Spatio-temporal dynamics of counterpropagating photorefractive self-trapped beams

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Abstract: We investigate spatio-temporal dynamics in the interaction of counterpropagating self-trapped beams in a photorefractive strontium barium niobate crystal. While the interaction of copropagating spatial optical solitons exhibits only transient dynamics, resulting in a final steady state, the counterpropagating geometry supports a dynamic instability mediated by intrinsic feedback. Experimental observations are compared to and found to be in qualitative agreement with numerical simulations. The threshold of the instability is examined numerically and period doubling beyond the bifurcation is demonstrated.

OCIS codes: 190.5530, 190.5330, 190.3100

Propagation of stable self-trapped beams (commonly called optical spatial solitons [1]) in photorefractive media has been the topic of intensive research in the last decade, primarily due to potential applications in all-optical switching. The majority of prior works concentrated on copropagating solitons, which exhibit characteristic interaction scenarios such as attraction, repulsion, fusion or birth of new solitons [2]. However, given one-sided boundary conditions, these nonlinear optical beams generally exhibit no dynamical behaviour beyond initial transient dynamics.

Several recent investigations on the formation of spatial solitons consisting of counterpropagating waves [3–7] indicate growing interest in this new field due to its applications for adaptive self-adjustment of beams. However, the counterpropagating geometry adds an intrinsic feedback to the soliton interaction. In general, counterpropagating waves coupled with feedback are often found to exhibit instabilities [8–10] and related phenomena, such as pattern formation [11]. Hence, one can expect qualitatively new properties to result from the interaction of counterpropagating solitons.

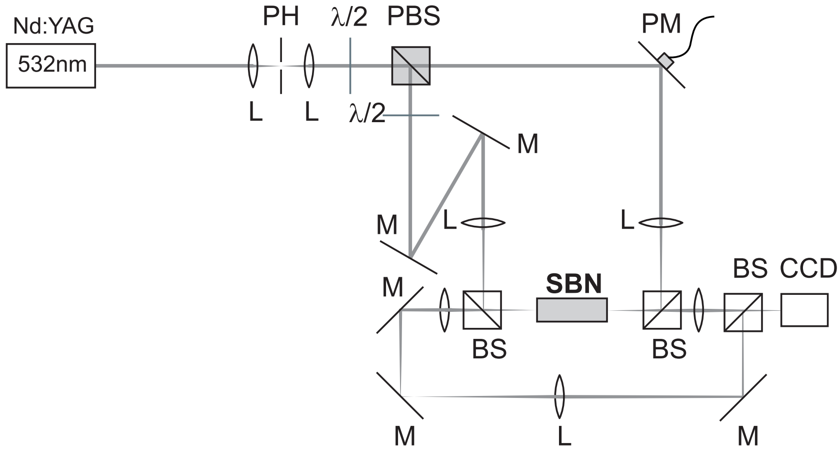

Fig. 1. Experimental setup. Two beams are rendered mutually incoherent using an oscillating piezo-mounted mirror (PM) and focused on opposite faces of a PR Ce:SBN60 crystal. Both crystal faces are imaged onto a CCD camera, allowing for synchronous observation of both exit beams and both input beams' reflections (M: mirror, L: lens, PH: pinhole, (P)BS: (polarizing) beam splitter).

In our contribution, we experimentally demonstrate the dynamic evolution of interacting counterpropagating self-trapped beams. Two qualitativly distinct scenarios are identified and interpreted by comparison with numerical simulations of a saturable Kerr model.

