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Optical solitons in birefringent fibers with quadratic–cubic refractive index by ϕ^6–model expansion

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ABSTRACT

This paper employs the new ϕ^6–model expansion to derive soliton solutions in birefringent fibers with quadratic-cubic law of refractive index. This scheme reveals bright, dark, singular and combo optical solitons to the model. Their respective existence criteria are also included.

1. Introduction

The study of optical solitons with quadratic–cubic (QC) law of refractive index was first proposed in 1994 [16]. After about a decade and a half of dormancy this law resurfaced during 2011 [15]. Subsequently, since 2017 till today, this law has gained extreme popularity and several results have flooded all across a variety of journals [1–21]. It has being studied in a variety of contexts that include polarization–preserving fibers, birefringent fibers, highly–dispersive solitons, Bragg gratings and several others. Apart from retrieving soliton solutions to the model [1–5,8–12,18], conservation laws have been retrieved [21], numerical simulations have also been reported using the methods of Adomian decomposition method, collective variables method [6]. While there are several integration algorithms available to retrieve soliton solutions [22–28], a new scheme that stands out is the ϕ^6–model expansion [29,30]. Today's work will be a re-visitation of QC nonlinearity to address optical solitons in birefringent fibers along using this lately reported scheme. In order to keep the model in perspective, the effect of four–wave mixing (4WM) is not discarded and consequently phase–matching condition is taken into account to permit integrability. The model of the governing nonlinear Schrödinger's equation (NLSE) is introduced and details of the solution procedure are inked in the subsequent sections.

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1.1. Governing model

The governing NLSE with QC nonlinearity for polarization-preserving fibers is written below [1–6,8,18,20,21]:

$$iq_t + aq_{xx} + (b_1 \,|q| + b_2 \,|q|^2)q = 0. \tag{1}$$

In Eq. (1), $q(x, t)$ is the complex–valued wave form that represents optical solitons in polarization–preserving fibers. The spatial and temporal independent variables are represented by x and t respectively, while $i = \sqrt{-1}$. The first term is linear temporal evolution, while a is the coefficient of group velocity dispersion, while b_1 and b_2 account for quadratic and cubic nonlinear terns that together represent QC nonlinearity. If $b_1 = 0$, Eq. (1) collapses to the familiar NLSE with Kerr law of refractive index.

Next, for birefringent fibers, when the pulses are split into two, the corresponding coupled vector NLSE frames as:

$$iq_t + a_1 q_{xx} + b_1 q\sqrt{|q|^2 + |r|^2 + qr^* + q^* r} + (c_1|q|^2 + d_1|r|^2)q + f_1 r^2 q^* = 0 \tag{2}$$

and

$$ir_t + a_2 r_{xx} + b_2 r\sqrt{|r|^2 + |q|^2 + q^* r + qr^*} + (c_2|r|^2 + d_2|q|^2)r + f_2 q^2 r^* = 0 \tag{3}$$

when the effect of 4WM is included. Here, in Eqs. (2) and (3), c_j for $j = 1, 2$ represents the coefficients of self–phase modulation (SPM) while d_j account for cross–phase modulation (XPM) and f_j gives the effect of 4WM for Kerr part of the nonlinearity. From the quadratic nonlinear form, the effects of SPM, XPM and 4WM are all embedded inside the radical sign.

The system (2)–(3) have been discussed earlier using extended trial function, dextended G'/G–expansion, extended Jacobi's elliptic function and F–expansion methods [9–12]. To the best of our knowledge, the coupled system (2)–(3) has not been discussed elsewhere using the above proposed technique of this paper.

This paper is organized as follows: In Sec. 2, mathematical analysis is given. In Sec. 3, we solve the system (2) and (3) using the above proposed technique. In Sec. 4, some conclusions are obtained.

2. Mathematical preliminaries

It is of interest to determine exact soliton solutions of the coupled system (1) and (2). To this aim, we introduce the transformation:

$$\begin{aligned} q(x, t) &= \varphi_1(\xi)\exp[i\psi(x, t)],\\ r(x, t) &= \varphi_2(\xi)\exp[i\psi(x, t)], \end{aligned} \tag{4}$$

and

$$\xi = B(x - vt), \quad \psi(x, t) = -kx + \omega t + \theta, \tag{5}$$

where B, v, k, ω and θ are all non zero constants to be determined which represent the width of the soliton, velocity of soliton, frequency of soliton, wave number and phase constants, respectively, while $\varphi_1(\xi)$, $\varphi_2(\xi)$ and $\psi(x, t)$ are real functions which represent the amplitudes portion of the solitons and the phase component of the soliton, respectively. Substituting (4) and (5) into the system (2) and (3), separating the real and the imaginary parts, we have

