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Optical solitons in fiber Bragg gratings via modified simple equation

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ABSTRACT

This paper implements modified simple equation to secure dark and singular optical solitons to fiber Bragg gratings. Five forms of nonlinear refractive index are considered. The existence criteria for such solitons are also enumerated.

1. Introduction

The main focus on the study of solitons is with optical fibers, PCF, metamaterials, magneto-optic waveguides, DWDM networks and several other devices [1–20]. However, inadvertently less attention is paid in addressing the dynamics of solitons in fiber Bragg gratings (FBGs). FBGs are considered excellent sensor elements, suitable for measuring various engineering parameters such as temperature, strain, pressure, tilt, displacement, acceleration, load, as well as the presence of various industrial, biomedical and chemical substances in both static and dynamic modes of operation. The FBG is also an excellent signal shaping and filtering element for a growing field of applications. This paper revisits soliton dynamics in FBG with dispersive reflectivity having five forms of nonlinear refractive index. While considerable results in FBG have been reported in the past, most of them are with Kerr and parabolic laws. In addition, a lot of results reported are from numerical perspective [1–5,12–20]. The current work focuses on analytical aspect of Bragg gratings, with dispersive reflectivity, by modified simple equation method. This leads to the retrieval of dark and singular solitons to the models. The results of the mathematical analysis are all displayed after a quick brush-up of the preliminary protocols. The details are jotted in subsequent sections.

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2. Preliminaries

Suppose we have a nonlinear evolution equation in the form

$$P(u, u_t, u_x, u_{xx}, u_{tt}, u_{tx}, \ldots) = 0, \tag{1}$$

where P is a polynomial in $u(x, t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we demonstrate the main steps of this method.

Step-1: We use the transformation

$$u(x, t) = u(\xi), \quad \xi = x - ct, \tag{2}$$

where c is a constant to be determined, to reduce Eq. (1) to the following ODE:

$$Q(u, u', u'', \ldots) = 0, \tag{3}$$

where Q is a polynomial in $u(\xi)$ and its total derivatives, while $' = \frac{d}{d\xi}$.

Step-2: We suppose that Eq. (3) has the formal solution.

$$u(\xi) = \sum_{l=0}^{N} a_l \left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^l, \tag{4}$$

where a_l are constants to be determined, such that $a_N \neq 0$, and $\psi(\xi)$ is an unknown function to be determined later.

Step-3: We determine the positive integer N in Eq. (4) by considering the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. (3).

Step-4: We substitute (4) into (3), then we calculate all the necessary derivatives u', u'', ... of the unknown function $u(\xi)$ and we account the function $\psi(\xi)$. As a result of this substitution, we get a polynomial of $\psi'(\xi)/\psi(\xi)$and its derivatives. In this polynomial, we gather all the terms of the same power of $\psi^{-j}(\xi)$, $j = 0, a, b, \ldots$ and its derivatives, and we equate with zero all the coefficients of this polynomial. This operation yields a system of equations which can be solved to find $a_k$and $\psi(\xi)$. Consequently, we can get the exact solutions of Eq. (1).

3. Application to Bragg gratings

3.1. Kerr law

The dimensionless form of the coupled NLSE in fiber Bragg gratings is given by [6,7]:

$$iq_t + a_1 r_{xx} + (b_1 |q|^2 + c_1 |r|^2)q + i\alpha_1 q_x + \beta_1 r = 0, \tag{5}$$

$$ir_t + a_2 q_{xx} + (b_2 |r|^2 + c_2 |q|^2)r + i\alpha_2 r_x + \beta_2 q = 0. \tag{6}$$

In Eqs. (5) and (6), $q(x, t)$ and $r(x, t)$ represent forward and backward propagating wave profiles while a_j for $j = 1, 2$ are the coefficients of dispersive reflectivity. Then, b_j are the coefficients of self-phase modulation terms; c_j represents cross-phase modulation; α_j represents the inter-modal dispersion and finally β_j are the detuning parameters. In order to solve this system, we introduce the following transformation

$$q(x, t) = P_1(x, t)e^{i\phi(x,t)}, \tag{7}$$

and

$$r(x, t) = P_2(x, t)e^{i\phi(x,t)}, \tag{8}$$

where $P_j(x, t)$ for $j = 1, 2$ are the amplitude parts of the waves while $\phi(x, t)$ is the phase component of both waves. One needs to use phase-matching condition to permit integrability. Therefore, the phase component is taken to be

$$\phi(x, t) = -\kappa x + \omega t + \theta, \tag{9}$$

where κ represents the soliton frequency and ω is the soliton wave number and finally, θ is the phase constant. Now, substituting (7) and (8) into (5) and (6) and decomposing into real and imaginary parts leads to the following set of coupled equations:

$$(\omega - \alpha_l\kappa)P_l + (a_l k^2 - \beta_l)P_{\bar{l}} - a_l\left(\frac{\partial^2 P_{\bar{l}}}{\partial x^2}\right) - (b_l P_l^2 + c_l P_{\bar{l}}^2)P_l = 0, \tag{10}$$

and

$$\left(\frac{\partial P_l}{\partial t}\right) - 2a_l\kappa\left(\frac{\partial P_{\bar{l}}}{\partial x}\right) + \alpha_l\left(\frac{\partial P_l}{\partial x}\right) = 0, \tag{11}$$

where $l = 1, 2$ and $\bar{l} = 3 - l$. From the imaginary part relation, the speed of the solitons is now given by:

$$(\omega - \alpha_l\kappa)P_l + (a_l\kappa^2 - \beta_l)P_{\bar{l}} - a_l P_{\bar{l}}'' - (b_l P_l^2 + c_l P_{\bar{l}}^2)P_l = 0, \tag{12}$$

and

$$(v + 2a_l\kappa + \alpha_l)P_l' = 0. \tag{13}$$

Form the last equation, one recovers that

$$v = \alpha_l - 2a_l\kappa. \tag{14}$$

Next, from (14), one obtains the following relation between the speed of the solitons:

$$(\alpha_1 - \alpha_2) - 2(a_l - a_2)\kappa = 0. \tag{15}$$

Setting the coefficients of independent parameters κ to zero brings about

$$a_j = a, \quad \alpha_j = \alpha, \tag{16}$$

for $j = 1, 2$, and finally the speed of the solitons changes to

$$v = \alpha - 2a\kappa. \tag{17}$$

Thus, Eqs. (5) and (6) transforms to

$$iq_t + ar_{xx} + (b_1\,|q|^2 + c_1\,|r|^2)q + i\alpha q_x + \beta_1 r = 0, \tag{18}$$

$$ir_t + aq_{xx} + (b_2\,|r|^2 + c_2\,|q|^2)r + i\alpha r_x + \beta_2 q = 0. \tag{19}$$

Also, real parts turn to

$$(\omega - \alpha\kappa)P_l + (a\kappa^2 - \beta_l)P_{\bar{l}} - a_l P_{\bar{l}}'' - (b_l P_l^2 + c_l P_{\bar{l}}^2)P_l = 0. \tag{20}$$