1. Experimental system and observations

We investigate the interaction of mutually incoherent counterpropagating self-trapped beams in a photorefractive cerium-doped strontium barium niobate (Ce:SBN:60) crystal (Fig. 1). The crystal is biased by an external dc field along the transverse x-direction, coinciding with the crystallographic c-axis. Both beams are obtained from a single laser source but rendered mutually incoherent by a mirror oscillating with a period significantly shorter than the relaxation time constant of the photofractive material. The beams' polarizations are also selected along the x-axis, taking advantage of the high electrooptic r_{33} coefficient of SBN. Propagating in +z and -z directions, both beams individually self-focus, as a result of the photorefractive screening of the external field [1], which has a value of 1.3 kV/cm. The diameter of each beam (x-axis) is 25 μm FWHM and their power is $1\,\mu$W each. To help the formation of photorefractive screening solitons, the interaction region is illuminated by white light. The beams' power and the level of nonlinearity are adjusted such that each of the beams individually forms a spatial soliton. In order to demonstrate both above- and below-threshold behaviour using a single crystal sample, we utilize two medium lengths by rotating the crystal about its c-axis, thus obtaining $L_1 = 5\,$mm and $L_2 = 23\,$mm respectively.

As the actual evolution of counterpropagating beams within the photorefractive medium is not accessible in this experiment, images of beam outputs at the crystal faces are recorded (Fig. 2). Besides each beam leaving the medium, a reflection of the counterpropagating input is recorded as a lateral reference. Initially, both beams are adjusted at a small angle such that their inputs and outputs overlap on both ends of the crystal, if propagating independently and in steady state, including the shift through beam bending. This configuration was chosen to minimize possible

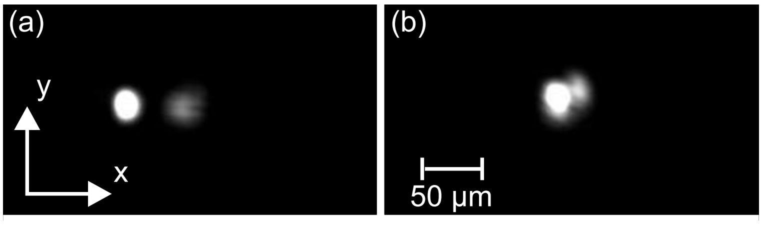

Fig. 2. Images of one exit face. **(a)** Separated beams: The beam leaving the crystal is visible as the bright spot. The second beam entering the crystal at this plane is visible in reflection (faint spot). **(b)** Strong interaction and the splitting of beams. While most of the output beam overlaps with the input beam, a fraction is split off into a second channel. Images (a) and (b) correspond to t=9 s and t=417 s of the time series displayed as Fig. 3(b).

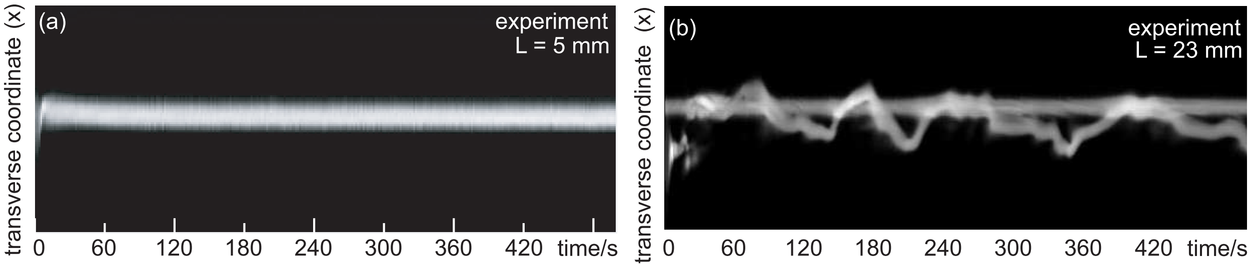

Fig. 3. Temporal plot of system dynamics in experiment. The vertical axis of the figure correponds to the x-axis of the experimental images which are reduced to this single transverse dimension by projection. Time is plotted along the horizontal axis of the figure. **(a)** In a short medium (L_1=5 mm), there exists a stable stationary final state, in which input and output beams fully overlap. **(b)** Using a long medium (L_2=23 mm), irregular dynamics are observed and there exists no stable stationary state. In fact, the overlapping state is now found to be metastable.

effects of beam bending [7] and to maximize the initial overlap of both beams.

In experiment, one usually starts with linear propagation of both beams and observes self focusing as the nonlinearity is switched on by activating the external field. Besides the primary effect of creating a self-induced waveguide, secondary effects such as beam bending (asymmetric refractive index change imposed by diffusive charge carrier transport) complicate the picture. However, for a single beam or multiple copropagating beams, a stable stationary state is generally reached for constant input conditions. This changes significantly if counterpropagating beams are considered, which interact through their respective effect on the medium's refractive index.

