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Optical soliton perturbation with quadratic-cubic nonlinearity using a couple of strategic algorithms

Anjan Biswas[a,b,c], Yakup Yıldırım[d], Emrullah Yaşar[*,d], Qin Zhou[e], Seithuti P. Moshokoa[c], Milivoj Belic[f]

[a] Department of Physics, Chemistry and Mathematics, Alabama A&M University, Normal, AL 35762, USA
[b] Department of Mathematics and Statistics, College of Science, Al-Imam Mohammad Ibn Saud Islamic University, Riyadh 13318, Saudi Arabia
[c] Department of Mathematics and Statistics, Tshwane University of Technology, Pretoria 0008, South Africa
[d] Department of Mathematics, Faculty of Arts and Sciences, Uludag University, Bursa, 16059, Turkey
[e] School of Electronics and Information Engineering, Wuhan Donghu University, Wuhan 430212, People's Republic of China
[f] Science Program, Texas A&M University at Qatar, Doha, PO Box 23874, Qatar

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ABSTRACT

In this work, we derive bright, dark and singular soliton solutions to quadratic-cubic nonlinear media with perturbation terms being present. We perform the modified simple and the trial equation algorithms to the considered model. In addition, periodic singular wave solutions will be constructed by the integration schemes.

1. Introduction

The research area of soliton theory is quite important in many physical fields such as fluid dynamics, optical fibers, quantum field theory, nuclear physics, etc. We observe many works on the construction of soliton solutions for various distinct mathematical models. For instance, in [1], Patel and Kumar obtain dark and kink soliton solutions for generalized ZK-BBM equation using the Adomian decomposition method (ADM) and variational iteration method (VIM). Yu et al. [2] build the N-soliton solutions in terms of the Wronskian of $(2+1)$ dimensional variable-coefficient Nizhnik–Novikov–Veselov system in an inhomogeneous medium. In [3], Tebue et al. retrieve kink and bell-shaped soliton solutions of modified Zakharov–Kuznetsov equation via Jacobi elliptical function method. In [4], Choi and Kim yield dark and bell-shaped soliton solutions of some space-time fractional nonlinear partial differential equations in the meaning of Jumarie's Riemann–Liouville fractional derivative operator.

Optical soliton perturbation is the backbone of soliton vibration deployment across trans–continental and trans–oceanic distances along diverse shapes of waveguides, for instance, optical fibers, PCF and metamaterials and metasurfaces. Several forms of nonlinear media come with these fibers and couplers. For the nonlinear Schrödinger's equation (NLSE) [5–14], we are going to consider quadratic-cubic (QC) media which was first worked in 1994 [8] and afterwards revisited in 2011 [11]. Also, several research results have gradually flooded in various journals all across. There are a variety of results that have been sequentially reported. The model equation was addressed using traveling wave hypothesis [7], the method of undetermined coefficients [6] and application of semi–inverse variational principle [8,9]. The conservation laws were uncovered to the model [14]. This paper will implement two forms of integration architecture in order to study the perturbed NLSE with QC nonlinearity that includes higher order dispersion terms. These are modified simple equation and trial equation methodology. Both dark and bright in additional singular soliton will be

* Corresponding author.
E-mail address: emrullah.yasar@gmail.com (E. Yaşar).

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found getting these couple of algorithms.

1.1. The model

The NLSE throughout QC nonlinearity and perturbation terms is presented by

$$iq_t + aq_{xx} + (b_1|q| + b_2|q|^2)q = i[\alpha q_x - \gamma q_{xxx} - i\sigma q_{xxxx} + \lambda(|q|^{2m}q)_x + \mu(|q|^{2m})_x q] \tag{1}$$

where $q(x, t)$ stands for complex function which means the nonlinear wave profile and the first term corresponds to the temporal evolution of the pulses. The two nonlinear terms are the coefficients of b_1 and b_2 that allude to quadratic media and cubic media separately when the coefficient of a implies group velocity dispersion. μ gives the coefficient of nonlinear dispersion, σ points out fourth order dispersion, λ indicates self-steepening term, α corresponds to inter–modal dispersion when γ comes from third order dispersion. Lastly, m comes from the full nonlinearity exponent.

2. Trial equation method

The fundamental stages of this method are enumerated by following steps [15,16]:
Step-1: Let's consider a nonlinear evolution equation (NLEE)

$$\Lambda(q, q_t, q_x, q_{tt}, q_{xt}, q_{xx}, ...) = 0. \tag{2}$$

The Eq. (2) transforms to ordinary differential equation (ODE)

$$\Delta(Q, Q', Q'', Q''', ...) = 0 \tag{3}$$

via wave variable $q(x, t) = Q(\zeta)$, $\zeta = x - vt$, where $Q = Q(\zeta)$ stands for an dependent function, Δ is a polynomial in Q and its subsequent derivatives.

Step-2: In what follows, we consider the following auxiliary first order ODE

$$(Q')^2 = H(Q) = \sum_{i=0}^{M} \delta_i Q^i \tag{4}$$

where $\delta_0,\delta_1,\delta_2,...,\delta_M$ unknown coefficients and will be later fixed. We have a polynomial expression $\Phi(Q)$ through by plugging Eq. (4) and necessary derivatives terms into Eq. (3). In Eq. (4), the positive value integer of M which is the order of the finite series can be attained by balancing rule. One comes up with a overdetermined system which contains unknown parameters under the condition of setting the coefficients of $\Phi(Q)$ to be zero. With help of symbolic computation softwares, the constants of v , δ_0, δ_1, ..., δ_M are fixed.