Now, balancing the terms P_l'' and P_l^5 in Eq. (20) lead to $N = \frac{1}{2}$. Set

$$P_l = U_l^{\frac{1}{2}}. \tag{21}$$

So that (20) transforms to

$$(\omega - \alpha\kappa)U + (a\kappa^2 - \beta)U - aU'' - (bU^2 + cU^2)U^3 = 0, \tag{22}$$

for this solution Eq. (22), assume

$$U(\xi) = \sum_{i=0}^{N} A_i\left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^i, \quad a_N \neq 0, \tag{23}$$

where A_i are all constants to be determined, Balancing U'' with U^3 in Eq. (22) given $N = 1$. Consequently, we arrive at

$$U(\xi) = A_0 + A_1\left(\frac{\psi'(\xi)}{\psi(\xi)}\right). \tag{24}$$

Substituting Eq. (24) in Eq. (22) and then setting the coefficients of $\psi^{-j}(j = 0, 1, 2, \ldots)$ to zero, then we obtain a set of algebraic equation involving A_0, A_1, κ, ω as follows:

ψ^{-3} Coeff.:

$$A_1(\psi')^3(2k^2a + A_1^2(b_l + c_l)) = 0, \tag{25}$$

ψ^{-2} Coeff.:

$$3A_1\psi'(A_0A_1\psi'(b_l + c_l) - k^2a\psi'') = 0, \tag{26}$$

ψ^{-1} Coeff.:

$$A_1(\psi'(\kappa^2(-a) + 3A_0^2(b_l + c_l) + \kappa\alpha + \beta_l - \omega) + k^2\psi''{}'a) = 0, \tag{27}$$

ψ^0 Coeff.:

$$A_0(\kappa^2(-a) + A_0^2(b_l + c_l) + \kappa\alpha + \beta_l - \omega) = 0. \tag{28}$$

Solving this system, we obtain

$$A_0 = \pm\sqrt{-\frac{\kappa(\alpha - \kappa a) + \beta_l - \omega}{b_l + c_l}}, \quad A_1 = \mp\sqrt{-\frac{2a}{b_l + c_l}}, \tag{29}$$

and

$$\psi'' = \pm A_0\sqrt{-\frac{2(b_l + c_l)}{a}}\,\psi', \tag{30}$$

$$\psi''' = -\frac{2A_0^2(b_l + c_l)}{a}\psi'. \tag{31}$$

From Eqs. (30) and (31), we can deduce that

$$\psi' = \pm\frac{1}{A_0}\sqrt{-\frac{a}{2(b_l + c_l)}}k_1 e^{\pm A_0\sqrt{-\frac{2(b_l+c_l)}{a_l}}\xi}, \tag{32}$$

$$\psi = -\frac{1}{A_0{}^2}\frac{a}{2(b_l + c_l)}k_1 e^{\pm A_0\sqrt{-\frac{2(b_l+c_l)}{a_l}}\xi} + k_2, \tag{33}$$

where k_1 and k_2 are constants of integration. Substituting Eqs. (32) and (33) into Eq. (24), we obtain following the following exact solution to Eqs. (5) and (6)

$$q(x,t) = \left(\pm\sqrt{-\frac{\kappa(\alpha - \kappa a) + \beta_1 - \omega}{b_1 + c_1}} + \frac{\frac{k_1 a}{A_0(b_1 + c_1)}e^{\pm A_0\sqrt{-\frac{2(b_1+c_1)}{a}}(x-\mathrm{v}t)}}{-\frac{1}{A_0{}^2}\frac{a}{2(b_1 + c_1)}k_1 e^{\pm A_0\sqrt{-\frac{2(b_1+c_1)}{a}}(x-\mathrm{v}t)} + k_2}\right)e^{i(-\kappa x+\omega t+\theta)}, \tag{34}$$

$$r(x,t) = \left(\pm\sqrt{-\frac{\kappa(\alpha - \kappa a) + \beta_2 - \omega}{b_2 + c_2}} + \frac{\frac{k_1 a}{A_0(b_2 + c_2)}e^{\pm A_0\sqrt{-\frac{2(b_2+c_2)}{a}}(x-\mathrm{v}t)}}{-\frac{1}{A_0{}^2}\frac{a}{2(b_2 + c_2)}k_1 e^{\pm A_0\sqrt{-\frac{2(b_2+c_2)}{a}}(x-\mathrm{v}t)} + k_2}\right)e^{i(-\kappa x+\omega t+\theta)}, \tag{35}$$

where v is given by (17) and ω is arbitrary.

If we set $k_1 = -\frac{2A_0{}^2(b_2 + c_2)}{a}$, $k_2 = \pm 1$, then we have

$$q(x,t) = \pm\sqrt{-\frac{\kappa(\alpha - \kappa a) + \beta_1 - \omega}{b_1 + c_1}}\tanh\left[\sqrt{\frac{\kappa(\alpha - \kappa a) + \beta_1 - \omega}{2a}}(x - \mathrm{v}t)\right]e^{i(-\kappa x+\omega t+\theta)}, \tag{36}$$

$$r(x,t) = \pm\sqrt{-\frac{\kappa(\alpha - \kappa a) + \beta_2 - \omega}{b_2 + c_2}}\tanh\left[\sqrt{\frac{\kappa(\alpha - \kappa a) + \beta_2 - \omega}{2a}}(x - \mathrm{v}t)\right]e^{i(-\kappa x+\omega t+\theta)}, \tag{37}$$

or

$$q(x,t) = \pm\sqrt{-\frac{\kappa(\alpha - \kappa a) + \beta_1 - \omega}{b_1 + c_1}}\coth\left[\sqrt{\frac{\kappa(\alpha - \kappa a) + \beta_1 - \omega}{2a}}(x - \mathrm{v}t)\right]e^{i(-\kappa x+\omega t+\theta)}, \tag{38}$$

$$r(x,t) = \pm\sqrt{-\frac{\kappa(\alpha - \kappa a) + \beta_2 - \omega}{b_2 + c_2}}\coth\left[\sqrt{\frac{\kappa(\alpha - \kappa a) + \beta_2 - \omega}{2a}}(x - \mathrm{v}t)\right]e^{i(-\kappa x+\omega t+\theta)}, \tag{39}$$

where ω is arbitrary constant.

These solution represent dark and singular soliton solution where

$$a\kappa(\alpha - \kappa a + \beta_l - \omega) > 0.$$

3.2. Parabolic law

The dimensionless form of the coupled NLSE in fiber Bragg gratings, having parabolic law of nonlinearity, is given by [8]. For Bragg grating equation with time dependent coefficient is

$$iq_t + a_1 r_{xx} + (b_1\ |q|\ 2 + c_1\ |r|^2)q + (\xi_1\ |q|^4 + \eta_1\ |q|^2|r|^2 + \zeta_1\ |r|^4)q + i\alpha_1 q_x + \beta_1 r = 0, \tag{40}$$

$$ir_t + a_2 q_{xx} + (b_2\ |r|^2 + c_2\ |q|^2)r + (\xi_2\ |r|^4 + \eta_2\ |q|^2|r|^2 + \zeta_2\ |q|^4)r + i\alpha_2 r_x + \beta_2 q = 0. \tag{41}$$

In Eqs. (40) and (41), $q(x, t)$ and $r(x, t)$ represent forward and backward propagating waves resprctively while a_j for $(j = 0, 1, \ldots)$ are coefficients of dispersive reflectivity. Next, b_j are the coefficients of SPM; c_j represent XPM for cubic nonlinearity portion. For quintic nonlinear part, ξ_j are coefficients of SPM while η_j and ξ_j are coefficients of XPM. Next, α_j represent inter-model dispersion and finally β_j are detuning parameters. All of the coefficients are real valued constant and $i = (\sqrt{-1})$.

