Optical solitons with quadratic nonlinearity by extended trial equation method

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This paper studies optical solitons with quadratic nonlinearity. The extended trial equation scheme is applied to the model to retrieve bright, dark and singular optical solitons as well as other forms of waves. The constraint conditions naturally emerged from the solution structure for the existence of these solitons and waves. The results of this paper are extended versions of previously reported versions.

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1. Introduction

Optical solitons is a very important and thrilling area of research in the field of nonlinear optics [1-31]. The non-linear effect in a quadratic media is the second harmonic generation (SHG). A pump wave at the fundamental harmonic (FH) generates a second harmonic (SH) with double frequency. This SHG phenomenon is recoverable from Maxwell's equation with quadratic non-linearity. The solitons in quadratic nonlinear media are studied in optical switching, optical routing, lasers with quadratic non-linear crystal, and others [1, 4].

1.1. Governing equation

For quadratic nonlinear media, with inter-modal dispersion (IMD) and spatio-temporal dispersion (STD) is given by [1]

$$iq_t + a_1 q_{xx} + b_1 q_{xt} + c_1 q + k_1 q^* r = i\alpha_1 q_x, \tag{1}$$

$$ir_t + a_2 r_{xx} + b_2 r_{xt} + c_2 r + k_2 q^2 = i\alpha_2 r_x. \tag{2}$$

Here, $q(x,t)$ and $r(x,t)$ represents the wave profile of the FH and SH components respectively. The independent variables are x and t that are spatial and temporal variables. The coefficients of group velocity dispersion (GVD) terms are a_l with $l = 1,2$ for the two components. Then, the coefficients of STD are b_l. The coefficients of the quadratic nonlinear terms are k_l while the IMD terms are on the right hand sides of the two components and are given by the coefficients of α_l. It was pointed out during 2011 that the inclusion of the STD makes the governing NLSE well-posed as opposed to the consideration of GVD alone, in which case, the model problem remains ill-posed [2, 3]. The first term for both components represents linear evolution.

During the past few years a lot of research was conducted on quadratic nonlinear media and therefore many results were reported during the past couple of decades [4–19]. Very recently, exact bright and singular 1-soliton solution was obtained for quadratic nonlinear media in presence of GVD only and also without IMD [4]. This paper is thus extension and generalization of those previously reported results. Extended trial equation method [20–24] will be the integration tool employed in this paper. Bright, dark and singular solitons will be obtained along with necessary constraint conditions that guarantees the existence of solitons and other form of waves.

1.2. Mathematical analysis

In order to integrate Eqs. (1) and (2), introduce the hypothesis [4, 25–27]

$$q(x,t) = P_1[\xi(x,t)]\exp[i\phi(x,t)], \tag{3}$$

$$r(x,t) = P_2[\xi(x,t)]\exp[2i\phi(x,t)], \tag{4}$$

where $P_l(\xi)$ for $l = 1,2$ represents the amplitude component of the soliton and $\xi(x,t) = B(x - vt)$, while $\phi(x,t)$ gives the phase with

$$\phi(x,t) = -\kappa x + \omega t + \theta. \tag{5}$$

Here, the constant coefficients of B, v, κ, ω, and θ are, respectively, inverse wave width, wave velocity, wave frequency, wave number, and phase constant. Substituting (3), (4) and (5) into (1) and (2) and decomposing into real and imaginary parts gives

$$\left(\omega + a_1\kappa^2 - b_1\kappa\omega + \alpha_1\kappa - c_1\right)P_1 + (b_1v - a_1)B^2P_{1''} - k_1P_1P_2 = 0, \tag{6}$$

and

$$v = \frac{2a_1\kappa - b_1\omega + \alpha_1}{b_1\kappa - 1}, \tag{7}$$

respectively, from the first component. From the second component, one obtains

$$\left(2\omega + 4a_2\kappa^2 - 4b_2\kappa\omega + 2\alpha_2\kappa - c_2\right)P_2 + (b_2v - a_2)B^2P_{2''} - k_2P_1^2 = 0, \tag{8}$$