Step-3: Reformulate Eq. (4) with an integral representation as

$$\pm(\zeta - \zeta_0) = \int \frac{dQ}{\sqrt{H(Q)}} \tag{5}$$

Based on the classification of the discriminants of the integrand, one categorizes the roots of $H(Q)$, and evaluate the integral Eq. (5). Thus, analytical solutions for Eq. (2) are recovered.

2.1. Application to NLSE

In order to solve the governing NLSE by help of the trial equation method, the solution form will be assumed as

$$q(x, t) = Q(\zeta)e^{i\phi(x,t)}, \tag{6}$$

where the parameter ζ is known as wave variable and can be represented by

$$\zeta = x - vt \tag{7}$$

while the phase function $\phi(x, t)$ is selected by means of

$$\phi(x, t) = -\kappa x + \omega t + \theta \tag{8}$$

where the parameters κ, ω and θ stand for the soliton frequency, soliton wave number and soliton phase respectively.

Plugging (6) into (1), the real and imaginary components give rise to

$$\sigma Q'''' - (a + 3\gamma\kappa + 6\kappa^2\sigma)Q'' + (\omega + a\kappa^2 + \alpha\kappa + \gamma\kappa^3 + \kappa^4\sigma)Q - b_1Q^2 - b_2Q^3 + \kappa\lambda Q^{2m+1} = 0, \tag{9}$$

and

$$(3\gamma\kappa^2 + 4\kappa^3\sigma + 2a\kappa + v + \alpha)Q' - (4\kappa\sigma + \gamma)Q''' + ((2m + 1)\lambda + 2m\mu)Q^{2m}Q' = 0. \tag{10}$$

and

$$(2m + 1)\lambda + 2m\mu = 0 \tag{12}$$

while the soliton speed emerges as

$$v = -(3\gamma\kappa^2 + 4\kappa^3\sigma + 2a\kappa + \alpha). \tag{13}$$

Case-1:

Balancing Q'''' between Q^3 along with $m = 1$ in Eq. (9), one gets $M = 3$. Pursuing the methedology of the trial equation approach gives rise to

Q^3 Coeff.:

$$15\sigma\delta_3^2 + 2(\kappa\lambda - b_2) = 0,$$

Q^2 Coeff.:

$$3(a + 3\gamma\kappa + 6\kappa^2\sigma)\delta_3 + 2b_1 = 0,$$

Q Coeff.:

$$\sigma\delta_2^2 - (a + 3\gamma\kappa + 6\kappa^2\sigma)\delta_2 + \omega + a\kappa^2 + \alpha\kappa + \gamma\kappa^3 + \kappa^4\sigma = 0,$$

Q^0 Coeff.:

$$(a + 3\gamma\kappa + 6\kappa^2\sigma)\delta_1 = 0.$$

Solving these equations by aid of Maple, we have

$$\delta_1 = 0, \quad \delta_3 = -\frac{2b_1}{3(a + 3\gamma\kappa + 6\kappa^2\sigma)},$$

$$\sigma\delta_2^2 - (a + 3\gamma\kappa + 6\kappa^2\sigma)\delta_2 + \omega + a\kappa^2 + \alpha\kappa + \gamma\kappa^3 + \kappa^4\sigma = 0,$$

$$b_2 = \frac{108\kappa^5\lambda\sigma^2 + 108\gamma\kappa^4\lambda\sigma + 36a\kappa^3\lambda\sigma + 27\gamma^2\kappa^3\lambda + 18a\gamma\kappa^2\lambda + 3a^2\kappa\lambda + 10\sigma b_1^2}{3(a + 3\gamma\kappa + 6\kappa^2\sigma)^2}.$$

Plugging these values into Eqs. (4) and (5), one has

$$\pm(\zeta - \zeta_0) = \int \frac{dQ}{\sqrt{\delta_0 + \delta_2 Q^2 - \frac{2b_1}{3(a + 3\gamma\kappa + 6\kappa^2\sigma)}Q^3}}. \tag{14}$$

Setting $\delta_0 = 0$ in Eq. (14) and integrating with respect to Q, the analytical solution of Eq. (1) are given by

$$q(x,t) = \pm\left\{\frac{3\delta_2(a + 3\gamma\kappa + 6\kappa^2\sigma)}{2b_1}\text{sech}^2\left[\frac{\sqrt{\delta_2}}{2}(x + (3\gamma\kappa^2 + 4\kappa^3\sigma + 2a\kappa + \alpha)t)\right]\right\}e^{i(-\kappa x + \omega t + \theta)}, \tag{15}$$

$$q(x,t) = \pm\left\{-\frac{3\delta_2(a + 3\gamma\kappa + 6\kappa^2\sigma)}{2b_1}\text{csch}^2\left[\frac{\sqrt{\delta_2}}{2}(x + (3\gamma\kappa^2 + 4\kappa^3\sigma + 2a\kappa + \alpha)t)\right]\right\}e^{i(-\kappa x + \omega t + \theta)}, \tag{16}$$

where Eqs. (15) and (16) corresponds to optical bright and optical singular soliton separately whenever