To integrate the couple NLSE (40) and (41), one introduces:

$$q(x,t) = P_1(\vartheta)e^{i\phi(x,t)}, \tag{42}$$

and

$$r(x,t) = P_2(\vartheta)e^{i\phi(x,t)}, \tag{43}$$

where P_j with $(j = 0, 1, \ldots)$ are the amplitude constituents of the solitons and

$$\vartheta = x - \mathrm{v}\,\mathrm{t}, \tag{44}$$

where v indicates the speed of the soliton. The phase component of both solitons is structured as:

$$\phi(x, t) = -\kappa x + \omega t + \theta, \tag{45}$$

where κ, ω and θ are the frequency, wave number and phase constant, respectively. Next, substitute (42) and (43) into (40) and (41). Thus, the real part is

$$(\alpha_l\kappa - \omega)P_l + (\beta_l - a_l\kappa^2)P_l + b_lP_l^3 + c_lP_lP_{\tilde{l}}^2 + \xi_lP_l^5 + \eta_lP_l^3P_{\tilde{l}}^2 + \zeta_lP_lP_{\tilde{l}}^4 + a_lP_{\tilde{l}}'' = 0, \tag{46}$$

while the imaginary part is

$$(v - \alpha_l)P_l' + 2a_l\kappa P_{\tilde{l}}' = 0, \tag{47}$$

where $l = 1, 2$ and $\tilde{l} = 3 - l$.Next, the balancing principle brings about

$$P_{\tilde{l}} = P_l,$$

and so (46) and (47) respectively gives

$$(a_l\kappa + \beta_l - \omega - a_l\kappa^2)P_l + (b_l + c_l)P_l^3 + (\xi_l + \eta_l + \zeta_l)P_l^5 + a_lP_l'' = 0, \tag{48}$$

and

$$(v + 2a_l\kappa + \alpha_l)P_l' = 0. \tag{49}$$

Form the last equation, one recovers

$$v = \alpha_l - 2a_l\kappa. \tag{50}$$

Next, from (50), one obtains the following relation between the speed of the solitons:

$$(\alpha_1 - \alpha_2) - 2(a_l - a_2)\kappa = 0. \tag{51}$$

Upon setting the coefficients of independent parameters κ to zero brings about

$$a_j = a, \quad \alpha_j = \alpha, \tag{52}$$

for $j = 1, 2$,and finally the speed of the solitons changes to

$$v = \alpha - 2a\kappa. \tag{53}$$

Thus, Eqs. (40) and (41) transforms to

$$iq_t + ar_{xx} + (b_1\,|q|\;2 + c_1\,|r|^2)q + (\xi_1\,|q|^4 + \eta_1\,|q|^2|r|^2 + \zeta_1\,|r|^4)q + i\alpha q_x + \beta r = 0, \tag{54}$$

$$ir_t + aq_{xx} + (b_2\,|r|^2 + c_2\,|q|^2)r + (\xi_2\,|r|^4 + \eta_2\,|q|^2|r|^2 + \zeta_2\,|q|^4)r + i\alpha r_x + \beta_2 q = 0. \tag{55}$$

Also, real parts turn to

$$(a_l\kappa + \beta_l - \omega - a_l\kappa^2)P_l + (b_l + c_l)P_l^3 + (\xi_l + \eta_l + \zeta_l)P_l^5 + a_lP_l'' = 0. \tag{56}$$

Now, balancing the terms P_l'' and P_l^5 in Eq. (56) lead to $N = \frac{1}{2}$. Set

$$P_l = U_l^{\frac{1}{2}}. \tag{57}$$

So that (56) transforms to

$$4(\alpha\kappa - a\kappa^2 - \omega - \beta_l)U_l^2 + 4(b_l + c_l)U_l^3 + 4(\xi_l + \eta_l + \zeta_l)U_l^4 + (2U_lU_l'' - (U_l')^2) = 0. \tag{58}$$

For the solution of Eq. (58), assume:

$$U(\xi) = \sum_{i=0}^{N} A_i\left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^i, \quad A_N \neq 0, \tag{59}$$

where a_i are all constants to be determined, Balancing UU'' with U^4 in Eq. (58) given $N = 1$. Consequently, we reach

$$U(\xi) = A_0 + A_1\left(\frac{\psi'(\xi)}{\psi(\xi)}\right). \tag{60}$$

Substituting Eq. (60) in Eq. (58) and then setting the coefficients of ψ^{-j} $(j = 0, 1, 2, \ldots)$ to zero, then we obtain a set of algebraic equation involving a_0, a_1, κ, ω_t as following

ψ^{-4} Coeff.:

$$A_1^2(3a + 4A_1^2(\zeta_l + \eta_l + \xi_l))(\psi')^4 = 0, \tag{61}$$

ψ^{-3} Coeff.:

$$4A_1(A_1(A_1\psi'(b_l+c_l)-a\psi'')+A_0\psi'(a+4A_1^2(\zeta_l+\eta_l+\xi_l)))(\psi')^2=0, \tag{62}$$

ψ^{-2} Coeff.:

$$A_1(A_1(-4(\psi')^2(a\kappa^2-3A_0(b_l+c_l)-6A_0^2(\zeta_l+\eta_l+\xi_l)-\kappa\alpha_l-\beta_l+\omega)-a(\psi'')^2+2a\psi^{(3)}\psi')-6aA_0\psi'\psi'')=0, \tag{63}$$

ψ^{-1} Coeff.:

$$2A_0A_1(\psi'(4(-a\kappa^2+\kappa\alpha_l+\beta_l-\omega)+6A_0(b_l+c_l)+8A_0^2(\zeta_l+\eta_l+\xi_l))+a\psi''')=0, \tag{64}$$

ψ^{0} Coeff.:

$$4A_0^2(-a\kappa^2+A_0(b_l+c_l)+A_0^2(\zeta_l+\eta_l+\xi_l)+\kappa\alpha_l+\beta_l-\omega)=0. \tag{65}$$

Solving this system, we obtain

$$A_0=-\frac{3(b_l+c_l)}{4(\zeta_l+\eta_l+\xi_l)},\quad A_1=\mp\sqrt{-\frac{3a}{4(\zeta_l+\eta_l+\xi_l)}}, \tag{66}$$

$$\omega=-\frac{16(\zeta_l+\eta_l+\xi_l)(\kappa(a\kappa-\alpha)-\beta_l)+3(b_l+c_l)^2}{16(\zeta_l+\eta_l+\xi_l)}, \tag{67}$$

and

$$\psi''=\pm\sqrt{-\frac{3(b_l+c_l)^2}{4a(\zeta_l+\eta_l+\xi_l)}}\psi', \tag{68}$$

$$\psi'''=-\frac{3(b_l+c_l)^2}{4a(\zeta_l+\eta_l+\xi_l)}\psi', \tag{69}$$

From Eqs. (68) and (69), we can deduce

$$\psi'=\pm\sqrt{-\frac{4a(\zeta_l+\eta_l+\xi_l)}{3(b_l+c_l)^2}}k_1e^{\pm\sqrt{-\frac{3(b_l+c_l)^2}{4a(\zeta_l+\eta_l+\xi_l)}}\xi}, \tag{70}$$

$$\psi=-\frac{4a(\zeta_l+\eta_l+\xi_l)}{3(b_l+c_l)^2}k_1e^{\pm\sqrt{-\frac{3(b_l+c_l)^2}{4a(\zeta_l+\eta_l+\xi_l)}}\xi}+k_2, \tag{71}$$

where k_1 and k_2 are constants of integration. Substituting Eqs. (70) and (71) into Eq. (60), we obtain following the following exact solution to Eqs. (54) and (55).