and

$$v = \frac{4a_2\kappa - 2b_2\omega + \alpha_2}{2b_2\kappa - 1}. \tag{9}$$

Equating the speed of the solitons of the two components from (7) and (9) gives

$$4\kappa^2(a_1b_2 - a_2b_1) + \kappa\{2\alpha_1b_2 - \alpha_2b_1 - 2(a_1 - 2a_2)\} + \omega(b_1 - 2b_2) + (\alpha_2 - \alpha_1) = 0. \tag{10}$$

Setting the coefficients of independent parameters ω and κ yields

$$a_1 = 2a_2, \tag{11}$$
$$b_1 = 2b_2, \tag{12}$$
$$\alpha_1 = \alpha_2. \tag{13}$$

Therefore, one can naturally define

$$a_1 = 2a, \quad a_2 = a \tag{14}$$
$$b_1 = 2b, \quad b_2 = b \tag{15}$$
$$\alpha_1 = \alpha_2 = \alpha. \tag{16}$$

Consequently, speed of the solitons for both components simplify to

$$v = \frac{4a\kappa - 2b\omega + \alpha}{2b\kappa - 1}. \tag{17}$$

Also, real part components modify to

$$\left(\omega + 2a\kappa^2 - 2b\kappa\omega + \alpha\kappa - c_1\right)P_1 + 2(bv - a)B^2P_{1''} - k_1P_1P_2 = 0, \tag{18}$$

and

$$\left(2\omega + 4a\kappa^2 - 4b\kappa\omega + 2\alpha\kappa - c_2\right)P_2 + (bv - a)B^2P_{2''} - k_2P_1^2 = 0, \tag{19}$$

respectively. Additionally, the governing model equations (1) and (2) simplify to

$$iq_t + 2aq_{xx} + 2bq_{xt} + c_1q + k_1q^*r = i\alpha q_x, \tag{20}$$

$$ir_t + ar_{xx} + br_{xt} + c_2r + k_2q^2 = i\alpha r_x. \tag{21}$$

These equations will now be analytically solved for bright, dark and singular soliton in the following section.

2. Extended trial equation method

This section will apply the extended trial equation method to handle Eqs. (20) and (21). For solutions to (18) and (19), the following assumptions for the soliton structure is made

$$P_1 = \sum_{i=0}^{\varsigma} \tau_i\Psi^i, \tag{22}$$

$$P_2 = \sum_{i=0}^{\tilde{\varsigma}} \tilde{\tau}_i\Psi^i, \tag{23}$$

where

$$(\Psi')^2 = \Lambda(\Psi) = \frac{\Phi(\Psi)}{\Upsilon(\Psi)} = \frac{\mu_\sigma\Psi^\sigma + \cdots + \mu_1\Psi + \mu_0}{\chi_\rho\Psi^\rho + \cdots + \chi_1\Psi + \chi_0}. \tag{24}$$

Using the relations (22)-(24), one can derive the terms $P_1^{''}$ and $P_2^{''}$ as below:

$$P_{1''} = \frac{\Phi'(\Psi)\Upsilon(\Psi) - \Phi(\Psi)\Upsilon'(\Psi)}{2\Upsilon^2(\Psi)}\left(\sum_{i=0}^{\varsigma} i\tau_i\Psi^{i-1}\right) + \frac{\Phi(\Psi)}{\Upsilon(\Psi)}\left(\sum_{i=0}^{\varsigma} i(i-1)\tau_i\Psi^{i-2}\right), \tag{25}$$

and

$$P_{2''}=\frac{\Phi'(\Psi)\Upsilon(\Psi)-\Phi(\Psi)\Upsilon'(\Psi)}{2\Upsilon^2(\Psi)}\left(\sum_{i=0}^{\tilde{\varsigma}}i\tilde{\tau}_i\Psi^{i-1}\right)+\frac{\Phi(\Psi)}{\Upsilon(\Psi)}\left(\sum_{i=0}^{\tilde{\varsigma}}i(i-1)\tilde{\tau}_i\Psi^{i-2}\right), \tag{26}$$