For illustration of the temporal evolution and for comparison with numerical simulations, experimental data are reduced to one transverse dimension, the images obtained on the exit faces of the crystal are projected onto the x-axis. As this data is plotted over time, one gets a representation of the dynamics of the beam exiting a crystal face (Fig. 3). Although changes parallel to the y-axis are not represented, most of the observable dynamics is confined to the x-axis, owing to the significance of the c-axis for the PR effect.

Investigating the short medium length ($L_1 = 5$mm), we observe transient dynamics towards a steady state as known from copropagating soliton cases. The output beams on both crystal surfaces initially shift their position (Fig. 3(a), t<20 s) converging to a steady state (t>20 s), where the inputs and outputs overlap on both faces of the photorefractive crystal. After that, no further dynamics are observed. If disturbed by external noise, both beams subsequently return to the steady state. The initial shift is mainly caused by beam bending, which is observed after the beam has already self focused.

In the case of the significantly longer medium ($L_2 = 23\,\mathrm{mm}$), the temporal evolution changes significantly. Again, the beams initially self-focus separately (Fig 3 (b), t<30 s). The initial transverse separation is now larger due to the longer medium which amplifies the angle selected compensating for the shift through beam bending. Nonetheless, the beams start to attract each other an overlap for more than half a minute (30 s<t<60 s). This state is no longer stable in this scenario and yields to irregular repetitions of repulsion and attraction, without any visible periodicity. Because dynamic behavior is observed for time spans that are orders of magnitude longer than the time constant of the system, it cannot be characterized as transient.

These two distinct scenarios which we observe are the result of a dynamic instability predicted earlier from numerical simulations [6]. A threshold value of the interaction length times coupling strength product ($L \cdot \Gamma$) separates the regime where stable stationary solutions exist from a regime where only nonstationary solutions can be found. The key to this instability is the bidirectional feedback provided by counterpropagation. A small transverse displacement in one beam can excite a displacement in the second beam through attraction. For a given coupling strength value, feedback allows for a mutual amplification of such displacements above a threshold longitudinal interaction length (i.e. medium's length in the propagation direction), which corresponds to the experimental observation of a threshold medium length separating stationary and dynamic regime.

2. Numerical analysis of a saturable Kerr model

To shed light on the bifurcation associated with the threshold, we numerically investigate a simplified system based on a dynamic one-dimensional saturable Kerr model [6, 12, 13],

$$i\partial_z F + \partial_x^2 F = \Gamma E_0 F \tag{1}$$

$$-i\partial_z B + \partial_x^2 B = \Gamma E_0 B \tag{2}$$

$$\tau \partial_t E_0 + E_0 = -\frac{|F|^2 + |B|^2}{1 + |F|^2 + |B|^2}, \tag{3}$$

where F and B are the counterpropagating wave envelopes, E_0 is the screening space charge field induced by the PR effect and Γ is the PR coupling constant. Using appropriate scaling, all variables are made dimensionless [6].

Commonly applied as an approximation to the PR nonlinearity, this model captures the basic dynamic effects of self-focusing and interaction of mutually incoherent CP waves in a medium with slow response time. The only interaction between both beams incorporated in this model is an attractive force. Beam bending and repulsive forces between solitons with a specific finite distance are not represented and are not required for explanantion of the effects observed in experiment.

We chose the parameters for the model to be in a range qualitatively agreeing with the experiment: Γ=3, input intensity 1.5 (normalized to background illumination), input beams had a lateral offest of 0.5 beam diameters to avoid tracking the unstable symmetric solution (both beams overlapping), after checking that the instability also develops from numerical noise under fully symmetric initial conditions.