$$q(x,t)=\left(-\frac{3(b_1+c_1)}{4(\zeta_1+\eta_1+\xi_1)}+\frac{\frac{a}{b_1+c_1}k_1e^{\pm\sqrt{-\frac{3(b_1+c_1)^2}{4a(\zeta_1+\eta_1+\xi_1)}}\xi}}{-\frac{4a(\zeta_1+\eta_1+\xi_1)}{3(b_1+c_1)^2}k_1e^{\pm\sqrt{-\frac{3(b_1+c_1)^2}{4a(\zeta_1+\eta_1+\xi_1)}}\xi}+k_2}\right)e^{i(-\kappa x+\omega t+\theta)}, \tag{72}$$

$$r(x,t)=\left(-\frac{3(b_2+c_2)}{4(\zeta_2+\eta_2+\xi_2)}+\frac{\frac{a}{b_2+c_2}k_1e^{\pm\sqrt{-\frac{3(b_2+c_2)^2}{4a(\zeta_2+\eta_2+\xi_2)}}\xi}}{-\frac{4a(\zeta_2+\eta_2+\xi_2)}{3(b_2+c_2)^2}k_1e^{\pm\sqrt{-\frac{3(b_2+c_2)^2}{4a(\zeta_2+\eta_2+\xi_2)}}\xi}+k_2}\right)e^{i(-\kappa x+\omega t+\theta)}, \tag{73}$$

If we set $k_1=-\frac{3(b_2+c_2)^2}{4a(\zeta_2+\eta_2+\xi_2)}$ and $k_2=\pm1$ we obtain

$$q(x,t)=-\frac{3(b_1+c_1)}{8(\zeta_1+\eta_1+\xi_1)}\left\{1\pm\tanh\left[\sqrt{-\frac{3(b_1+c_1)^2}{16a(\zeta_1+\eta_1+\xi_1)}}(x-\mathrm{v}t)\right]\right\}e^{i(-\kappa x+\omega t+\theta)}, \tag{74}$$

$$r(x,t)=-\frac{3(b_2+c_2)}{8(\zeta_2+\eta_2+\xi_2)}\left\{1\pm\tanh\left[\sqrt{-\frac{3(b_2+c_2)^2}{16a(\zeta_2+\eta_2+\xi_2)}}(x-\mathrm{v}t)\right]\right\}e^{i(-\kappa x+\omega t+\theta)}, \tag{75}$$

or

$$q(x,t)=-\frac{3(b_1+c_1)}{8(\zeta_1+\eta_1+\xi_1)}\left\{1\pm\coth\left[\sqrt{-\frac{3(b_1+c_1)^2}{16a(\zeta_1+\eta_1+\xi_1)}}(x-\mathrm{v}t)\right]\right\}e^{i(-\kappa x+\omega t+\theta)}, \tag{76}$$

$$r(x,t) = -\frac{3(b_2+c_2)}{8(\zeta_2+\eta_2+\xi_2)}\left\{1 \pm \coth\left[\sqrt{-\frac{3(b_2+c_2)^2}{16a(\zeta_2+\eta_2+\xi_2)}}(x-\text{v}t)\right]\right\}e^{i(-\kappa x+\omega t+\theta)}, \quad (77)$$

where ω is given by Eq. (67).

These solution represent dark and singular soliton solution where

$$a(\zeta_1+\eta_1+\xi_l) < 0.$$

3.3. Polynomial law

The dimensionless form of the coupled NLSE in fiber Bragg gratings, having polynomial law of nonlinearity, is given by [9]

$$\begin{aligned} &iq_t + ar_{xx} + (b_1\,|q|^2 + c_1\,|r|^2)q + (\xi_1\,|q|^4 + \eta_1\,|q|^2|r|^2 + \zeta_1\,|r|^4)q \\ &+ (l_1\,|q|^6 + m_1\,|q|^4|r|^2 + n_1\,|q|^2|r|^4 + p_1\,|r|^6)q + i\alpha q_x + \beta_1 r = 0, \end{aligned} \quad (78)$$

$$\begin{aligned} &ir_t + aq_{xx} + (b_2\,|r|^2 + c_2\,|q|^2)r + (\xi_2\,|r|^4 + \eta_2\,|r|^2|q|^2 + \zeta_2\,|q|^4)r \\ &+ (l_2\,|r|^6 + m_2\,|r|^4|q|^2 + n_2\,|r|^2|q|^4 + p_2\,|q|^6)r + i\alpha r_x + \beta_2 q = 0, \end{aligned} \quad (79)$$

To integrate (78) and (79), the considered hypothesis is

$$q(x,t) = P_1(\xi)e^{i\phi(x,t)}, \quad (80)$$

$$r(x,t) = P_2(\xi)e^{i\phi(x,t)}, \quad (81)$$

where

$$\xi = x - \text{v t}, \quad (82)$$

and v stands for the soliton velocity. From the phase

$$\phi(x,t) = -\kappa x + \omega t + \theta, \quad (83)$$

where κ, ω and θ give the soliton frequency, its wave number and phase center, respectively. Put (80) and (81) into (78) and (79). The real part causes

$$(\alpha_j\kappa - \omega)P_j + \beta_j - a_jk^2P_{\bar{j}} + b_jP_j^3 + c_jP_jP_{\bar{j}}^2 + \xi_jP_j^5 + \eta_jP_j^3P_{\bar{j}}^2 + \zeta_jP_jP_{\bar{j}}^4 + l_jP_j^7 + m_jP_j^5P_{\bar{j}}^2 + n_jP_j^3P_{\bar{j}}^4 + p_jP_jP_{\bar{j}}^6 + a_jP_{\bar{j}}'' = 0, \quad (84)$$

while the imaginary part gives

$$(v-\alpha_j)P_j' + 2a_j\kappa P_{\bar{j}}' = 0, \quad (85)$$

where $j = 1, 2$ and $\tilde{j} = 3 - j$. According to the balancing principle, one has

$$P_{\bar{j}} = P_j, \quad (86)$$

and so (78) and (79) transform into

$$(\alpha_j\kappa + \beta_j - \omega - a_j\kappa^2)P_j + (b_j + c_j)P_j^3 + (\xi_j + \eta_j + \zeta_j)P_j^5 + (l_j + m_j + n_j + p_j)P_j^7 + a_jP_j'' = 0, \quad (87)$$