$$a_1 B^2 \varphi_1'' - (\omega + a_1 k^2)\varphi_1 + b_1\varphi_1^2 + b_1\varphi_1\varphi_2 + c_1\varphi_1^3 + (d_1 + f_1)\varphi_1\varphi_2^2 = 0, \tag{6}$$

$$a_2 B^2 \varphi_2'' - (\omega + a_2 k^2)\varphi_2 + b_2\varphi_2^2 + b_2\varphi_1\varphi_2 + c_2\varphi_2^3 + (d_2 + f_2)\varphi_2\varphi_1^2 = 0, \tag{7}$$

and

$$B(v + 2a_1 k)\varphi_1' = 0, \tag{8}$$

$$B(v + 2a_2 k)\varphi_2' = 0. \tag{9}$$

From Eqs.(8) and (9), one can obtain the velocity of the soliton as

$$v = -2a_1 k = -2a_2 k. \tag{10}$$

Consequently, we have

$$a_1 = a_2 = a, \tag{11}$$

and then velocity of the solitons is

$$v = -2ak. \tag{12}$$

Therefore, the coupled two Eqs. (6) and (7) can be rewritten as

$$aB^2\varphi_1'' - (\omega + ak^2)\varphi_1 + b_1\varphi_1^2 + b_1\varphi_1\varphi_2 + c_1\varphi_1^3 + (d_1 + f_1)\varphi_1\varphi_2^2 = 0, \tag{13}$$

and

$$aB^2\varphi_2'' - (\omega + ak^2)\varphi_2 + b_2\varphi_2^2 + b_2\varphi_1\varphi_2 + c_2\varphi_2^3 + (d_2 + f_2)\varphi_2\varphi_1^2 = 0. \tag{14}$$

Setting

$$\varphi_2(\xi) = \lambda\varphi_1(\xi), \tag{15}$$

where λ is a non zero constant, such that $\lambda \neq 1$. Consequently, Eqs. (13) and (14) reduce to

$$aB^2\varphi_1'' - (\omega + ak^2)\varphi_1 + b_1(1 + \lambda)\varphi_1^2 + [c_1 + \lambda^2(d_1 + f_1)]\varphi_1^3 = 0, \tag{16}$$

and

$$aB^2\varphi_1'' - (\omega + ak^2)\varphi_1 + b_2(1 + \lambda)\varphi_1^2 + [\lambda^2 c_2 + (d_2 + f_2)]\varphi_1^3 = 0. \tag{17}$$

Eqs.(16) and (17) have the same form under the constraint condition:

$$\frac{aB^2}{aB^2} = \frac{(\omega + ak^2)}{(\omega + ak^2)} = \frac{b_1(1 + \lambda)}{b_2(1 + \lambda)} = \frac{[c_1 + \lambda^2(d_1 + f_1)]}{[\lambda^2 c_2 + (d_2 + f_2)]}, \tag{18}$$

which leads to the following relations

$$b_1 = \qquad b_2 = b, \tag{19}$$

$$\lambda^2 = \qquad \frac{d_2 + f_2 - c_1}{d_1 + f_1 - c_2} > 0. \tag{20}$$

Now, we will solve Eq.(16) using the following technique.

3. ϕ^6–Model expansion

To this aim, balancing φ_1'' with φ_1^3 in Eq.(16), yields the balance number $N = 1$. According to the new ϕ^6- expansion method [28–30], we assume that Eq.(16) has the formal solution:

$$\varphi_1(\xi) = \beta_0 + \beta_1\phi(\xi) + \beta_2\phi^2(\xi), \tag{21}$$

where β_0, β_1 and β_2 are constants to be determined, such that $\beta_2 \neq 0$, while the function $\phi(\xi)$ satisfies the well-known auxiliary nonlinear ODE:

$$\begin{cases}\phi'^2(\xi) = h_0 + h_2\phi^2(\xi) + h_4\phi^4(\xi) + h_6\phi^6(\xi),\\ \phi''(\xi) = h_2\phi(\xi) + 2h_4\phi^3(\xi) + 3h_6\phi^5(\xi)\end{cases} \tag{22}$$