where $\Phi(\Psi)$ and $\Upsilon(\Psi)$ are polynomials of Ψ. One can reduce Eq. (24) to the elementary integral form

$$\pm(\xi-\xi_0)=\int\frac{d\Psi}{\sqrt{\Lambda(\Psi)}}=\int\sqrt{\frac{\Upsilon(\Psi)}{\Phi(\Psi)}}d\Psi. \tag{27}$$

Using the balance principle, One determines a relation of σ, ρ and ς as

$$\varsigma=\tilde{\varsigma}=\sigma-\rho-2. \tag{28}$$

Case-1: When $\sigma=3$, $\rho=0$ and $\varsigma=\tilde{\varsigma}=1$ in Eq. (28), we gain

$$P_1=\tau_0+\tau_1\Psi, \tag{29}$$

$$P_2=\tilde{\tau}_0+\tilde{\tau}_1\Psi, \tag{30}$$

and

$$P_{1'}=\frac{\tau_1(3\mu_3\Psi^2+2\mu_2\Psi+\mu_1)}{2\chi_0}, \tag{31}$$

$$P_{2'}=\frac{\tilde{\tau}_1(3\mu_3\Psi^2+2\mu_2\Psi+\mu_1)}{2\chi_0}, \tag{32}$$

where $\mu_3\neq 0$ and $\chi_0\neq 0$. Substituting Eqs. (29)-(32) into (18) and (19), and solving the resulting system of algebraic equations one recovers

$$\begin{aligned}
\mu_1&=\frac{\tau_0^2\tilde{\tau}_1\chi_0(2c_1-c_2)}{2\tau_1B^2(a-bv)(\tau_1\tilde{\tau}_0-\tau_0\tilde{\tau}_1)},\\
\mu_2&=\frac{\tau_0\tilde{\tau}_1\chi_0(2c_1-c_2)}{2B^2(a-bv)(\tau_1\tilde{\tau}_0-\tau_0\tilde{\tau}_1)},\\
\mu_3&=\frac{\tau_1\tilde{\tau}_1\chi_0(2c_1-c_2)}{6B^2(a-bv)(\tau_1\tilde{\tau}_0-\tau_0\tilde{\tau}_1)},\\
\omega&=\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)},\\
k_1&=\frac{\tau_1(c_2-2c_1)}{2(\tau_1\tilde{\tau}_0-\tau_0\tilde{\tau}_1)},\\
k_2&=\frac{\tilde{\tau}_1^2(c_2-2c_1)}{4\tau_1(\tau_1\tilde{\tau}_0-\tau_0\tilde{\tau}_1)},\\
\mu_0&=\mu_0,\quad \tau_0=\tau_0,\\
\tau_1&=\tau_1,\tilde{\tau}_0=\tilde{\tau}_0,\quad \tilde{\tau}_1=\tilde{\tau}_1.
\end{aligned} \tag{33}$$

Substituting these results into Eqs. (24) and (27), we obtain

$$\pm(\xi-\xi_0)=\sqrt{W_1}\int\frac{d\Psi}{\sqrt{\Lambda(\Psi)}}, \tag{34}$$

where

$$\Lambda(\Psi)=\Psi^3+\frac{\mu_2}{\mu_3}\Psi^2+\frac{\mu_1}{\mu_3}\Psi+\frac{\mu_0}{\mu_3},\quad W_1=\frac{\chi_0}{\mu_3}. \tag{35}$$

Integrating (34), one obtains the traveling wave solutions to Eqs. (20) and (21) as the following:

When $\Lambda(\Psi)=(\Psi-\lambda_1)^3$, plane wave solutions are:

$$q(x,t)=\left\{\tau_0+\tau_1\lambda_1+\frac{4\tau_1W_1}{\left(B\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]-\xi_0\right)^2}\right\}\exp\left[i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right], \tag{36}$$

$$r(x,t)=\left\{\tilde{\tau}_0+\tilde{\tau}_1\lambda_1+\frac{4\tilde{\tau}_1W_1}{\left(B\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]-\xi_0\right)^2}\right\}\exp\left[2i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right]. \tag{37}$$

When $\Lambda(\Psi)=(\Psi-\lambda_1)^2(\Psi-\lambda_2)$ and $\lambda_2>\lambda_1$, one recovers dark solitons:

$$q(x,t)=\left\{\tau_0+\tau_1\lambda_2+\tau_1(\lambda_1-\lambda_2)\tanh^2\left[\frac{1}{2}\sqrt{\frac{\lambda_1-\lambda_2}{W_1}}\left(B\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]-\xi_0\right)\right]\right\}\times\exp\left[i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right], \tag{38}$$

$$r(x,t)=\left\{\tilde{\tau}_0+\tilde{\tau}_1\lambda_2+\tilde{\tau}_1(\lambda_1-\lambda_2)\tanh^2\left[\frac{1}{2}\sqrt{\frac{\lambda_1-\lambda_2}{W_1}}\left(B\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]-\xi_0\right)\right]\right\}\times\exp\left[2i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right]. \tag{39}$$

When $\Lambda(\Psi)=(\Psi-\lambda_1)(\Psi-\lambda_2)^2$ and $\lambda_1>\lambda_2$, singular solitons are recovered:

$$q(x,t)=\left\{\begin{array}{l}\tau_0+\tau_1\lambda_1+\tau_1(\lambda_1-\lambda_2)\operatorname{cosech}^2\\ \left[\left[\frac{B}{2}\sqrt{\frac{\lambda_1-\lambda_2}{W_1}}\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]\right]\right]\end{array}\right\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right], \tag{40}$$

$$r(x,t)=\left\{\begin{array}{l}\tilde{\tau}_0+\tilde{\tau}_1\lambda_1+\tilde{\tau}_1(\lambda_1-\lambda_2)\operatorname{cosech}^2\\ \left[\left[\frac{B}{2}\sqrt{\frac{\lambda_1-\lambda_2}{W_1}}\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]\right]\right]\end{array}\right\}$$
$$\times\exp\left[2i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right]. \tag{41}$$

When $\Lambda(\Psi)=(\Psi-\lambda_1)(\Psi-\lambda_2)(\Psi-\lambda_3)$ and $\lambda_1>\lambda_2>\lambda_3$, Jacobi elliptic function solutions are:

$$q(x,t)=\left\{\begin{array}{l}\tau_0+\tau_1\lambda_3+\tau_1(\lambda_2-\lambda_3)sn^2\\ \left[\left[\mp\frac{1}{2}\sqrt{\frac{\lambda_1-\lambda_3}{W_1}}\left(B\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]-\xi_0\right),\frac{\lambda_2-\lambda_3}{\lambda_1-\lambda_3}\right]\right]\end{array}\right\}$$
$$\times\exp\left[i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right], \tag{42}$$

$$r(x,t)=\left\{\begin{array}{l}\tilde{\tau}_0+\tilde{\tau}_1\lambda_3+\tilde{\tau}_1(\lambda_2-\lambda_3)sn^2\\ \left[\left[\mp\frac{1}{2}\sqrt{\frac{\lambda_1-\lambda_3}{W_1}}\left(B\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]-\xi_0\right),\frac{\lambda_2-\lambda_3}{\lambda_1-\lambda_3}\right]\right]\end{array}\right\}$$
$$\times\exp\left[2i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right]. \tag{43}$$

