b_l v)B^2\frac{\mathrm{d}^2 U_l}{\mathrm{d}\tau^2} + U_l\left(b_l\omega\kappa - \omega - a_l\kappa^2\right) + \xi_l\,U_l^3 + \eta_l\,U_l^5 - \sum_{m=1}^{N} k_{lm}U_m = 0. \tag{211}$$

We then assume that Eq. (211) has the following formal solution:

$$U_l(\tau) = A_l\,(G(\tau))^{\frac{1}{2}}, \quad A_l \neq 0 \tag{212}$$

where A_l are constants to be determined later and G satisfies Eq. (83). Thus, we obtain

$$(a_l - b_l v)B^2\left(\frac{3}{4}A_l G^{\frac{3}{2}} - A_l\delta G^{\frac{1}{2}} + \frac{\delta^2}{4}A_l\right) + A_l\left(b\omega\kappa - \omega - a\kappa^2\right) + \xi_l A_l^3 G^{\frac{1}{2}} + \eta_l A_l^5 G^{\frac{3}{2}} - \sum_{m=1}^{N} k_{lm}A_m = 0. \tag{213}$$

Then, equating the coefficient of each power of G to zero, we obtain a system of nonlinear algebraic equations, and by solving it, we get

$$v = \frac{4\eta_l A_l^4 + 3B^2 a_l}{3B^2 b_l}, \tag{214}$$
$$\delta = -\frac{3\xi_l}{4A_l^2\eta_l}, \tag{215}$$
$$\omega = \frac{3\xi_l^2 A_l + 16A_l a_l\kappa^2\eta_l + 16\eta_l\sum_{m=1}^{N} k_{lm}A_m}{16A_l\eta_l\,(b_l\kappa - 1)} \tag{216}$$

where $B,\ \kappa,\ A_l,\ k_{lm}$ are arbitrary constants.

Equating the two expressions for the soliton speed v from (116) and (214) implies

$$B = \pm\frac{2A_l^2\sqrt{\eta_l}\,(1-b_l\kappa)}{\sqrt{3\left(b_l^2\omega - a_l b_l\kappa - a_l\right)}} \tag{217}$$

for

$$\eta_l\,(1-b_l\kappa)\left(b_l^2\omega - a_l b_l\kappa - a_l\right) > 0. \tag{218}$$

Thus, we obtain the exact traveling wave solution of Eq. (210) as

$$q^{(l)}(x,t) = \left\{-\frac{3\xi_l}{8\eta_l}\left[1 \pm \tanh \times\left(\frac{\xi_l}{4}\sqrt{\frac{3(1-b_l\kappa)}{\eta_l(b_l^2\omega - a_l b_l\kappa - a_l)}} \times\left(x - \left(\frac{b_l\omega - 2a_l\kappa}{1-b_l\kappa}\right)t\right)\right)\right]\right\}^{\frac{1}{2}} e^{i(-\kappa x+\omega t+\theta)}, \tag{219}$$

and

$$q^{(l)}(x,t) = \left\{-\frac{3\xi_l}{8\eta_l}\left[1 \pm \coth \times\left(\frac{\xi_l}{4}\sqrt{\frac{3(1-b_l\kappa)}{\eta_l(b_l^2\omega - a_l b_l\kappa - a_l)}} \times\left(x - \left(\frac{b_l\omega - 2a_l\kappa}{1-b_l\kappa}\right)t\right)\right)\right]\right\}^{\frac{1}{2}} e^{i(-\kappa x+\omega t+\theta)}, \tag{220}$$

which are dark and singular 1-soliton solutions respectively, where ω is given by Eq. (216).