where $h_i\,(i = 0, 2, 4, 6)$ are arbitrary real constants to be determined later. It is well-known [23–25] that Eq.(22) has the solutions:

$$\phi(\xi) = \frac{U(\xi)}{\sqrt{f\,U^2(\xi) + g}}, \tag{23}$$

provided that $(fU^2(\xi) + g) > 0$ and $U(\xi)$ is the solution of the Jacobi elliptic equation (see the Table in [24]):

$$U'^2(\xi) = l_0 + l_2U^2(\xi) + l_4U^4(\xi), \tag{24}$$

and $l_j\,(j = 0,\ 2,\ 4)$ are constants to be determined later, while f and g are constants given by

$$f = \qquad \frac{h_4(l_2 - h_2)}{(l_2 - h_2)^2 + 3l_0l_4 - 2l_2(l_2 - h_2)}, \tag{25}$$

$$g = \qquad \frac{3l_0h_4}{(l_2 - h_2)^2 + 3l_0l_4 - 2l_2(l_2 - h_2)}, \tag{26}$$

under the constraint condition

$$h_4^2(l_2 - h_2)[9l_0l_4 - (l_2 - h_2)(2l_2 + h_2)] + 3h_6[3l_0l_4 - (l_2^2 - h_2^2)]^2 = 0. \tag{27}$$

Substituting (21) along with (22) into Eq. (16), collecting the coefficients of each power $\phi^i(\xi)(\phi'(\xi))^j$, $(i = 0, 1, 2, ..,6, j = 0, 1)$, and setting these coefficients to zero, we obtain the following system of algebraic equations:

$$\begin{aligned}
&\phi^6(\xi): A\beta_2^3 + 8\mathrm{aB}^2\beta_2 h_6 = 0,\\
&\phi^5(\xi): 3A\beta_1\beta_2^2 + 3\mathrm{aB}^2\beta_1 h_6 = 0,\\
&\phi^4(\xi): 3A\beta_0\beta_2^2 + 3A\beta_1^2\beta_2 + 6\mathrm{aB}^2\beta_2 h_4 + b\beta_2^2(1+\lambda) = 0,\\
&\phi^3(\xi): 6A\beta_0\beta_1\beta_2 + A\beta_1^3 + 2\mathrm{aB}^2\beta_1 h_4 + 2b\beta_1\beta_2(1+\lambda) = 0\\
&\phi^2(\xi): 3A\beta_0^2\beta_2 + 3A\beta_0\beta_1^2 + 4\mathrm{a}B^2\beta_2 h_2 + b(1+\lambda)[2\beta_0\beta_2 + \beta_1^2] - \mathrm{ak}^2\beta_2 - \omega\beta_2 = 0,\\
&\phi(\xi): \mathrm{aB}^2\beta_1 h_2 + 3A\beta_0^2\beta_1 + 2b\beta_0\beta_1(1+\lambda) - \mathrm{ak}^2\beta_1 - \omega\beta_1 = 0,\\
&\phi^0(\xi): 2\mathrm{aB}^2\beta_2 h_0 + A\beta_0^3 + b\beta_0^2(1+\lambda) - \mathrm{ak}^2\beta_0 - \omega\beta_0 = 0.
\end{aligned} \tag{28}$$

On solving the above algebraic Eq. (28) using Maple to obtain the following results:

$$\begin{aligned}
&\omega = 4\mathrm{ah}_2B^2 + 3A\beta_0^2 + 2(1+\lambda)b\beta_0 - \mathrm{ak}^2,\ \beta_0 = \beta_0,\quad \beta_1 = 0,\ \beta_2 = \frac{\beta_0[4\mathrm{ah}_2B^2 + 2A\beta_0^2 + (1+\lambda)b\beta_0]}{2\mathrm{ah}_0B^2},\\
&h_4 = \frac{-\beta_0[4\mathrm{ah}_2B^2 + 2A\beta_0^2 + (1+\lambda)b\beta_0][3A\beta_0 + b(1+\lambda)]}{12a^2h_0B^4},\ h_6 = \frac{-A\beta_0^2(4\mathrm{ah}_2B^2 + 2A\beta_0^2 + (1+\lambda)b\beta_0)^2}{32a^3h_0^2B^6},
\end{aligned} \tag{29}$$

where $A = [c_1 + \lambda^2(d_1 + f_1)]$ and $h_0 \neq 0$. Substituting (29) into (21) along with (23), we get:

$$\varphi_1(\xi) = \beta_0\left[1 + \frac{4\mathrm{ah}_2B^2 + 2A\beta_0^2 + (1+\lambda)b\beta_0}{2\mathrm{ah}_0B^2}\right]\left[\frac{U^2(\xi)}{\mathrm{fU}^2(\xi) + g}\right], \tag{30}$$

where λ is given by (20). From (4), (15) and (30), we have the following new results:

(1) If $l_0 = 1$, $l_2 = -(1+m^2)$, $l_4 = m^2$, $0 < m < 1$, then $U(\xi) = \mathrm{sn}(\xi)$ or $U(\xi) = \mathrm{cd}(\xi)$. In this case, we have the Jacobi elliptic function solutions for Eqs. (2) and (3):

$$q(x,t) = \beta_0 - \frac{6\mathrm{aB}^2}{3A\beta_0 + b(1+\lambda)}\left[\frac{(m^4 - m^2 - h_2^2 + 1)\mathrm{sn}^2(\xi)}{(1 + m^2 + h_2)\mathrm{sn}^2(\xi) - 3}\right]e^{i(-\mathrm{kx} + \omega t + \theta)}, \tag{31}$$

$$r(x,t) = \beta_0\lambda - \frac{6\mathrm{aB}^2\lambda}{3A\beta_0 + b(1+\lambda)}\left[\frac{(m^4 - m^2 - h_2^2 + 1)\mathrm{sn}^2(\xi)}{(1 + m^2 + h_2)\mathrm{sn}^2(\xi) - 3}\right]e^{i(-\mathrm{kx} + \omega t + \theta)}, \tag{32}$$

or

$$q(x,t) = \beta_0 - \frac{6\mathrm{aB}^2}{3A\beta_0 + b(1+\lambda)}\left[\frac{(m^4 - m^2 - h_2^2 + 1)\mathrm{cd}^2(\xi)}{(1 + m^2 + h_2)\mathrm{cd}^2(\xi) - 3}\right]e^{i(-\mathrm{kx} + \omega t + \theta)}, \tag{33}$$

$$r(x,t) = \beta_0\lambda - \frac{6\mathrm{aB}^2\lambda}{3A\beta_0 + b(1+\lambda)}\left[\frac{(m^4 - m^2 - h_2^2 + 1)\mathrm{cd}^2(\xi)}{(1 + m^2 + h_2)\mathrm{cd}^2(\xi) - 3}\right]e^{i(-\mathrm{kx} + \omega t + \theta)}, \tag{34}$$

under the constraint condition

$$-h_4^2(1 + m^2 + h_2)[9m^2 - (1 + m^2 + h_2)(2 + 2m^2 - h_2)] + 3h_6[3m^2 - (1 + m^2)^2 + h_2^2]^2 = 0, \tag{35}$$

where h_4 and h_6 are given by (29). In particular, if $m \to 1$, then we have the dark soliton solution:

$$q(x,t) = \beta_0 - \frac{6\mathrm{aB}^2}{3A\beta_0 + b(1+\lambda)}\left[\frac{(h_2^2 - 1)\tanh^2(\xi)}{3 - (h_2 + 2)\tanh^2(\xi)}\right]e^{i(-\mathrm{kx} + \omega t + \theta)}, \tag{36}$$

$$r(x,t) = \beta_0\lambda - \frac{6\mathrm{aB}^2\lambda}{3A\beta_0 + b(1+\lambda)}\left[\frac{(h_2^2 - 1)\tanh^2(\xi)}{3 - (h_2 + 2)\tanh^2(\xi)}\right]e^{i(-\mathrm{kx} + \omega t + \theta)}, \tag{37}$$

under the constraint condition

$$-h_4^2(h_2 + 2) + 3h_6(h_2 + 1)^2 = 0, \tag{38}$$

while if $m \to 0$, then we have the periodic solution:

$$q(x,t) = \beta_0 - \frac{6\mathrm{aB}^2}{3A\beta_0 + b(1+\lambda)}\left[\frac{(1 - h_2^2)}{(h_2 + 1) - 3\csc^2(\xi)}\right]e^{i(-\mathrm{kx} + \omega t + \theta)}, \tag{39}$$

$$r(x,t) = \beta_0\lambda - \frac{6\mathrm{aB}^2\lambda}{3A\beta_0 + b(1+\lambda)}\left[\frac{(1 - h_2^2)}{(h_2 + 1) - 3\csc^2(\xi)}\right]e^{i(-\mathrm{kx} + \omega t + \theta)}, \tag{40}$$