Note that λ_i $(i=1,2,3)$ are the roots of the polynomial equation

$$\Lambda(\Psi)=0. \tag{44}$$

When $\tau_0=-\tau_1\lambda_1$, $\tilde{\tau}_0=-\tilde{\tau}_1\lambda_1$ and $\xi_0=0$, solutions (36) - (41) are reduced to plane wave solutions

$$q(x,t)=\left\{\frac{A_1}{B\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]}\right\}^2 \tag{45}$$
$$\exp\left[i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right],$$

$$r(x,t)=\left\{\frac{\tilde{A}_1}{B\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]}\right\}^2 \tag{46}$$
$$\exp\left[2i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right],$$

bright 1-soliton solutions are:

$$q(x,t)=\left\{\frac{A_2}{\cosh^2\left(B_1\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]\right)}\right\} \tag{47}$$
$$\exp\left[i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right],$$

$$r(x,t)=\left\{\frac{\tilde{A}_2}{\cosh^2\left(B_1\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]\right)}\right\} \tag{48}$$
$$\exp\left[2i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right],$$

and singular 1-soliton solutions

$$q(x,t)=\left\{\frac{A_3}{\sinh^2\left(B_1\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]\right)}\right\} \tag{49}$$
$$\exp\left[i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right],$$

$$r(x,t)=\left\{\frac{\tilde{A}_3}{\sinh^2\left(B_1\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]\right)}\right\} \exp\left[2i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right], \tag{50}$$

where

$$A_1=2\sqrt{\tau_1 W_1},\quad A_2=\tau_1(\lambda_2-\lambda_1),\quad A_3=\tau_1(\lambda_1-\lambda_2),$$
$$\tilde{A}_1=2\sqrt{\tilde{\tau}_1 W_1},\quad \tilde{A}_2=\tilde{\tau}_1(\lambda_2-\lambda_1),\quad \tilde{A}_3=\tilde{\tau}_1(\lambda_1-\lambda_2),$$
$$B_1=\frac{B}{2}\sqrt{\frac{\lambda_1-\lambda_2}{W_1}}. \tag{51}$$

Here, A_2, A_3 and $\tilde{A}_2$, $\tilde{A}_3$ are respectively the amplitudes of 1-solitons and singular solitons, while B_1 is the inverse width of the solitons. So, the solitons exist for $\tau_1>0$ and $\tilde{\tau}_1>0$. Furthermore, when $\tau_0=-\tau_1\lambda_3$, $\tilde{\tau}_0=-\tilde{\tau}_1\lambda_3$ and $\xi_0=0$, the solutions (42) and (43) are simplified as

$$q(x,t)=A_4 sn^2\left(B_j\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right],\frac{\lambda_2-\lambda_3}{\lambda_1-\lambda_3}\right) \exp\left[i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right], \tag{52}$$

$$r(x,t)=\tilde{A}_4 sn^2\left(B_j\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right],\frac{\lambda_2-\lambda_3}{\lambda_1-\lambda_3}\right) \exp\left[2i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right], \tag{53}$$

where

$$A_4=\tau_1(\lambda_2-\lambda_3),\quad \tilde{A}_4=\tilde{\tau}_1(\lambda_2-\lambda_3),$$
$$B_j=\frac{(-1)^j B}{2}\sqrt{\frac{\lambda_1-\lambda_3}{W_1}},\quad (j=2,3). \tag{54}$$

Remark-1: When the modulus $l\to 1$, dark soliton solutions emerge:

$$q(x,t)=A_4\tanh^2\left(B_j\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]\right) \exp\left[i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right], \tag{55}$$

$$r(x,t)=\tilde{A}_4\tanh^2\left(B_j\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]\right) \exp\left[2i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right], \tag{56}$$

where $\lambda_1=\lambda_2$.