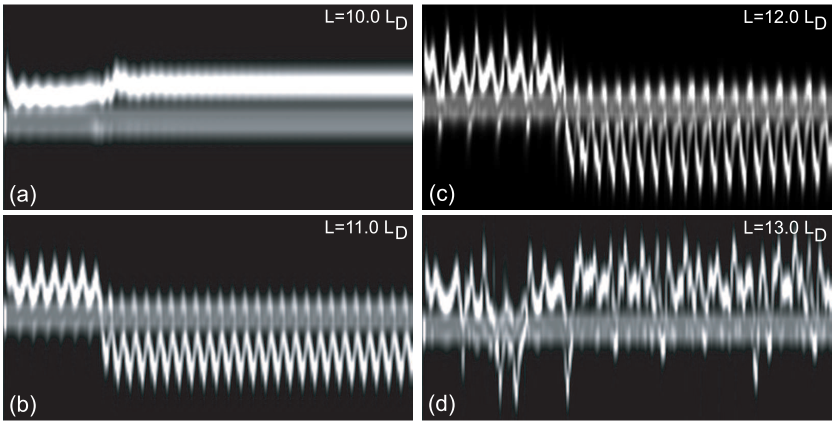

Fig. 4. Temporal plot of the evolution of one beam's output plane for four different interaction lengths, similar to Fig. 3. The vertical axes correspond to the transverse coordinate and time is plotted on the horizontal axis. The second beam's input is added in grey as a lateral reference. At $t = 0$, the nonlinearity is turned on in every image, where self focussing results in a brief period of overlapping beams before the instability sets in. **(a)** The interaction length is still below the dynamic instability threshold, but above one for spatial separatino of the beams [13]. Relaxation oscillations towards a stationary state are observed. **(b)** Slightly above threshold: Continous oscillations are observed. **(c)** Further above threshold: period doubling behavior. **(d)** Far above threshold: oscillations become irregular and aperiodic.

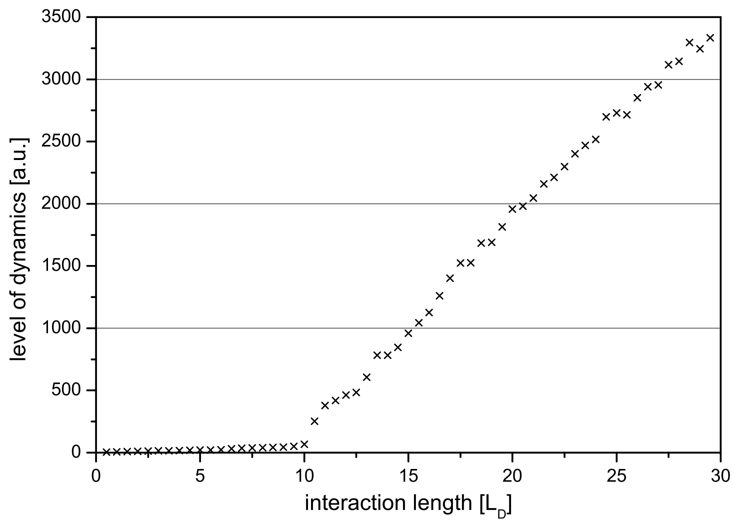

Fig. 5. Bifurcation diagram illustrating the dynamics of the system depending on the interaction length. The change of intensity over a temporal step is integrated over transverse space and it's mean value for a large number of time steps is given in arbitrary units. The diagram is robust in that it does not change with an increase in the number of time steps.