5.4 Dual-power law nonlinearity

For dual-power law nonlinearity, the governing equation is [6–9,43]

$$iq_t^{(l)} + a_l q_{xx}^{(l)} + b_l q_{xt}^{(l)} + \left(\xi_l \, |q^{(l)}|^{2n} + \eta_l \, |q^{(l)}|^{4n}\right) q^{(l)} = \sum_{m=1}^{N} k_{lm} q^{(m)}. \tag{221}$$

where $1 \leq l \leq N$. The real part equation therefore is

$$(a_l - b_l v)B^2 \frac{d^2U_l}{d\tau^2} + U_l\left(b_l\omega\kappa - \omega - a_l\kappa^2\right) + \xi_l \, U_l^{2n+1} + \eta_l \, U_l^{4n+1} - \sum_{m=1}^{N} k_{lm} U_m = 0. \tag{222}$$

We then assume that Eq. (222) has the following formal solution:

$$U_l(\tau) = A_l G^{\frac{1}{2n}}(\tau), \quad A_l \neq 0 \tag{223}$$

where A_l are constants to be determined later and G satisfies Eq. (83). Thus, we obtain

$$(a_l - b_l v)B^2\left\{\frac{1+2n}{4n^2}A_l G^2 - \frac{A_l\delta(1+n)}{2n^2}G + \frac{\delta^2}{4n^2}A_l\right\} + A_l\left(b_l\omega\kappa - \omega - a_l\kappa^2\right) + \xi_l A_l^{2n+1}G + \eta_l A_l^{4n+1}G^2 - \sum_{m=1}^{N} k_{lm} A_m = 0. \tag{224}$$

Then, equating the coefficient of each power of G to zero, we obtain a system of nonlinear algebraic equations, and by solving it, we get

$$v = \frac{4n^2\eta_l A_l^{4n} + (1+2n)B^2 a_l}{(1+2n)B^2 b_l}, \tag{225}$$

$$\delta = -\frac{(1+2n)\xi_l}{2(1+n)A_l^{2n}\eta_l}, \tag{226}$$

$$\mu = 0, \tag{227}$$

$$\omega = \frac{(1+2n)\xi_l^2 A_l + 4(1+n)^2 A_l a_l \kappa^2 \eta_l + 4(1+n)^2 \eta_l \sum_{m=1}^{N} k_{lm} A_m}{4(1+n)^2 A_l \eta_l \,(b_l\kappa - 1)} \tag{228}$$

where $B, \ \kappa, \ A_l, \ k_{lm}$ are arbitrary constants.

Next, equating the two values of the speed v from the imaginary part Eq. (116) and real part equation (225) gives the free parameter

$$B = \pm\frac{2nA_l^{2n}\sqrt{\eta_l\,(1-b_l\kappa)}}{\sqrt{(1+2n)\left(b_l^2\omega - a_l b_l\kappa - a_l\right)}} \tag{229}$$

as long as

$$\eta_l\,(1-b_l\kappa)\left(b_l^2\omega - a_l b_l\kappa - a_l\right) > 0. \tag{230}$$

Thus, we recover exact dark and singular 1-soliton solutions of Eq. (221), respectively, as

$$q^{(l)}(x,t) = \left\{-\frac{(1+2n)\xi_l}{4(1+n)\eta_l}\left[1 \pm \tanh \times\left(\frac{n\xi_l}{2(1+n)}\sqrt{\frac{(1+2n)(1-b_l\kappa)}{\eta_l(b_l^2\omega - a_l b_l\kappa - a_l)}} \times (x - vt)\right)\right]\right\}^{\frac{1}{2n}} e^{i(-\kappa x+\omega t+\theta)}, \tag{231}$$

and

$$q^{(l)}(x,t) = \left\{-\frac{(1+2n)\xi_l}{4(1+n)\eta_l}\left[1 \pm \coth \times\left(\frac{n\xi_l}{2(1+n)}\sqrt{\frac{(1+2n)(1-b_l\kappa)}{\eta_l(b_l^2\omega - a_l b_l\kappa - a_l)}} \times (x - vt)\right)\right]\right\}^{\frac{1}{2n}} e^{i(-\kappa x+\omega t+\theta)}, \tag{232}$$

where ω is given by Eq. (228).