$$\delta_2 > 0.$$

and

$$q(x,t) = \pm\left\{-\frac{3\delta_2(a + 3\gamma\kappa + 6\kappa^2\sigma)}{2b_1}\sec^2\left[\frac{\sqrt{-\delta_2}}{2}(x + (3\gamma\kappa^2 + 4\kappa^3\sigma + 2a\kappa + \alpha)t)\right]\right\}e^{i(-\kappa x + \omega t + \theta)}, \tag{17}$$

$$q(x,t) = \pm\left\{-\frac{3\delta_2(a + 3\gamma\kappa + 6\kappa^2\sigma)}{2b_1}\csc^2\left[\frac{\sqrt{-\delta_2}}{2}(x + (3\gamma\kappa^2 + 4\kappa^3\sigma + 2a\kappa + \alpha)t)\right]\right\}e^{i(-\kappa x + \omega t + \theta)}, \tag{18}$$

where Eqs. (17) and (18) correspond to singular periodic solutions under the condition of

$$\delta_2 < 0.$$

Case-2: Comparing Q'''' between Q^5 along with $m = 2$ in Eq. (9), one can conclude that the order of the finite series must be $M = 4$. Pursuing the methedology of the trial equation approach gives rise to

Q^3 Coeff.:

$$15\sigma\delta_3^2 - 4(a + 3\gamma\kappa + 6\kappa^2\sigma)\delta_4 - 2b_2 = 0,$$

Q^2 Coeff.:

$$3(a + 3\gamma\kappa + 6\kappa^2\sigma)\delta_3 + 2b_1 = 0,$$

Q Coeff.:

$$\sigma\delta_2^2 - (a + 3\gamma\kappa + 6\kappa^2\sigma)\delta_2 + \omega + a\kappa^2 + \alpha\kappa + \gamma\kappa^3 + \kappa^4\sigma = 0,$$

Q^0 Coeff.:

$$(a + 3\gamma\kappa + 6\kappa^2\sigma)\delta_1 = 0.$$

Solving these equations by help of Maple, we have

$$\delta_1 = 0, \quad \delta_3 = -\frac{2b_1}{3(a + 3\gamma\kappa + 6\kappa^2\sigma)}, \quad \lambda = -\frac{2\sigma\rho_1}{3\kappa\rho_2},$$

$$\sigma\delta_2^2 - (a + 3\gamma\kappa + 6\kappa^2\sigma)\delta_2 + \omega + a\kappa^2 + \alpha\kappa + \gamma\kappa^3 + \kappa^4\sigma = 0,$$

$$\delta_4 = -\frac{108\kappa^4\sigma^2 b_2 + 108\gamma\kappa^3\sigma b_2 + 36a\kappa^2\sigma b_2 + 27\gamma^2\kappa^2 b_2 + 18a\gamma\kappa b_2 + 3a^2 b_2 - 10\sigma b_1^2}{6(216\kappa^6\sigma^3 + 324\gamma\kappa^5\sigma^2 + 108a\kappa^4\sigma^2 + 162\gamma^2\kappa^4\sigma + 108a\gamma\kappa^3\sigma + 27\gamma^3\kappa^3 + 18a^2\kappa^2\sigma + 27a\gamma^2\kappa^2 + 9a^2\gamma\kappa + a^3)},$$

$$\begin{aligned}\rho_1 = {}& 11664\kappa^8\sigma^4 b_2^2 + 23328\gamma\kappa^7\sigma^3 b_2^2 + 7776a\kappa^6\sigma^3 b_2^2 + 17496\gamma^2\kappa^6\sigma^2 b_2^2 + 11664a\gamma\kappa^5\sigma^2 b_2^2 + 5832\gamma^3\kappa^5\sigma b_2^2 + 1944a^2\kappa^4\sigma^2 b_2^2\\ &+ 5832a\gamma^2\kappa^4\sigma b_2^2 + 729\gamma^4\kappa^4 b_2^2 - 2160\kappa^4\sigma^3 b_1^2 b_2 + 1944a^2\gamma\kappa^3\sigma b_2^2 + 972a\gamma^3\kappa^3 b_2^2 - 2160\gamma\kappa^3\sigma^2 b_1^2 b_2 + 216a^3\kappa^2\sigma b_2^2\\ &+ 486a^2\gamma^2\kappa^2 b_2^2 - 720a\kappa^2\sigma^2 b_1^2 b_2 - 540\gamma^2\kappa^2\sigma b_1^2 b_2 + 108a^3\gamma\kappa b_2^2 - 360a\gamma\kappa\sigma b_1^2 b_2 + 9a^4 b_2^2 - 60a^2\sigma b_1^2 b_2 + 100\sigma^2 b_1^4,\end{aligned}$$

$$\begin{aligned}\rho_2 = {}& 46656\kappa^{12}\sigma^6 + 139968\gamma\kappa^{11}\sigma^5 + 46656a\kappa^{10}\sigma^5 + 174960\gamma^2\kappa^{10}\sigma^4 + 116640a\gamma\kappa^9\sigma^4 + 116640\gamma^3\kappa^9\sigma^3 + 19440a^2\kappa^8\sigma^4\\ &+ 116640a\gamma^2\kappa^8\sigma^3 + 43740\gamma^4\kappa^8\sigma^2 + 38880a^2\gamma\kappa^7\sigma^3 + 58320a\gamma^3\kappa^7\sigma^2 + 8748\gamma^5\kappa^7\sigma + 4320a^3\kappa^6\sigma^3 + 29160a^2\gamma^2\kappa^6\sigma^2\\ &+ 14580a\gamma^4\kappa^6\sigma + 729\gamma^6\kappa^6 + 6480a^3\gamma\kappa^5\sigma^2 + 9720a^2\gamma^3\kappa^5\sigma + 1458a\gamma^5\kappa^5 + 540a^4\kappa^4\sigma^2 + 3240a^3\gamma^2\kappa^4\sigma + 1215a^2\gamma^4\kappa^4\\ &+ 540a^4\gamma\kappa^3\sigma + 540a^3\gamma^3\kappa^3 + 36a^5\kappa^2\sigma + 135a^4\gamma^2\kappa^2 + 18a^5\gamma\kappa + a^6.\end{aligned}$$