Case-2: When $\sigma=4$, $\rho=0$ and $\varsigma=2$ in Eq. (28), one obtains

$$P_1=\tau_0+\tau_1\Psi+\tau_2\Psi^2, \tag{57}$$

$$P_2=\tilde{\tau}_0+\tilde{\tau}_1\Psi+\tilde{\tau}_2\Psi^2, \tag{58}$$

and

$$P_{1''}=\frac{(\tau_1+2\tau_2\Psi)(4\mu_4\Psi^3+3\mu_3\Psi^2+2\mu_2\Psi+\mu_1)}{2\chi_0}+\frac{4\tau_2(\mu_4\Psi^4+\mu_3\Psi^3+\mu_2\Psi^2+\mu_1\Psi+\mu_0)}{2\chi_0}, \tag{59}$$

$$P_{2''}=\frac{(\tilde{\tau}_1+2\tilde{\tau}_2\Psi)(4\mu_4\Psi^3+3\mu_3\Psi^2+2\mu_2\Psi+\mu_1)}{2\chi_0}+\frac{4\tilde{\tau}_2(\mu_4\Psi^4+\mu_3\Psi^3+\mu_2\Psi^2+\mu_1\Psi+\mu_0)}{2\chi_0}, \tag{60}$$

where $\mu_4\neq 0$ and $\chi_0\neq 0$. Substituting Eqs. (57) - (60) into (18) and (19), and solving the resulting system of algebraic equations, one recovers the following results:

$$\mu_1=-\frac{4\mu_0\tau_2\tilde{\tau}_1\tilde{\tau}_2\left(\tau_2\tilde{\tau}_1^2-12\tau_0\tilde{\tau}_2^2\right)}{\tau_2^2\tilde{\tau}_1^4-12\tau_0\tau_2\tilde{\tau}_1^2\tilde{\tau}_2^2+48\tau_0^2\tilde{\tau}_2^4},$$
$$\mu_2=\frac{12\mu_0\tau_2\tilde{\tau}_2^2\left(\tau_2\tilde{\tau}_1^2+4\tau_0\tilde{\tau}_2^2\right)}{\tau_2^2\tilde{\tau}_1^4-12\tau_0\tau_2\tilde{\tau}_1^2\tilde{\tau}_2^2+48\tau_0^2\tilde{\tau}_2^4},$$
$$\mu_3=\frac{32\mu_0\tau_2^2\tilde{\tau}_1\tilde{\tau}_2^3}{\tau_2^2\tilde{\tau}_1^4-12\tau_0\tau_2\tilde{\tau}_1^2\tilde{\tau}_2^2+48\tau_0^2\tilde{\tau}_2^4},$$
$$\mu_4=\frac{16\mu_0\tau_2^2\tilde{\tau}_2^4}{\tau_2^2\tilde{\tau}_1^4-12\tau_0\tau_2\tilde{\tau}_1^2\tilde{\tau}_2^2+48\tau_0^2\tilde{\tau}_2^4},$$
$$\chi_0=-\frac{96\mu_0\tilde{\tau}_2^5B^2(a-bv)}{k_2\left(\tau_2^2\tilde{\tau}_1^4-12\tau_0\tau_2\tilde{\tau}_1^2\tilde{\tau}_2^2+48\tau_0^2\tilde{\tau}_2^4\right)},$$
$$\omega=\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)},\quad k_1=\frac{2k_2\tau_2^2}{\tilde{\tau}_2^2},$$
$$\mu_0=\mu_0,\quad \tau_0=\tau_0,\quad \tau_2=\tau_2,\quad \tilde{\tau}_1=\tilde{\tau}_1,\tilde{\tau}_2=\tilde{\tau}_2,$$
$$\tau_1=\frac{\tau_2\tilde{\tau}_1}{\tilde{\tau}_2},\quad \tilde{\tau}_0=\frac{\tilde{\tau}_2\left[4k_2\tau_0\tau_2+\tilde{\tau}_2\left(c_2-2c_1\right)\right]}{4k_2\tau_2^2}. \tag{61}$$

Substituting these results into Eqs. (24) and (27), we have

$$\pm(\xi-\xi_0)=W_2\int\frac{d\Psi}{\sqrt{\Lambda(\Psi)}}, \tag{62}$$

where

$$\Lambda(\Psi)=\Psi^4+\frac{\mu_3}{\mu_4}\Psi^3+\frac{\mu_2}{\mu_4}\Psi^2+\frac{\mu_1}{\mu_4}\Psi+\frac{\mu_0}{\mu_4},\quad W_2=\sqrt{\frac{\chi_0}{\mu_4}}. \tag{63}$$