and

$$(v - \alpha_j + 2a_j\kappa)P_j' = 0, \quad (88)$$

respectively. From imaginary portion,

$$v = \alpha_j - 2a_j\kappa, \quad (89)$$

and this implies a relation as

$$2(a_1 - a_2)\kappa - (\alpha_1 - \alpha_2) = 0, \quad (90)$$

By virtue of (90), one can observe

$$a_j = a \quad \alpha_j = \alpha, \quad (91)$$

and then

$$v = \alpha - 2a\kappa. \quad (92)$$

In this case, the governing model given by (78) and (79) modifies to

$$\begin{aligned} &iq_t + ar_{xx} + (b_1\,|q|^2 + c_1\,|r|^2)q + (\xi_1\,|q|^4 + \eta_1\,|q|^2|r|^2 + \zeta_1\,|r|^4)q \\ &+ (l_1\,|q|^6 + m_1\,|q|^4|r|^2 + n_1\,|q|^2|r|^4 + p_1\,|r|^6)q + i\alpha q_x + \beta_1 r = 0, \end{aligned} \quad (93)$$

$$ir_t + aq_{xx} + (b_2\,|r|^2 + c_2\,|q|^2)r + (\xi_2\,|r|^4 + \eta_2\,|r|^2|q|^2 + \zeta_2\,|q|^4)r$$
$$+ (l_2\,|r|^6 + m_2\,|r|^4|q|^2 + n_2\,|r|^2|q|^4 + p_2\,|q|^6)r + i\alpha r_x + \beta_2 q = 0. \quad (94)$$

In addition, real parts transform to

$$(\alpha\kappa + \beta_j - \omega - a\kappa^2)P_j + (b_j + c_j)P_j^3 + (\xi_j + \eta_j + \zeta_j)P_j^5 + (l_j + m_j + n_j + p_j)P_j^7 + aP_j^{\prime\,\prime} = 0. \quad (95)$$

Next, balancing the terms P_j'' and P_j^7 in Eq. (95) brings about $N = \frac{1}{3}$. Set

$$P_j = U_j^{\frac{1}{3}}, \quad (96)$$

So that Eq. (95) transforms to

$$9(\alpha\kappa + \beta_j - \omega - a\kappa^2)U_j^2 + 9(b_j + c_j)U_j^{\frac{5}{3}} + 9(\xi_j + \eta_j + \zeta_j)U_j^{\frac{10}{3}} + 9(l_j + m_j + n_j + p_j)U_j^4 - 2a(U')_j^2 + 3aU_jU_j'' = 0. \quad (97)$$

It needs to be taken the following equalities for integrability target:

$$b_j + c_j = 0, \quad (98)$$

$$\zeta_j + \eta_j + \xi_j = 0, \quad (99)$$

So, Eq.(97) is re-casted as

$$9(\alpha\kappa + \beta_j - \omega - ak^2)U_j^2 + 9(l_j + m_j + n_j + p_j)U_j^4 + a(3UU'' - 2(U')^2) = 0, \quad (100)$$

for this solution of Eq. (100), assume

$$U(\xi) = \sum_{i=0}^{N} A_i\left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^i, \quad A_N \neq 0, \quad (101)$$

where a_i are all constants to be determined, Balancing UU'' with U^4 in Eq. (100) given $N = 1$. Consequently, we reach at

$$U(\xi) = A_0 + A_1\left(\frac{\psi'(\xi)}{\psi(\xi)}\right). \quad (102)$$

Substituting Eq. (102) in Eq. (100) and then setting the coefficients of ψ^{-j} $(j = 0, 1, 2, \ldots)$ to zero, then we obtain a set of algebraic equation involving a_0, a_1, κ, ω as following

ψ^{-4} Coeff.:

$$A_1^2(\psi')^4(4a + 9A_1^2(l_j + m_j + n_j + p_j)) = 0, \quad (103)$$

ψ^{-3} Coeff.:

$$A_1(\psi')^2(6A_0\psi'(a + 6A_1^2(l_j + m_j + n_j + p_j)) - 5aA_1\psi'') = 0, \quad (104)$$

ψ^{-2} Coeff.:

$$A_1(A_1(9(\psi')^2(-a\kappa^2 + \alpha\kappa + 6A_0^2(l_j + m_j + n_j + p_j) + \beta_j - \omega) - 2a(\psi'')^2 + 3a\psi''^{\prime}\psi') - 9aA_0\psi'\psi'') = 0, \quad (105)$$

ψ^{-1} Coeff.:

$$3A_0A_1(6\psi'(-a\kappa^2 + \alpha\kappa + 2A_0^2(l_j + m_j + n_j + p_j) + \beta_j - \omega) + a\psi''^{\prime}) = 0, \quad (106)$$

ψ^0 Coeff.:

$$9A_0^2(-a\kappa^2 + \alpha\kappa + A_0^2(l_j + m_j + n_j + p_j) + \beta_j - \omega) = 0. \quad (107)$$

Solving this system, we obtain

$$A_0 = 0, \quad A_1 = \sqrt{-\frac{4a}{9(l_j + m_j + n_j + p_j)}}, \quad (108)$$

and

$$\psi'' = 0, \quad (109)$$

$$\psi''^{\prime} = \frac{3(a\kappa^2 - \alpha\kappa - \beta_j + \omega)}{a}\psi'. \quad (110)$$

Eqs. (109) and (110) lead to a trivial solution. Thus, modified simple equation fails to secure dark or singular solitons to FBGs maintaining polynomial law nonlinearity!

3.4. Quadratic-cubic law

The dimensionless form of the coupled NLSE in fiber Bragg gratings, with quadratic-cubic (QC) nonlinearity, is given by [10]

$$iq_t + ar_{xx} + b_1 q\sqrt{|q|^2 + |r|^2 + qr^* + q^*r} + (c_1\,|q|^2 + d_1\,|r|^2)q + p_1 r^2 q^* + i\alpha q_x + \beta_1 r = 0, \tag{111}$$

$$ir_t + aq_{xx} + b_2 q\sqrt{|r|^2 + |q|^2 + qr^* + qr^*} + (c_2\,|r|^2 + d_2\,|q|^2)r + p_2 q^2 r^* + i\alpha r_x + \beta_2 q = 0, \tag{112}$$