Plugging these values into Eqs. (4) and (5), one has

$$\pm(\zeta - \zeta_0) = \int \frac{dQ}{\sqrt{\delta_0 + \delta_2 Q^2 - \frac{2b_1}{3(a + 3\gamma\kappa + 6\kappa^2\sigma)}Q^3 + \delta_4 Q^4}}. \tag{19}$$

Setting $\delta_0 = 0$ and $\delta_2 = \frac{b_1^2}{9\delta_4(a + 3\gamma\kappa + 6\kappa^2\sigma)^2}$ in Eq. (19) and integrating with respect to Q, the exact solution of the NLSE are given by means of

$$q(x,t) = \frac{b_1}{6\delta_4(a + 3\gamma\kappa + 6\kappa^2\sigma)}\left\{1 \pm \tanh\left[\sqrt{\frac{b_1^2}{36\delta_4(a + 3\gamma\kappa + 6\kappa^2\sigma)^2}}(x + (3\gamma\kappa^2 + 4\kappa^3\sigma + 2a\kappa + \alpha)t)\right]\right\}$$
$$\times e^{i(-\kappa x + \omega t + \theta)}, \tag{20}$$

$$q(x,t) = \frac{b_1}{6\delta_4(a + 3\gamma\kappa + 6\kappa^2\sigma)}\left\{1 \pm \coth\left[\sqrt{\frac{b_1^2}{36\delta_4(a + 3\gamma\kappa + 6\kappa^2\sigma)^2}}(x + (3\gamma\kappa^2 + 4\kappa^3\sigma + 2a\kappa + \alpha)t)\right]\right\}$$
$$\times e^{i(-\kappa x + \omega t + \theta)}, \tag{21}$$

where Eqs. (20) and (21) correspond to optical dark and optical singular soliton whenever

$$\delta_4 > 0.$$

3. Modified simple equation method

Let's consider a NLEE

$$\Lambda(q, q_t, q_x, q_{tt}, q_{xt}, q_{xx}, \ldots) = 0. \tag{22}$$

The fundamental points of this method are highlighted by following steps [17,18]:

where c is unknown coefficient which must be found. Substituting the Eq. (23) to (22) gives rise to the following ODE:

$$\Delta(Q, Q', Q'', Q''', \ldots) = 0 \tag{24}$$

where Δ is a polynomial including $Q(\zeta)$ and its partial derivatives.

Step-2: Eq. (24) permits the formal solution

$$Q(\zeta) = \sum_{l=0}^{M} \delta_l \left(\frac{\psi'(\zeta)}{\psi(\zeta)} \right)^l, \tag{25}$$

where δ_l are constants whose values must be detected, and $\psi(\zeta)$ will be obtained subsequently.

Step-3: In Eq. (25), the positive value integer of M which is the order of the finite series can be attaine by homogenous balancing rule.

Step-4: Plugging (25) and all the essential derivatives of the function $Q(\zeta)$ into (24), a polynomial which contains $\psi'(\zeta)/\psi(\zeta)$ and its derivatives is obtained. Setting all of the coefficients which can be achieved by collecting the same terms of $\psi^0(\zeta)$, $\psi^{-1}(\zeta)$, $\psi^{-2}(\zeta)$,… .to zero comes up a algebraic equation system. Solving this highly complicated system by assistance of Maple, the δ_k and $\psi(\zeta)$ are determined. Thus, analytical solutions for Eq. (22) are recovered.

3.1. Implementation to NLSE

Case-1:

Balancing of Q'''' with Q^3 along with $m = 1$ in Eq. (9), one ensure $M = 2$. As a result, Eq. (25) reduces to

$$Q(\zeta) = \delta_0 + \delta_1 \left(\frac{\psi(\zeta)}{\psi(\zeta)} \right) + \delta_2 \left(\frac{\psi'(\zeta)}{\psi(\zeta)} \right)^2. \tag{26}$$

Plugging (26) into Eq. (9) and setting the coefficients of ψ^{-6}, ψ^{-5}, ψ^{-4}, ψ^{-3}, ψ^{-2}, ψ^{-1}, ψ^0 equal to zero give rise to:

ψ^{-6} coeff.:

$$\delta_2(\psi')^6(\kappa\lambda\delta_2^2 - b_2\delta_2^2 + 120\sigma) = 0, \tag{27}$$

ψ^{-5} coeff.:

$$3(\psi')^4((\kappa\lambda\delta_1\delta_2^2 - \delta_1\delta_2^2 b_2 + 8\sigma\delta_1)\psi' - 112\sigma\delta_2\psi'') = 0, \tag{28}$$