Integrating (62) and taking $\xi_0=0$, we obtain the traveling wave solutions to Eqs. (20) and (21) in the forms:

When $\Lambda(\Psi)=(\Psi-\lambda_1)^4$,

$$q(x,t)=\sum_{j=0}^{2}\tau_j\left(\lambda_1\pm\frac{W_2}{B\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]}\right)^j \exp\left[i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right], \tag{64}$$

$$r(x,t)=\sum_{j=0}^{2}\tilde{\tau}_j\left(\lambda_1\pm\frac{W_2}{B\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]}\right)^j \exp\left[2i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right]. \tag{65}$$

When $\Lambda(\Psi)=(\Psi-\lambda_1)^3(\Psi-\lambda_2)$ and $\lambda_2>\lambda_1$,

$$q(x,t)=\sum_{j=0}^{2}\tau_j\left(\lambda_1+\frac{4W_2^2(\lambda_2-\lambda_1)}{4W_2^2-\left(B(\lambda_1-\lambda_2)\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]\right)^2}\right)^j \exp\left[i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right], \tag{66}$$

$$r(x,t)=\sum_{j=0}^{2}\tilde{\tau}_j\left(\lambda_1+\frac{4W_2^2(\lambda_2-\lambda_1)}{4W_2^2-\left(B(\lambda_1-\lambda_2)\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]\right)^2}\right)^j \exp\left[2i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right]. \tag{67}$$

When $\Lambda(\Psi)=(\Psi-\lambda_1)^2(\Psi-\lambda_2)^2$,

$$q(x,t)=\sum_{j=0}^{2}\tau_j\left(\lambda_2+\frac{\lambda_2-\lambda_1}{\exp\left(\frac{B(\lambda_1-\lambda_2)}{W_2}\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]\right)-1}\right)^j \exp\left[i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right], \tag{68}$$

$$r(x,t)=\sum_{j=0}^{2}\tilde{\tau}_j\left(\lambda_2+\frac{\lambda_2-\lambda_1}{\exp\left(\frac{B(\lambda_1-\lambda_2)}{W_2}\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]\right)-1}\right)^j \exp\left[2i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right], \tag{69}$$

and

$$q(x,t)=\sum_{j=0}^{2}\tau_j\left(\lambda_1+\frac{\lambda_1-\lambda_2}{\exp\left(\frac{B(\lambda_1-\lambda_2)}{W_2}\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]\right)-1}\right)^j \exp\left[i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right], \tag{70}$$

$$r(x,t)=\sum_{j=0}^{2}\tilde{\tau}_j\left(\lambda_1+\frac{\lambda_1-\lambda_2}{\exp\left(\frac{B(\lambda_1-\lambda_2)}{W_2}\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]\right)-1}\right)^j \exp\left[2i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right]. \tag{71}$$

When $\Lambda(\Psi)=(\Psi-\lambda_1)^2(\Psi-\lambda_2)(\Psi-\lambda_3)$ and $\lambda_1>\lambda_2>\lambda_3$,

$$q(x,t)=\sum_{j=0}^{2}\tau_j\left(\lambda_1-\frac{2(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}{2\lambda_1-\lambda_2-\lambda_3+(\lambda_3-\lambda_2)}\cdot\frac{1}{\cosh\left(\frac{B\sqrt{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}}{W_2}\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]\right)}\right)^j \times\exp\left[i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right], \quad (72)$$

$$r(x,t)=\sum_{j=0}^{2}\tilde{\tau}_j\left(\lambda_1-\frac{2(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}{2\lambda_1-\lambda_2-\lambda_3+(\lambda_3-\lambda_2)}\cdot\frac{1}{\cosh\left(\frac{B\sqrt{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)}}{W_2}\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]\right)}\right)^j \times\exp\left[2i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right]. \quad (73)$$