6 Conclusions

This paper secures soliton solutions to nonlinear directional couplers that come with four forms of nonlinearity. They are Kerr law, power law, parabolic law and dual-power law. There are two integration schemes adopted in this paper. These lead to bright soliton solution (for Kerr and power laws only) as well as dark and singular soliton solutions (for parabolic and dual-power laws of nonlinearity). The appropriate constraint conditions are enumerated that secures existence of these solitons. Additionally, for Kerr and power laws, singular periodic solutions are obtained as a by-product since these give periodic blow-ups. The results of this paper come with a lot of scope and hope.

Later, several perturbation terms will be taken into account, and thus, the governing equation will be extended which will lead to soliton solutions with a generalized structure. These results are awaited at this time although it is on the horizon. Additionally, numerical simulations will be carried out using several advanced techniques such as pseudospectral approximations, Jacobi-tau approximations, shifted Legendre polynomials, collocation method and others [2–5,13]. These results will soon be available and reported.

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References

1. Alshaery, A.A., Hilal, E.M., Banaja, M.A., Alkhateeb, S.A., Moraru, L., Biswas, A.: Optical solitons in multiple-core couplers. J. Optoelectron. Adv. Mater. **16**(5–6), 750–758 (2014)
2. Bhrawy, A.H.: An efficient Jacobi pseudo-spectral approximation for nonlinear complex generalized Zakharov system. Appl. Math. Comput. **247**, 30–46 (2014)
3. Bhrawy, A.H., Zaky, M.A.: A method based on Jacobi tau approximation for solving multi-term time–space fractional partial differential equations. J. Comput. Phys. **281**, 876–895 (2015)
4. Bhrawy, A.H., Zaky, M.A.: Numerical simulation for two-dimensional variable-order fractional nonlinear cable equation. Nonlinear Dyn. **80**(1), 101–116 (2015)
5. Bhrawy, A.H., Doha, E.H., Ezz-Eldien, S.S., Abdelkawy, M.A.: A numerical technique based on the shifted Legendre polynomials for solving the time-fractional coupled KdV equation. Calcolo. doi:10.1007/s10092-014-0132-x
6. Biswas, A.: Solitons in multiple-core couplers. J. Nonlinear Opt. Phys. Mater. **10**(3), 329–336 (2001)
7. Biswas, A.: Theory of optical couplers. Opt. Quantum Electron. **35**(3), 221–235 (2003)
8. Biswas, A.: Dispersion-managed solitons in optical couplers. J. Nonlinear Opt. Phys. Mater. **12**(1), 45–74 (2003)
9. Biswas, A., Lott, D.A., Sutton, B., Khan, K.R., Mahmood, M.F.: Optical Gaussons in nonlinear directional couplers. J. Electromagn. Waves Appl. **27**(15), 1976–1985 (2013)