ψ^{-4} coeff.:

$$\begin{aligned} &-(\psi')^2((-3\kappa\lambda\delta_0\delta_2^2 - 3\kappa\lambda\delta_1^2\delta_2 + 36\kappa^2\sigma\delta_2 + 3\delta_0\delta_2^2 b_2 + 3\delta_1^2\delta_2 b_2 + 18\gamma\kappa\delta_2 + \delta_2^2 b_1 + 6a\delta_2)(\psi')^2 \\ &+ 60\sigma\delta_1\psi'\psi'' - 96\sigma\delta_2\psi'\psi''' - 234\sigma\delta_2(\psi'')^2) = 0, \end{aligned} \tag{29}$$

ψ^{-3} coeff.:

$$\begin{aligned} &(6\kappa\lambda\delta_0\delta_1\delta_2 + \kappa\lambda\delta_1^3 - 12\kappa^2\sigma\delta_1 - 6\delta_0\delta_1\delta_2 b_2 - \delta_1^3 b_2 - 6\gamma\kappa\delta_1 - 2\delta_1\delta_2 b_1 - 2a\delta_1)(\psi')^3 \\ &+ (60\kappa^2\sigma\delta_2 + 30\gamma\kappa\delta_2 + 10a\delta_2)\psi''(\psi')^2 + 20\sigma\delta_1\psi'''(\psi')^2 - 18\sigma\delta_2\psi''''(\psi')^2 \\ &+ 30\sigma\delta_1\psi'(\psi'')^2 - 88\sigma\delta_2\psi'\psi''\psi''' - 24\sigma\delta_2(\psi'')^3 = 0, \end{aligned} \tag{30}$$

ψ^{-2} coeff.:

$$\begin{aligned} &(3\delta_0^2\delta_2\lambda\kappa + 3\delta_0\delta_1^2\lambda\kappa - 2b_1\delta_0\delta_2 + \delta_2\sigma\kappa^4 + \delta_2\gamma\kappa^3 + \delta_2 a\kappa^2 + \delta_2\alpha\kappa - 3\delta_0^2\delta_2 b_2 - 3\delta_0\delta_1^2 b_2 - b_1\delta_1^2 + \delta_2\omega)(\psi')^2 \\ &+ (18\kappa^2\sigma\delta_1 + 9\gamma\kappa\delta_1 + 3a\delta_1)\psi'\psi'' - (12\kappa^2\sigma\delta_2 + 6\gamma\kappa\delta_2 + 2a\delta_2)\psi'\psi''' - 5\sigma\delta_1\psi'\psi'''' + 2\sigma\delta_2\psi'\psi''''' \\ &- (12\kappa^2\sigma\delta_2 + 6\gamma\kappa\delta_2 + 2a\delta_2)(\psi'')^2 - 10\sigma\delta_1\psi''\psi''' + 8\sigma\delta_2\psi''\psi'''' + 6\sigma\delta_2(\psi''')^2 = 0, \end{aligned} \tag{31}$$

ψ^{-1} coeff.:

$$\delta_1((3\delta_0^2\lambda\kappa + \sigma\kappa^4 + \gamma\kappa^3 + a\kappa^2 + \alpha\kappa - 2b_1\delta_0 - 3\delta_0^2 b_2 + \omega)\psi' - (6\kappa^2\sigma + 3\gamma\kappa + a)\psi''' + \sigma\psi''''') = 0, \tag{32}$$

ψ^0 coeff.:

$$\delta_0(\omega - b_1\delta_0 - \delta_0^2 b_2 + \sigma\kappa^4 + a\kappa^2 + \alpha\kappa + \delta_0^2\lambda\kappa + \gamma\kappa^3) = 0. \tag{33}$$

Solving this system with help of Maple, one conclude that

$$\delta_0 = 0, \quad \omega = -\frac{\kappa^4\sigma\delta_2^4 + \gamma\kappa^3\delta_2^4 + a\kappa^2\delta_2^4 - 6\kappa^2\sigma\delta_1^2\delta_2^2 + \alpha\kappa\delta_2^4 - 3\gamma\kappa\delta_1^2\delta_2^2 - a\delta_1^2\delta_2^2 + \sigma\delta_1^4}{\delta_2^4},$$

$$, \quad 6(6\kappa^2\sigma\delta_2^2 + 3\gamma\kappa\delta_2^2 + a\delta_2^2 - 5\sigma\delta_1^2) \quad , \quad \kappa\lambda\delta_2^2 + 120\sigma$$

$$\psi''=\pm\sqrt{\frac{\delta_1^2(b_2-\kappa\lambda)}{120\sigma}}\psi', \tag{35}$$

$$\psi'''=\frac{\delta_1^2(b_2-\kappa\lambda)}{120\sigma}\psi'. \tag{36}$$

From Eqs. (35) and (36), we can deduce that

$$\psi'=\pm\sqrt{\frac{120\sigma}{\delta_1^2(b_2-\kappa\lambda)}}k_1 e^{\pm\sqrt{\frac{\delta_1^2(b_2-\kappa\lambda)}{120\sigma}}\zeta}, \tag{37}$$

and

$$\psi=\frac{120\sigma}{\delta_1^2(b_2-\kappa\lambda)}k_1 e^{\pm\sqrt{\frac{\delta_1^2(b_2-\kappa\lambda)}{120\sigma}}\zeta}+k_2, \tag{38}$$

where k_1 and k_2 are integration constants. The analytical solution to Eq. (1) are reached by plugging Eq. (37) and (38) into Eq. (26):