To investigate soliton solutions to Eqs. (111) and (112), the starting hypothesis is

$$q(x, t) = P_1(\zeta)e^{i\phi(x,t)}, \tag{113}$$

and

$$r(x, t) = P_2(\zeta)e^{i\phi(x,t)}, \tag{114}$$

where P_j for $j = 1, 2$ are the amplitudes of the waves,

$$\zeta = x - \mathrm{v}\,\mathrm{t}, \tag{115}$$

where v is the velocity of the solitary wave. The phase of the waves is given by

$$\phi(x, t) = -\kappa x + \omega t + \theta, \tag{116}$$

where κ, ω and θ are respectively the frequency, wave number and phase constant. Next, insert (113) and (114) into (111) and (112). The real part is

$$(\alpha_l\kappa - \omega)P_l + (\beta_l - a_l k^2)P_{\tilde{l}} + b_l P_l^2 + b_l P_l P_{\tilde{l}}\zeta_l P_l^3 + (\eta_l + \xi_l)P_l P_{\tilde{l}}^2 + a_l P_{\tilde{l}}'' = 0, \tag{117}$$

while the imaginary part gives

$$(v - \alpha_l)P_l' + 2a_l\kappa P_{\tilde{l}}' = 0, \tag{118}$$

where $l = 1, 2$ and $\tilde{l} = 3 - l$. According to the balancing principle, one has

$$P_{\tilde{l}} = P_l, \tag{119}$$

and thus (111) and (112) turn into, respectively

$$(\alpha_l\kappa + \beta_l - \omega - a_l\kappa^2)P_l + 2b_l P_l^2 + (\zeta_l + \eta_l + \xi_l)P_l^3 + a_l P_l'' = 0, \tag{120}$$

and

$$(v - \alpha_l + 2a_l\kappa)P_l' = 0. \tag{121}$$

From imaginary portion,

$$v = \alpha_l - 2a_l\kappa, \tag{122}$$

and from here the speed v of the solitons implies

$$2(a_1 - a_2)\kappa - (\alpha_1 - \alpha_2) = 0. \tag{123}$$

Also, from the coefficients of independent parameters κ, it is easy to observe that

$$a_l = a, \quad \alpha_l = \alpha. \tag{124}$$

In view of this observation, the speed of the solitons is

$$v = \alpha - 2a\kappa. \tag{125}$$

Thus, the model (111) and (112) alternate to

$$iq_t + ar_{xx} + b_1 q\sqrt{|q|^2 + |r|^2 + qr^* + q^*r} + (c_1\,|q|^2 + d_1\,|r|^2)q + p_1 r^2 q^* + i\alpha q_x + \beta_1 r = 0, \tag{126}$$

$$ir_t + aq_{xx} + b_2 q\sqrt{|r|^2 + |q|^2 + qr^* + qr^*} + (c_2\,|r|^2 + d_2\,|q|^2)r + p_2 q^2 r^* + i\alpha r_x + \beta_2 q = 0. \tag{127}$$

In addition, real parts transform to

$$(\alpha_l\kappa + \beta_l - \omega - a_l\kappa^2)P_l + 2b_l P_l^2 + (\zeta_l + \eta_l + \xi_l)P_l^3 + a_l P_l^{''} = 0, \tag{128}$$

for this solution of Eq. (128), assume

$$P(\xi) = \sum_{i=0}^{N} A_i\left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^i, \quad A_N \neq 0, \tag{129}$$

where a_i are all constants to be determined, Balancing U'' with U^3 in Eq. (130) given $N = 1$. Consequently, we reach

$$P(\xi) = A_0 + A_1\left(\frac{\psi'(\xi)}{\psi(\xi)}\right). \tag{130}$$

Substituting Eq. (130) in Eq. (128) and then setting the coefficients of ψ^{-j} $(j = 0, 1, 2, \ldots)$ to zero, then we obtain a set of algebraic equation involving a_0, a_1, κ, ω as following

ψ^{-3} Coeff.:

$$A_1(\psi')^3(2a + A_1^2(c_l + d_l + p_l)) = 0, \tag{131}$$

ψ^{-2} Coeff.:

$$A_1\psi'(A_1\psi'(3A_0(c_l + d_l + p_l) + 2b_l) - 3a\psi'') = 0, \tag{132}$$

ψ^{-1} Coeff.:

$$A_1(\psi'(-a\kappa^2 + \alpha\kappa + 4A_0b_l + 3A_0^2(c_l + d_l + p_l) + \beta_j - \omega) + a\psi''') = 0, \tag{133}$$

ψ^0 Coeff.:

$$A_0(-a\kappa^2 + \alpha\kappa + 2A_0b_l + A_0^2(c_l + d_l + p_l) + \beta_j - \omega) = 0. \tag{134}$$

Solving this system, we obtain

$$A_0 = 0, \quad A_1 = \pm\sqrt{-\frac{2a}{c_l + d_l + p_l}}, \tag{135}$$

and

$$\psi'' = \sqrt{-\frac{8b_l^2}{9a(c_l + d_l + p_l)}}\,\psi', \tag{136}$$

$$\psi''' = \frac{\kappa(a\kappa - \alpha) - \beta_j + \omega}{\alpha}\psi'. \tag{137}$$

From Eqs. (136) and (137), we can deduce

$$\psi' = \mp\sqrt{\frac{-8b_l^2}{9a(c_l + d_l + p_l)}}\,k_1 e^{\mp\sqrt{\frac{-9(c_l+d_l+p_l)(\kappa(a\kappa-\alpha)+\omega-\beta_j)^2}{8ab_1^2}}} \tag{138}$$

$$\psi = \frac{-8b_l^2}{9a(c_l + d_l + p_l)(\kappa(a\kappa - \alpha) + \omega - \beta_j)^2}k_1 e^{\mp\sqrt{\frac{-9(c_l+d_l+p_l)(\kappa(a\kappa-\alpha)+\omega-\beta_j)^2}{8ab_1^2}}} + k_2 \tag{139}$$

where k_1 and k_2 are constants of integration. Substituting Eqs. (138) and (139) into Eq. (130), we obtain following the following exact solution to Eqs. (126) and (127).

$$q(x,t) = \left(\frac{-\frac{4b_1}{c_1+d_1+p_1}k_1 e^{\sqrt{-\frac{9(c_1+d_1+p_1)(\kappa(a\kappa-\alpha)+\omega-\beta_1)^2}{8ab_1^2}}}}{-\frac{8b_1^2}{9(c_1+d_1+p_1)(\kappa(a\kappa-\alpha)+\omega-\beta_j)}k_1 e^{\sqrt{-\frac{9(c_1+d_1+p_1)(\kappa(a\kappa-\alpha)+\omega-\beta_1)^2}{8ab_1^2}}} + k_2}\right)e^{i(-\kappa x+\omega t+\theta)}, \tag{140}$$

$$r(x,t) = \left(\frac{-\frac{4b_2}{c_2+d_2+p_2}k_1 e^{\sqrt{-\frac{9(c_2+d_2+p_2)(\kappa(a\kappa-\alpha)+\omega-\beta_2)^2}{8ab_2^2}}}}{-\frac{8b_2^2}{9(c_2+d_2+p_2)(\kappa(a\kappa-\alpha)+\omega-\beta_2)}k_1 e^{\sqrt{-\frac{9(c_2+d_2+p_2)(\kappa(a\kappa-\alpha)+\omega-\beta_2)^2}{8ab_2^2}}} + k_2}\right)e^{i(-\kappa x+\omega t+\theta)}. \tag{141}$$