when $\Lambda(\Psi)=(\Psi-\lambda_1)(\Psi-\lambda_2)(\Psi-\lambda_3)(\Psi-\lambda_4)$ and $\lambda_1>\lambda_2>\lambda_3>\lambda_4$,

$$q(x,t)=\sum_{j=0}^{2}\tau_j\left(\lambda_2+\frac{(\lambda_1-\lambda_2)(\lambda_4-\lambda_2)}{\lambda_4-\lambda_2+(\lambda_1-\lambda_4)sn^2}\cdot\frac{1}{\left(\pm\frac{B\sqrt{(\lambda_1-\lambda_3)(\lambda_2-\lambda_4)}}{2W_2}\right)}\cdot\frac{1}{\left(\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right],\frac{(\lambda_2-\lambda_3)(\lambda_1-\lambda_4)}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_4)}\right)}\right)^j \times\exp\left[i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right], \quad (74)$$

$$r(x,t)=\sum_{j=0}^{2}\tilde{\tau}_j\left(\lambda_2+\frac{(\lambda_1-\lambda_2)(\lambda_4-\lambda_2)}{\lambda_4-\lambda_2+(\lambda_1-\lambda_4)sn^2}\cdot\frac{1}{\left(\pm\frac{B\sqrt{(\lambda_1-\lambda_3)(\lambda_2-\lambda_4)}}{2W_2}\right)}\cdot\frac{1}{\left(\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right],\frac{(\lambda_2-\lambda_3)(\lambda_1-\lambda_4)}{(\lambda_1-\lambda_3)(\lambda_2-\lambda_4)}\right)}\right)^j \times\exp\left[2i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right]. \quad (75)$$

Note that λ_i $(i=1,\ldots,4)$ are the roots of the polynomial equation

$$\Lambda(\Psi)=0. \quad (76)$$

Remark-2: When the modulus $l\to 1$, singular solitons emerge:

$$q(x,t)=\sum_{j=0}^{2}\tau_j\left(\lambda_2+\frac{(\lambda_1-\lambda_2)(\lambda_4-\lambda_2)}{\lambda_4-\lambda_2+(\lambda_1-\lambda_4)\tanh^2}\cdot\frac{1}{\left(\pm\frac{B\sqrt{(\lambda_1-\lambda_3)(\lambda_2-\lambda_4)}}{2W_2}\right)}\cdot\frac{1}{\left(\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]\right)}\right)^j \times\exp\left[i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right], \quad (77)$$

$$r(x,t)=\sum_{j=0}^{2}\tilde{\tau}_j\left(\lambda_2+\frac{(\lambda_1-\lambda_2)(\lambda_4-\lambda_2)}{\lambda_4-\lambda_2+(\lambda_1-\lambda_4)\tanh^2}\cdot\frac{1}{\left(\pm\frac{B\sqrt{(\lambda_1-\lambda_3)(\lambda_2-\lambda_4)}}{2W_2}\right)}\cdot\frac{1}{\left(\left[x-\left(\frac{4a\kappa-2b\omega+\alpha}{2b\kappa-1}\right)t\right]\right)}\right)^j \times\exp\left[2i\left\{-\kappa x+\left(\frac{2\kappa(\alpha+2a\kappa)-c_2}{2(2b\kappa-1)}\right)t+\theta\right\}\right], \quad (78)$$

where $\lambda_3=\lambda_4$.

3. Conclusions

This paper obtained solitons and other relevant solutions to quadratic nonlinear media. The integration algorithm is extended trial equation approach. Bright, dark and singular soliton solutions are recovered. In addition, plane waves and snoidal wave solutions are also presented in this paper. These additional results are being reported for the first time in this paper. The results come with constraint conditions that guarantee the existence of these variety of waves. This principle will be further explored later to other areas of nonlinear optics such as couplers, birefringence, metamaterials, liquid crystals and others. The results of those research are awaited at this time.

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