$$q(x,t)=\left\{\delta_1\left(\frac{\pm\sqrt{\frac{120\sigma}{\delta_1^2(b_2-\kappa\lambda)}}k_1 e^{\pm\sqrt{\frac{\delta_1^2(b_2-\kappa\lambda)}{120\sigma}}(x+(3\gamma\kappa^2+4\kappa^3\sigma+2a\kappa+\alpha)t)}}{\frac{120\sigma}{\delta_1^2(b_2-\kappa\lambda)}k_1 e^{\pm\sqrt{\frac{\delta_1^2(b_2-\kappa\lambda)}{120\sigma}}(x+(3\gamma\kappa^2+4\kappa^3\sigma+2a\kappa+\alpha)t)}+k_2}\right)\right.$$
$$\left.\pm\sqrt{\frac{120\sigma}{b_2-\kappa\lambda}}\left(\frac{\pm\sqrt{\frac{120\sigma}{\delta_1^2(b_2-\kappa\lambda)}}k_1 e^{\pm\sqrt{\frac{\delta_1^2(b_2-\kappa\lambda)}{120\sigma}}(x+(3\gamma\kappa^2+4\kappa^3\sigma+2a\kappa+\alpha)t)}}{\frac{120\sigma}{\delta_1^2(b_2-\kappa\lambda)}k_1 e^{\pm\sqrt{\frac{\delta_1^2(b_2-\kappa\lambda)}{120\sigma}}(x+(3\gamma\kappa^2+4\kappa^3\sigma+2a\kappa+\alpha)t)}+k_2}\right)^2\right\}e^{i(-\kappa x+\omega t+\theta)}$$

If we set

$$k_1=\frac{\delta_1^2(b_2-\kappa\lambda)}{120\sigma}e^{\pm\sqrt{\frac{\delta_1^2(b_2-\kappa\lambda)}{120\sigma}}\zeta_0},\quad k_2=\pm1,$$

then

$$q(x,t)=\pm\frac{\delta_1^2}{4}\sqrt{\frac{b_2-\kappa\lambda}{120\sigma}}\operatorname{sech}^2\left[\sqrt{\frac{\delta_1^2(b_2-\kappa\lambda)}{480\sigma}}(x+(3\gamma\kappa^2+4\kappa^3\sigma+2a\kappa+\alpha)t+\zeta_0)\right]e^{i(-\kappa x+\omega t+\theta)}, \tag{39}$$

$$q(x,t)=\pm\frac{\delta_1^2}{4}\sqrt{\frac{b_2-\kappa\lambda}{120\sigma}}\operatorname{csch}^2\left[\sqrt{\frac{\delta_1^2(b_2-\kappa\lambda)}{480\sigma}}(x+(3\gamma\kappa^2+4\kappa^3\sigma+2a\kappa+\alpha)t+\zeta_0)\right]e^{i(-\kappa x+\omega t+\theta)}, \tag{40}$$

where Eqs. (39) and (40) represent bright and singular soliton solutions respectively under the condition of

$\sigma(b_2-\kappa\lambda)>0.$

$$q(x,t)=\pm\frac{\delta_1^2}{4}\sqrt{\frac{b_2-\kappa\lambda}{120\sigma}}\sec^2\left[\sqrt{-\frac{\delta_1^2(b_2-\kappa\lambda)}{480\sigma}}(x+(3\gamma\kappa^2+4\kappa^3\sigma+2a\kappa+\alpha)t+\zeta_0)\right]e^{i(-\kappa x+\omega t+\theta)}, \tag{41}$$

$$q(x,t)=\pm\frac{\delta_1^2}{4}\sqrt{\frac{b_2-\kappa\lambda}{120\sigma}}\csc^2\left[\sqrt{-\frac{\delta_1^2(b_2-\kappa\lambda)}{480\sigma}}(x+(3\gamma\kappa^2+4\kappa^3\sigma+2a\kappa+\alpha)t+\zeta_0)\right]e^{i(-\kappa x+\omega t+\theta)}, \tag{42}$$

where Eqs. (41) and (42) present singular periodic solutions under the condition of

$\sigma(b_2-\kappa\lambda)<0.$

Case-2:

Balancing of Q'''' with Q^5 along with $m=2$ in Eq. (9), one ensures $M=1$. Therefore, Eq. (25) reduces to

$$Q(\zeta)=\delta_0+\delta_1\left(\frac{\psi'(\zeta)}{\psi(\zeta)}\right). \tag{43}$$

Plugging (43) into Eq. (9) and equating the coefficients of ψ^{-5}, ψ^{-4}, ψ^{-3}, ψ^{-2}, ψ^{-1}, ψ^{0} to zero give rise to:

ψ^{-5} coeff.:

$$5\delta_1(\psi')^3(\kappa\lambda\delta_0\delta_1^3\psi' - 12\sigma\psi'') = 0, \tag{45}$$

ψ^{-3} coeff.:

$$-\delta_1(\psi')((-10\kappa\lambda\delta_0^2\delta_1^2 + 12\kappa^2\sigma + \delta_1^2 b_2 + 6\gamma\kappa + 2a)(\psi')^2 - 20\sigma\psi'\psi''' - 30\sigma(\psi'')^2) = 0, \tag{46}$$

ψ^{-2} coeff.:

$$\delta_1((10\kappa\lambda\delta_0^3\delta_1 - 3\delta_0\delta_1 b_2 - \delta_1 b_1)(\psi')^2 + 3(a + 3\gamma\kappa + 6\kappa^2\sigma)\psi'\psi'' - 5\sigma\psi'\psi'''' - 10\sigma\psi''\psi''') = 0, \tag{47}$$