If we set $k_1 = \frac{9(c_l + d_l + p_l)(\kappa(a\kappa - \alpha) + \omega - \beta_j)}{8ab_2^2}$ and $k_2 = \pm1$, we obtain

$$q(x,t) = \frac{9(\kappa(a\kappa - \alpha) + \omega - \beta_1)}{4b_l}\left\{1 \pm \tanh\left[\sqrt{\frac{-9(c_1 + d_1 + p_1)(\kappa(a\kappa - \alpha) + \omega + \beta_1)}{8ab_1^2}}(x - vt)\right]\right\}e^{i(-\kappa x+\omega t+\theta)}, \tag{142}$$

$$r(x,t) = \frac{9(\kappa(a\kappa - \alpha) + \omega - \beta_2)}{4b_2}\left\{1 \pm \tanh\left[\sqrt{\frac{-9(c_2 + d_2 + p_2)(\kappa(a\kappa - \alpha) + \omega + \beta_2)}{8ab_2^2}}(x - vt)\right]\right\}e^{i(-\kappa x+\omega t+\theta)}, \tag{143}$$

or

$$q(x,\,t)=\frac{9(\kappa(a\kappa-\alpha)+\omega-\beta_1)}{4b_l}\{1\pm\coth[\sqrt{\frac{-9(c_1+d_1+p_1)(\kappa(a\kappa-\alpha)+\omega+\beta_1)}{8ab_1^2}}\,(x-\upsilon t)]\}e^{i(-\kappa x+\omega t+\theta)},\tag{144}$$

$$r(x,\,t)=\frac{9(\kappa(a\kappa-\alpha)+\omega-\beta_2)}{4b_2}\{1\pm\coth[\sqrt{\frac{-9(c_2+d_2+p_2)(\kappa(a\kappa-\alpha)+\omega+\beta_2)}{8ab_2^2}}\,(x-\upsilon t)]\}e^{i(-\kappa x+\omega t+\theta)},\tag{145}$$

where ω is arbitrary constant.

These solution represent dark and singular soliton solutions and remain valid whenever

$$a(c_l+d_l+p_l)(\kappa(a\kappa-\alpha)+\omega+\beta_2)<0.$$

3.5. Parabolic-nonlocal combo law

The dimensionless form of the coupled NLSE in FBG, having parabolic-nonlocal combo nonlinearity, is given by [11]

$$iq_t+a_1r_{xx}+(b_1\,|q|^2+c_1\,|r|^2)_{xx}q+(\xi_1\,|q|^4+\eta_1\,|q|^2|r|^2+\zeta_1\,|r|^4)q+i\alpha_1q_x+\beta_1r=0,\tag{146}$$

$$ir_t+a_2q_{xx}+(b_2\,|r|^2+c_2\,|q|^2)_{xx}r+(\xi_2\,|r|^4+\eta_2\,|r|^2|q|^2+\zeta_2\,|q|^4)r+i\alpha_2r_x+\beta_2q=0.\tag{147}$$

In Eqs. (146) and (147), $q(x,\,t)$ and $(x,\,t)$ represent forward and backward propagating waves respectively while a_j for $j=1,2$ are coefficients of dispersive reflectivity. Next, b_j are the coefficients of self-phase modulation (SPM); c_j represents the cross-phase modulation (XPM) for non-local nonlinearity. Next, α_j represents inter-modal dispersion and finally β_j are detuning parameters. All of the coefficients are real valued constants and $i=\sqrt{-1}$.

In order to integrate (146) and (147), the assumption is:

$$q(x,\,t)=P_1(\xi)e^{i\phi(x,t)},\tag{148}$$

$$r(x,\,t)=P_2(\xi)e^{i\phi(x,t)},\tag{149}$$

where

$$\xi=x-\mathrm{v}\,\mathrm{t},\tag{150}$$

and v stands for the soliton velocity. From the phase

$$\phi(x,\,t)=-\kappa x+\omega t+\theta,\tag{151}$$

where κ, ω and θ give the soliton frequency, its wave number and phase center, respectively. The real and imaginary parts yield respectively

$$(\alpha_j\kappa-\omega)P_j+(\beta_j-a_j\kappa^2)P_{\bar{j}}+\xi_jP_j^5+\eta_jP_j^3P_{\bar{j}}^2+\zeta_jP_jP_{\bar{j}}^4+2b_jP_j(P_j')^2+2c_jP_j\left(P_{\bar{j}}'\right)^2+2b_jP_j^2P_j''+a_jP_{\bar{j}}''+2c_jP_jP_{\bar{j}}P_{\bar{j}}''=0,\tag{152}$$

and

$$(\upsilon-\alpha_j)P_j'+2a_j\kappa P_{\bar{j}}'=0,\tag{153}$$

where $j=1,2$ and $\tilde{j}\,=3-j$, after inserting (148) and (149) into (146) and (147). Next, the balancing principle causes

$$P_{\bar{j}}=P_j,\tag{154}$$

and thus (152) and (153) are reshaped as

$$(\alpha_j\kappa+\beta_j-\omega-a_j\kappa^2)P_j+(\zeta_j+\eta_j+\xi_j)P_j^5+2(b_j+c_j)P_j(P_j')^2+a_jP_j^{\prime\prime}+2(b_j+c_j)P_j^2P_j'',\tag{155}$$

and

$$(\upsilon-\alpha_j+2a_j\kappa)P_j'=0,\tag{156}$$

respectively. From (156) one observes the speed of soliton as

$$\upsilon=\alpha_j-2a_j\kappa,\tag{157}$$

and this speed brings about

$$2(a_l-a_2)\kappa-(\alpha_1-\alpha_2)=0.\tag{158}$$

With the help of (158)

$$a_j=a,\quad \alpha_j=\alpha,\tag{159}$$

and then

$$v=\alpha-2a\kappa.\tag{160}$$

In view of these results, (146) and (147) are re-casted as

$$iq_t + a_1 r_{xx} + (b_1\,|q|^2 + c_1\,|r|^2)_{xx} q + (\xi_1\,|q|^4 + \eta_1\,|q|^2|r|^2 + \zeta_1\,|r|^4) q + i\alpha_1 q_x + \beta_1 r = 0, \tag{161}$$

$$ir_t + a_2 q_{xx} + (b_2\,|r|^2 + c_2\,|q|^2)_{xx} r + (\xi_2\,|r|^4 + \eta_2\,|r|^2|q|^2 + \zeta_2\,|q|^4) r + i\alpha_2 r_x + \beta_2 q = 0. \tag{162}$$

Therefore, real parts imply

$$(\alpha_j\kappa + \beta_j - \omega - a_j\kappa^2)P_j + (\zeta_j + \eta_j + \xi_j)P_j^5 + 2(b_j + c_j)P_j(P_j')^2 + a_jP_j'' + 2(b_j + c_j)P_j^2P_j''. \tag{163}$$

For this solution of Eq. (163), assume

$$P(\xi) = \sum_{i=0}^{N} A_i\left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^i, \quad A_N \neq 0, \tag{164}$$

where a_i are all constants to be determined, Balancing P^2P'' with P^5 in Eq.(164) given $N = 1$. Consequently, we reach at

$$P(\xi) = A_0 + A_1\left(\frac{\psi'(\xi)}{\psi(\xi)}\right). \tag{165}$$

Substituting Eq. (165) in Eq. (163) and then setting the coefficients of $\psi^{-j}(j = 0, 1, 2, \ldots)$ to zero, then we obtain a set of algebraic equation involving a_0, a_1, κ, ω as following

ψ^{-5} Coeff.:

$$A_1^3(\psi')^5(A_1^2(\zeta_j + \eta_j + \xi_j) + 6b_j + 6c_j) = 0, \tag{166}$$

ψ^{-4} Coeff.:

$$5A_1^2(\psi')^3(A_0\psi'(A_1^2(\zeta_j + \eta_j + \xi_j) + 2b_j + 2c_j) - 2A_1\psi''(b_j + c_j)) = 0, \tag{167}$$