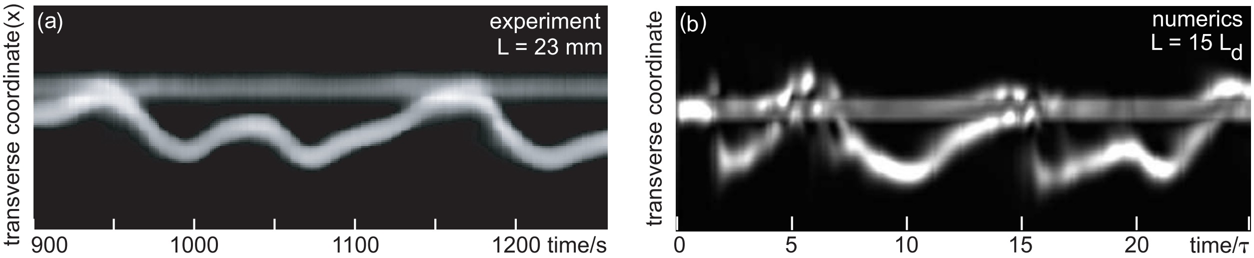

Fig. 6. Temporal plot of system dynamics. **(a)** Closeup of later development of an experiment similar to Fig. 3(b) starting at t=15 min. **(b)** Numerical simulation qualitatively corresponding to experimental parameters for (c), $\Gamma L = 45$, other parameters as before. Fast oscillations where the beams overlap result from confinement to one transverse dimension and can be considered numerical artifact. In it's place, experimental observations feature a splitting of the beams into two parts (cmp. Fig. 2).

Similar to the experimental system, the model demonstrates a threshold medium length beyond which no stable temporally stationary solutions could be found, in contrast to the usual interaction behaviour observed in copropagating solitons. The qualitative change through the threshold is illustrated by Fig. 4, that presents a temporal plot of one (transversely one dimensional) output beam.

In Fig. 4(a), the interaction length is still below the threshold of the dynamic instability: Initially, both beams self focus into a common waveguide for a very brief period of time. They immediately separate [13] and relax into a metastable state which again falls into the final state with relaxation oscillations. In the final stationary state, both beams are separated at the output plane. In Fig. 4(b), the interaction length is increased beyond the threshold. While the principal evolution is comparable to the previous case, the amplitude of the oscillations is larger and there is no relaxation. Here the final state is dynamic instead of stationary. As the interaction length is further increased (c), the oscillations gain im amplitude and show distinct signs of period doubling behaviour. And finally, the oscillation depicted in Fig. 4(d) cannot be characterized as regular any more. The change of the dynamics throughout the threshold indicates a period doubling bifurcation and hence a potentially chaotic regime on the far side of the threshold.

Finally, the threshold is visualized by a bifurcation diagram showing a measure for the variability of the system state as a function of the interaction length [Fig. 5]. Transient dynamics result in a low value while oscillations and more complex states result in high values. A distinct threshold is found at approximately $10 < L_{Dth} < 10.5$, corresponding with the onset of oscillations in Fig. 4.

3. Discussion and conclusion

In Figure 6, we compare experimental and numerical dynamics for systems above the threshold. Similar to the experiment, initial overlap and subsequent repetitions of attraction and repulsion are observed in the numerical system. The time scale agrees well with the experimental data. Both experimental and numerical data display a transverse asymmetry which is a result of the symmetry breaking nature of the instability [12]: the transversely symmetric state becomes unstable above threshold. The actual preferred direction manifested in a given experiment or numerical run is sensitive to several parameters such as the exact initial beam configuration, medium length, displacement by the beam bending effect and noise effects such as medium inhomogeneities.

As we can rule out external causes for the observed aperiodicity in experiment, the observations combined with the numerical data confirm the existence of a dynamic instability in the interaction of counterpropagating self-trapped beams, and we propose, based on the numerical bifurcation scenario, that the dynamics observed in experiment [Fig. 3(b)] may be chaotic in nature.

In summary, we demonstrated a dynamic instability in the interaction of counterpropagating localized optical beams in an experimental and corresponding numerical system. Each beam individually forms an optical spatial soliton and converges to a steady state after transient dynamics. Despite mutual attraction, both beams do not necessarily form either a common vector soliton or any other stationary waveguide structure. Instead, a dynamic instability enforces spatial separation of the localized beams while rendering such separate states unstable. A threshold interaction length is found, beyond which the interaction leads to non-transient dynamics, experimentally observable on the exit faces of the crystal. Qualitatively, experimental observations are found to be in good agreement with numerical simulations. Above and below threshold states are clearly distinguishable. The numerical investigation of the bifurcation shows a period doubling scenario, potentially leading to a chaotic regime. Further quantitative investigation of the threshold, the inclusion of beam bending and repulsive forces observed between photorefractive solitons into the analysis, and a determination whether the experimentally observed dynamics is chaotic, are challenging experimental tasks and subject of ongoing research.

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