ψ^{-1} coeff.:

$$\delta_1((5\kappa\lambda\delta_0^4 + \sigma\kappa^4 + \gamma\kappa^3 + a\kappa^2 + \alpha\kappa - 2b_1\delta_0 - 3b_2\delta_0^2 + \omega)\psi' - (a + 3\gamma\kappa + 6\kappa^2\sigma)\psi''' + \sigma\psi'''''') = 0, \tag{48}$$

ψ^0 coeff.:

$$\delta_0(\omega - b_1\delta_0 - \delta_0^2 b_2 + \sigma\kappa^4 + a\kappa^2 + \alpha\kappa + \delta_0^4\lambda\kappa + \gamma\kappa^3) = 0. \tag{49}$$

Solving above system with help of Maple, one conclude that

$$b_1 = 0, \quad \kappa\lambda\delta_1^4 + 24\sigma = 0, \quad \delta_0 = \pm\sqrt{-\frac{(12\kappa^2\sigma + \delta_1^2 b_2 + 6\gamma\kappa + 2a)\delta_1^2}{40\sigma}},$$

$$\omega = \frac{116\kappa^4\sigma^2 + 6\kappa^2\sigma\delta_1^2 b_2 - \delta_1^4 b_2^2 + 116\gamma\kappa^3\sigma + 3\gamma\kappa\delta_1^2 b_2 - 28a\kappa^2\sigma + a\delta_1^2 b_2 + 54\gamma^2\kappa^2 + 36a\gamma\kappa + 6a^2 - 100\omega\sigma}{100\kappa\sigma} \tag{50}$$

and

$$\psi'' = \pm\sqrt{-\frac{12\kappa^2\sigma + \delta_1^2 b_2 + 6\gamma\kappa + 2a}{10\sigma}}\,\psi', \tag{51}$$

$$\psi''' = -\frac{12\kappa^2\sigma + \delta_1^2 b_2 + 6\gamma\kappa + 2a}{10\sigma}\psi'. \tag{52}$$

With $b_1 = 0$, we have Eq. (1) reduces to Kerr law nonlinear medium. From Eqs. (51) and (52), we can derive

$$\psi' = \pm\sqrt{-\frac{10\sigma}{12\kappa^2\sigma + \delta_1^2 b_2 + 6\gamma\kappa + 2a}}\,k_1 e^{\pm\sqrt{-\frac{12\kappa^2\sigma + \delta_1^2 b_2 + 6\gamma\kappa + 2a}{10\sigma}}\zeta}, \tag{53}$$

and

$$\psi = -\frac{10\sigma}{12\kappa^2\sigma + \delta_1^2 b_2 + 6\gamma\kappa + 2a}k_1 e^{\pm\sqrt{-\frac{12\kappa^2\sigma + \delta_1^2 b_2 + 6\gamma\kappa + 2a}{10\sigma}}\zeta} + k_2, \tag{54}$$

where k_1 and k_2 are integration constants. The analytical solution to Eq. (1) are reached by plugging Eq. (53) and (54) into Eq. (43):

$$q(x, t) = \Bigg\{\pm\sqrt{-\frac{(12\kappa^2\sigma + \delta_1^2 b_2 + 6\gamma\kappa + 2a)\delta_1^2}{40\sigma}}$$

$$+\delta_1\left(\frac{\pm\sqrt{-\frac{10\sigma}{12\kappa^2\sigma + \delta_1^2 b_2 + 6\gamma\kappa + 2a}}\,k_1 e^{\pm\sqrt{-\frac{12\kappa^2\sigma + \delta_1^2 b_2 + 6\gamma\kappa + 2a}{10\sigma}}(x + (3\gamma\kappa^2 + 4\kappa^3\sigma + 2a\kappa + \alpha)t)}}{-\frac{10\sigma}{12\kappa^2\sigma + \delta_1^2 b_2 + 6\gamma\kappa + 2a}k_1 e^{\pm\sqrt{-\frac{12\kappa^2\sigma + \delta_1^2 b_2 + 6\gamma\kappa + 2a}{10\sigma}}(x + (3\gamma\kappa^2 + 4\kappa^3\sigma + 2a\kappa + \alpha)t)} + k_2}\right)\Bigg\} e^{i(-\kappa x + \omega t + \theta)}$$

If we set

$$k_1 = -\frac{12\kappa^2\sigma + \delta_1^2 b_2 + 6\gamma\kappa + 2a}{10\sigma}e^{\pm\sqrt{-\frac{12\kappa^2\sigma + \delta_1^2 b_2 + 6\gamma\kappa + 2a}{10\sigma}}\zeta_0}, \quad k_2 = \pm 1,$$

we get

$$q(x, t) = \pm\sqrt{-\frac{(12\kappa^2\sigma + \delta_1^2 b_2 + 6\gamma\kappa + 2a)\delta_1^2}{40\sigma}}\tanh\left[\sqrt{-\frac{12\kappa^2\sigma + \delta_1^2 b_2 + 6\gamma\kappa + 2a}{40\sigma}}(x + (3\gamma\kappa^2 + 4\kappa^3\sigma + 2a\kappa + \alpha)t + \zeta_0)\right]$$
$$\times e^{i(-\kappa x + \omega t + \theta)}, \tag{55}$$