ψ^{-3} Coeff.:

$$2A_1\psi'((\psi')^2(a + A_0^2(5A_1^2(\zeta_j + \eta_j + \xi_j) + 2b_j + 2c_j)) + A_1^2(\psi'')^2(b_j + c_j) + A_1\psi'(A_1\psi''' - 8A_0\psi'')(b_j + c_j)) = 0, \tag{168}$$

ψ^{-2} Coeff.:

$$A_1(-3a\psi'\psi'' - 6A_0^2\psi'\psi''(b_j + c_j) + 2A_1A_0(b_j + c_j)((\psi'')^2 + 2\psi'''\psi') + 10A_1A_0^3(\psi')^2(\zeta_j + \eta_j + \xi_j)) = 0, \tag{169}$$

ψ^{-1} Coeff.:

$$A_1(\psi^{(3)}(a + 2A_0^2(b_j + c_j)) + \psi'(-a\kappa^2 + \alpha\kappa + 5A_0^4(\zeta_j + \eta_j + \xi_j) + \beta_j - \omega)) = 0, \tag{170}$$

ψ^0 Coeff.:

$$A_0(-a\kappa^2 + \alpha\kappa + A_0^4(\zeta_j + \eta_j + \xi_j) + \beta_j - \omega) = 0, \tag{171}$$

Solving this system, we obtain

$$A_0 = -\sqrt{\frac{a}{4(b_j + c_j)}}, \quad A_1 = \mp\sqrt{-\frac{6(b_j + c_j)}{\zeta_j + \eta_j + \xi_j}}, \tag{172}$$

$$\omega = \frac{a^2(\zeta_j + \eta_j + \xi_j) + 16(b_j + c_j)^2(\kappa(\alpha - a\kappa) + \beta_j)}{16(b_j + c_j)^2}, \tag{173}$$

and

$$\psi'' = \pm\sqrt{-\frac{a(\zeta_j + \eta_j + \xi_j)}{6(b_j + c_j)^2}}\psi', \tag{174}$$

$$\psi''' = -\frac{a(\zeta_j + \eta_j + \xi_j)}{6(b_j + c_j)^2}\psi'. \tag{175}$$

From Eqs. (174) and (175), we can deduce

$$\psi' = -\sqrt{\frac{6(b_j + c_j)^2}{a(\zeta_j + \eta_j + \xi_j)}}k_1 e^{\sqrt{-\frac{a(\zeta_j+\eta_j+\xi_j)}{6(b_j+c_j)^2}}\xi}, \tag{176}$$

$$\psi = -\frac{6(b_j + c_j)^2}{a(\zeta_j + \eta_j + \xi_j)}k_1 e^{\sqrt{-\frac{a(\zeta_j+\eta_j+\xi_j)}{6(b_j+c_j)^2}}\xi} + k_2, \tag{177}$$

where k_1 and k_2 are constants of integration. Substituting Eqs. (176) and (177) into Eq. (165), we obtain following the following exact solution to Eqs. (146) and (147).

$$q(x,t)=\left(-\sqrt{\frac{a}{4(b_j+c_j)}}\mp\sqrt{-\frac{6(b_j+c_j)}{\zeta_j+\eta_j+\xi_j}}\frac{\pm\sqrt{-\frac{6(b_1+c_1)^2}{a(\zeta_1+\eta_1+\xi_1)}}k_1e^{\sqrt{-\frac{a(\zeta_1+\eta_1+\xi_1)}{6(b_1+c_1)^2}}\xi}}{-\frac{6(b_1+c_1)^2}{a(\zeta_1+\eta_1+\xi_1)}k_1e^{\sqrt{-\frac{a(\zeta_1+\eta_1+\xi_1)}{6(b_1+c_1)^2}}\xi}+k_2}\right)e^{i(-\kappa x+\omega t+\theta)}, \tag{178}$$

$$r(x,t)=\left(-\sqrt{\frac{a}{4(b_j+c_j)}}\mp\sqrt{-\frac{6(b_j+c_j)}{\zeta_j+\eta_j+\xi_j}}\frac{\pm\sqrt{-\frac{6(b_2+c_2)^2}{a(\zeta_2+\eta_2+\xi_2)}}k_1e^{\sqrt{-\frac{a(\zeta_2+\eta_2+\xi_2)}{6(b_2+c_2)^2}}\xi}}{-\frac{6(b_2+c_2)^2}{a(\zeta_2+\eta_2+\xi_2)}k_1e^{\sqrt{-\frac{a(\zeta_2+\eta_2+\xi_2)}{6(b_2+c_2)^2}}\xi}+k_2}\right)e^{i(-\kappa x+\omega t+\theta)}. \tag{179}$$

If we set $k_1=\frac{-a(\zeta_j+\eta_j+\xi_j)}{6(b_j+c_j)^2}$ and $k_2=\pm 1$, we obtain

$$q(x,t)=\pm\sqrt{\frac{a}{4(b_1+c_1)}}\left\{\tanh\left[\sqrt{-\frac{a(\zeta_1+\eta_1+\xi_1)}{6(b_1+c_1)}}(x-vt)\right]\right\}e^{i(-\kappa x+\omega t+\theta)}, \tag{180}$$

$$r(x,t)=\pm\sqrt{\frac{a}{4(b_2+c_2)}}\left\{\tanh\left[\sqrt{-\frac{a(\zeta_2+\eta_2+\xi_2)}{6(b_2+c_2)}}(x-vt)\right]\right\}e^{i(-\kappa x+\omega t+\theta)}, \tag{181}$$

or

$$q(x,t)=\pm\sqrt{\frac{a}{4(b_1+c_1)}}\left\{\coth\left[\sqrt{-\frac{a(\zeta_1+\eta_1+\xi_1)}{6(b_1+c_1)}}(x-vt)\right]\right\}e^{i(-\kappa x+\omega t+\theta)}, \tag{182}$$

$$r(x,t)=\pm\sqrt{\frac{a}{4(b_2+c_2)}}\left\{\coth\left[\sqrt{-\frac{a(\zeta_2+\eta_2+\xi_2)}{6(b_2+c_2)}}(x-vt)\right]\right\}e^{i(-\kappa x+\omega t+\theta)}, \tag{183}$$

where ω is given by Eq. (173).

These solutions represent dark and singular solitons respectively, whenever

$$a(\zeta_j+\eta_j+\xi_j)(b_j+c_j)>0.$$

4. Conclusions

Optical solitons in FBGs is studied in details by the aid of modified simple equation. This led to the retrieval of dark and singular soliton solutions to the models that are considered with five different nonlinearity structures. This algorithm failed to secure soliton solutions for polynomial law nonlinearity. Another limitation to this scheme is that it is unable to recover bright solitons to the models. Thus, a complete spectrum of soliton solutions was not obtained using today's algorithm. Therefore, one is encouraged to address these models with additional schemes. This gives way to open ended problems. Later, Lie symmetry analysis, Painleve analysis, Kudryashov's scheme and others will yield other solutions, hopefully. Such research activities are under way. Their results will be revealed with time.

Conflict of interest

The authors also declare that there is no conflict of interest.

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