10. Chen, M., Liu, Z., Lu, R., Yang, Y., Liang, M., Tang, T.-T.: An ultracompact optical directional coupler based on lithium niobate photonic wires. Optik **124**(14), 1974–1976 (2013)
11. Dai, C.Q., Wang, X.G., Zhao, G.Q.: Stable light-bullet solutions in the harmonic and parity-time symmetric potentials. Phys. Rev. A **89**, 013834 (2014)
12. Dai, C.Q., Wang, Y.Y.: Superposed Akhmediev breather of the (3+1)-dimensional generalized nonlinear Schrödinger equation with external potential. Ann. Phys. **341**, 142–152 (2014)
13. Doha, E.H., Bhrawy, A.H., Abdelkawy, M.A., Van Gorder, R.A.: Jacobi–Gauss–Lobatto collocation method for the numerical solution of 1+1 nonlinear Schrödinger equations. J. Comput. Phys. **261**, 244–255 (2014)
14. Eslami, M., Mirzazadeh, M., Biswas, A.: Soliton solutions of the resonant nonlinear Schrödinger's equation in optical fibers with time-dependent coefficients by simplest equation approach. J. Mod. Opt. **60**(19), 1627–1636 (2013)
15. Eslami, M., Mirzazadeh, M., Vajargah, B.F., Biswas, A.: Optical solitons for the resonant nonlinear Schrödinger's equation with time-dependent coefficients by the first integral method. Optik **125**(9), 3107–3116 (2014)
16. Fang, Y., Zhou, J.: Effects of third-order dispersion on soliton switching in fiber nonlinear directional couplers. Optik **119**(2), 86–89 (2008)
17. Geng, X., Lv, Y.: Darboux transformation for an integrable generalization of the nonlinear Schrödinger equation. Nonlinear Dyn. **69**(4), 1621–1630 (2012)
18. Govindaraji, A., Mahalingam, A., Uthayakumar, A.: Dark soliton switching in nonlinear fiber couplers with gain. Opt. Laser Technol. **60**, 18–21 (2014)
19. Gupta, R.K., Kumar, V., Jiwari, R.: Exact and numerical solutions of coupled short-pulse equation with time-dependent coefficients. Nonlinear Dyn. **79**(1), 455–464 (2015)
20. Guo, R., Hao, H.-Q.: Breathers and localized solitons for the Hirota–Maxwell–Bloch system on constant backgrounds in erbium doped fibers. Ann. Phys. **344**, 10–16 (2014)
21. Guo, R., Hao, H.-Q., Zhang, L.-L.: Dynamic behaviors of the breather solutions for the AB system in fluid mechanics. Nonlinear Dyn. **74**, 701–709 (2013)
22. Guo, R., Liu, Y.-F., Hao, H.-Q., Qi, F.-H.: Coherently coupled solitons, breathers and rogue waves for polarized optical waves in an isotropic medium. Nonlinear Dyn. **80**, 1221–1230 (2015)
23. Guo, R., Liu, Y.-F.: The canonical AB system: conservation laws and soliton solutions. Appl. Math. Comput. **259**, 153–163 (2015)
24. He, X., Xie, K., Yang, H.: Gain-induced soliton switching in fiber nonlinear directional coupler. Optik **123**(24), 2247–2249 (2012)