$$q(x, t) = \pm\sqrt{-\frac{(12\kappa^2\sigma + \delta_1^2 b_2 + 6\gamma\kappa + 2a)\delta_1^2}{40\sigma}}\coth\left[\sqrt{-\frac{12\kappa^2\sigma + \delta_1^2 b_2 + 6\gamma\kappa + 2a}{40\sigma}}(x + (3\gamma\kappa^2 + 4\kappa^3\sigma + 2a\kappa + \alpha)t + \zeta_0)\right]$$

where (55) and (56) represent optical dark and optical singular soliton respectively whenever

$$\sigma(12\kappa^2\sigma + \delta_1^2 b_2 + 6\gamma\kappa + 2a) < 0$$

and

$$q(x, t) = \pm\sqrt{\frac{(12\kappa^2\sigma + \delta_1^2 b_2 + 6\gamma\kappa + 2a)\delta_1^2}{40\sigma}} \tan\left[\sqrt{\frac{12\kappa^2\sigma + \delta_1^2 b_2 + 6\gamma\kappa + 2a}{40\sigma}}(x + (3\gamma\kappa^2 + 4\kappa^3\sigma + 2a\kappa + \alpha)t + \zeta_0)\right] \times e^{i(-\kappa x + \omega t + \theta)}, \tag{57}$$

$$q(x, t) = \pm\sqrt{\frac{(12\kappa^2\sigma + \delta_1^2 b_2 + 6\gamma\kappa + 2a)\delta_1^2}{40\sigma}} \cot\left[\sqrt{\frac{12\kappa^2\sigma + \delta_1^2 b_2 + 6\gamma\kappa + 2a}{40\sigma}}(x + (3\gamma\kappa^2 + 4\kappa^3\sigma + 2a\kappa + \alpha)t + \zeta_0)\right] \times e^{i(-\kappa x + \omega t + \theta)}, \tag{58}$$

where (57) and (58) stand for singular periodic solutions under the condition of

$$\sigma(12\kappa^2\sigma + \delta_1^2 b_2 + 6\gamma\kappa + 2a) > 0.$$

4. Dynamical analysis

Now that these soliton solutions are explicitly available in presence of perturbation terms with full nonlinearity, for the model, the groundwork for future activities has been laid. These soliton solutions will be of great asset for a variety of research activities in the field of fiber optics research. The soliton solutions will help to study soliton perturbation theory with additional perturbation terms that are of non-Hamiltonian as well as non-local type. These quasi-monochromatic perturbation terms will lead to the effect of soliton cooling that is a very important phenomena in optical fibers and PCF. Later quasi-stationary solitons will also be retrieved for the model with additional perturbation terms. The quasi-particle theory will also be developed to suppress intra-channel collision which is a major hindrance in telecommunication system. Then for inter-channel communications, collision-induced timing jitter will be developed and studied along with BER for its communication dynamics. The results will be extended to birefringent fibers and eventually it will model parallel communication with DWDM topology. The basic soliton solutions of this paper will serve as a platform for such developments.

In addition, several additional aspects can be easily ventured upon. These include formulation of soliton parameter dynamics using the variational principle after constructing the Lagrangian. These will be studied in polarization preserving fibers, birefringent fibers as well as with DWDM topology. The parameter dynamics will again form the basic ingredients for further advances such as studying the dynamics of soliton fission as well as developing supercontinuum generation. One other issue is the study of stochastic perturbation terms in addition to deterministic ones. In this context, soliton perturbation theory can be implemented to address the corresponding Langevin equation and compute the mean free velocity of the soliton in presence of such perturbations. One final issue is the consideration of the model with fractional temporal evolution. This will aid us in addressing the Internet bottleneck problem. Thus Internet traffic flow can be controlled and regulated smoothly at an Internet hub. This will allow a global flow of pulses without any traffic disruption. The basic soliton solutions presented in the paper, are thus encouraging, to address dynamical applications in fiber optic industry.

5. Conclusions

This paper performed two distinct integration algorithms to solve and yield dark, bright with singular soliton solutions for the NLSE that appeared with perturbation terms as well as QC nonlinearity. In addition, periodic singular waves naturally emerged from the reversed restriction requirements on the constants. Comparing our work with the works done on the NLSE with QC nonlinearity [8–14], we conclude some important aspects. Firstly, the perturbation terms known as inter-modal dispersion, self-steepening, higher-order dispersion, third order dispersion, fourth order dispersion throughout the full nonlinearity effect m are considered to keep the model from a generalized standpoint to the optical soliton propagation dynamics. However, the works of Aslan and Co-workers [5,9,11,12,14] consider only unperturbed NLSE. Secondly, only the full nonlinearity parameter $m = 1$ is investigated in [6,7,13] which includes special case of Eq. (1) while different values of the full nonlinearity parameter namely both $m = 1$ and $m = 2$ are employed in order to reveal soliton solutions by means of modifed simple equation method and trial equation method. This is an interesting conclusion as these integration schemes indicates. Thirdly, only bright soliton type solution is imparted in [8] whilst bright, dark and singular solitons in additional to singular periodic solutions are yielded in this paper. Also, all of the obtained solutions by these effective methods are new comparing with the obtained solutions in [8]. Lastly, Eq. (1) is an extended model of the models in [5–7,9,11–14] and its (15–18) and (39–42) type solutions are new and to best of our knowledge is reported firstly in this paper.

These results will serve as a steppping stone to additional investigation on this model. We emphasize that the obtained results are under way and their results will be revealed with time.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.cjph.2018.09.009.

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