25. Irawan, D., Saktioto, T., Ali, J., Fadhali, M.: Birefringence analysis of directional fiber coupler induced by fusion and coupling parameters. Optik **124**(17), 3063–3066 (2013)
26. Khan, K.R., Wu, T., Christodoulides, D.N., Stegeman, G.I.: Soliton switching and multi-frequency generation in nonlinear photonic crystal fiber. Opt. Express **16**(13), 9417–9428 (2008)
27. Li, J., Cao, J., Xu, x: Effects of phase errors on phase locking of all-fiber laser arrays. Opt. Laser Technol. **47**, 372–378 (2013)
28. Li, H., Dong, X., Li, E., Liu, Z., Bai, Y.: Highly compact 2 × 2 multiport interference coupler in silicon photonic nanowires for array waveguide grating demodulation integration microsystem. Opt. Laser Technol. **47**, 366–371 (2013)
29. Lu, X., Peng, M.: Painlevé integrability and explicit solutions of the generalized two-coupled nonlinear Schrödinger system in the optical fibers communications. Nonlinear Dyn. **73**, 405–410 (2013)
30. Mandal, P., Midda, S.: All optical method of developing OR and NAND logic system based on nonlinear fiber couplers. Optik **122**(20), 1795–1798 (2011)
31. Mandal, P.: Method of developing all optical half-adder based on nonlinear directional coupler. Opt. Photonics Lett. **6**(1), 1350001 (2013)
32. Mirzazadeh, M., Eslami, M., Milovic, D., Biswas, A.: Topological solitons of resonant nonlinear Schrödinger's equation with dual-power law nonlinearity using G'/G-expansion technique. Optik **125**(19), 5480–5489 (2014)
33. Mirzazadeh, M., Eslami, M., Vajargah, B.F., Biswas, A.: Optical solitons and optical rogons of generalized resonant dispersive nonlinear Schrödinger's equation with power law nonlinearity. Optik **125**(9), 4246–4256 (2014)
34. Potasek, M.J., Yang, Y.: Multiterabit-per-second all-optical switching in a nonlinear directional coupler. IEEE J. Sel. Top. Quantum Electron. **8**, 714–721 (2002)
35. Pu, S., Hou, C., Yuan, C.: Soliton switching in inhomogeneous nonlocal media. Optik **125**(3), 1075–1078 (2014)
36. Pustelny, T., Struk, P.: Numerical analyses of optical couplers for planar waveguides. Optoelectron. Lett. **20**(3), 201–206 (2012)
37. Rahmati, A.T., Granpayeh, N.: Design and simulation of a switch based on nonlinear directional coupler. Optik **121**(18), 1631–1634 (2010)
38. Rahmati, A.T., Granpayeh, N.: Kerr nonlinear switch based on ultra-compact photonic crystal directional coupler. Optik **122**(6), 502–505 (2011)
39. da Silva, M.G., Bastos, A.M., Sobrinho, C.S., Lima, J.L.S., de Almelda, E.F., Sombra, A.S.B.: Optical crosstalk in a periodically inhomogeneous nonlinear dispersion directional fiber coupler. Opt. Fiber Technol. **11**(2), 180–192 (2005)
40. Sarma, A.K.: A comparative study of soliton switching in two- and three-core coupler with TOD and IMD. Optik **120**(8), 390–394 (2009)
41. Sarma, A.K.: Dark soliton switching in an NLDC in presence of higher order perturbative effects. Opt. Laser Technol. **41**(3), 247–250 (2009)
42. Sarma, A.K.: Vector soliton switching in a fiber nonlinear directional coupler. Opt. Commun. **284**(1), 186–190 (2011)
43. Savescu, M., Bhrawy, A.H., Alshaery, A.A., Hilal, E.M., Khan, K.R., Mahmood, M.F., Biswas, A.: Optical soliton in nonlinear directional couplers with spatio-temporal dispersion. J. Mod. Opt. **61**(5), 441–458 (2014)
44. Wang, Y.Y., Dai, C.Q., Wang, X.G.: Stable localized spatial solitons in PT-symmetric potentials with power law nonlinearity. Nonlinear Dyn. **77**, 1323–1330 (2014)
45. Wang, Y.Y., Dai, C.Q., Wang, X.G.: Spatio-temporal localized modes in PT-symmetric optical media. Ann. Phys. **348**, 289–296 (2014)
46. Xiang, J.J., Jiang, H.J., Wang, Y.Y., Dai, C.Q.: Nonautonomous bright soliton solutions on continuous wave and cnoidal wave backgrounds on blood vessels. Nonlinear Dyn. **75**, 201–207 (2014)
47. Zhou, Q., Zhou, Q., Biswas, A.: Optical solitons in birefringent fibers with parabolic law nonlinearity. Opt. Appl. **41**(3), 399–409 (2014)
48. Zhou, Q., Zhou, Q., Liu, Y., Yu, H., Yao, P., Biswas, A.: Thirring optical solitons in birefringent fibers with spatio-temporal dispersion and Kerr law nonlinearity. Laser Phys. **25**(1), 014502 (2015)
49. Zhou, Q., Zhou, Q., Liu, Y., Yu, H., Wei, C., Yao, P., Bhrawy, A.H., Biswas, A.: Bright, dark and singular optical solitons in cascaded system. Laser Phys. **25**(2), 025402 (2015)
50. Zhu, H.P.: Nonlinear tunneling of controllable rogue waves in two-dimensional graded-index waveguides. Nonlinear Dyn. **72**, 873–882 (2013)