or

$$q(x, t) = \beta_0 - \frac{6aB^2}{3A\beta_0 + b(1+\lambda)}\left[\frac{(1-h_2^2)}{(h_2+1) - 3\sec^2(\xi)}\right]e^{i(-kx+\omega t+\theta)}, \tag{41}$$

$$r(x, t) = \beta_0\lambda - \frac{6aB^2\lambda}{3A\beta_0 + b(1+\lambda)}\left[\frac{(1-h_2^2)}{(h_2+1) - 3\sec^2(\xi)}\right]e^{i(-kx+\omega t+\theta)}, \tag{42}$$

under the constraint condition

$$-h_4^2(h_2-2) + 3h_6(h_2-1)^2 = 0. \tag{43}$$

(2) If $l_0 = 1 - m^2$, $l_2 = 2m^2 - 1$, $l_4 = -m^2$, $0 < m < 1$, then $U(\xi) = \text{cn}(\xi)$.In this case, we have the Jacobi elliptic function solutions for Eqs.(2) and (3):

$$q(x, t) = \beta_0 - \frac{6aB^2}{3A\beta_0 + b(1+\lambda)}\left[\frac{(m^4 - m^2 - h_2^2 + 1)\text{cn}^2(\xi)}{-(2m^2 - 1 - h_2)\text{cn}^2(\xi) + 3(m^2 - 1)}\right]e^{i(-kx+\omega t+\theta)}, \tag{44}$$

$$r(x, t) = \beta_0\lambda - \frac{6aB^2\lambda}{3A\beta_0 + b(1+\lambda)}\left[\frac{(m^4 - m^2 - h_2^2 + 1)\text{cn}^2(\xi)}{-(2m^2 - 1 - h_2)\text{cn}^2(\xi) + 3(m^2 - 1)}\right]e^{i(-kx+\omega t+\theta)}, \tag{45}$$

under the constraint condition

$$-h_4^2(2m^2 - 1 - h_2)[9m^2(1-m^2) + (2m^2 - 1 - h_2)(4m^2 - 2 + h_2)] + 3h_6[-3m^2(1-m^2) - (2m^2-1)^2 + h_2^2]^2 = 0. \tag{46}$$

In particular, if $m \to 0$, then we have the same periodic solution (41) and (42).

(3) If $l_0 = m^2 - 1$, $l_2 = 2 - m^2$, $l_4 = -1$, $0 < m < 1$, then $U(\xi) = \text{dn}(\xi)$.In this case, we have the Jacobi elliptic function solutions for Eqs.(2) and (3):

$$q(x, t) = \beta_0 + \frac{6aB^2}{3A\beta_0 + b(1+\lambda)}\left[\frac{(m^4 - m^2 - h_2^2 + 1)\text{dn}^2(\xi)}{(2 - m^2 - h_2)\text{dn}^2(\xi) + 3(m^2 - 1)}\right]e^{i(-kx+\omega t+\theta)}, \tag{47}$$

$$r(x, t) = \beta_0\lambda + \frac{6aB^2\lambda}{3A\beta_0 + b(1+\lambda)}\left[\frac{(m^4 - m^2 - h_2^2 + 1)\text{dn}^2(\xi)}{(2 - m^2 - h_2)\text{dn}^2(\xi) + 3(m^2 - 1)}\right]e^{i(-kx+\omega t+\theta)}, \tag{48}$$

under the constraint condition

$$h_4^2(2 - m^2 - h_2)[9(m^2-1) + (2 - m^2 - h_2)(4 - 2m^2 + h_2)] + 3h_6[3(m^2-1) + (2-m^2)^2 - h_2^2]^2 = 0. \tag{49}$$

(4) If $l_0 = m^2$, $l_2 = -(1 + m^2)$, $l_4 = 1$, $0 < m < 1$, then $U(\xi) = \text{ns}(\xi)$ or $U(\xi) = \text{dc}(\xi)$.In this case, we have the Jacobi elliptic function solutions of Eqs.(2) and (3):

$$q(x, t) = \beta_0 - \frac{6aB^2}{3A\beta_0 + b(1+\lambda)}\left[\frac{(m^4 - m^2 - h_2^2 + 1)\text{ns}^2(\xi)}{(1 + m^2 + h_2)\text{ns}^2(\xi) - 3m^2}\right]e^{i(-kx+\omega t+\theta)}, \tag{50}$$

$$r(x, t) = \beta_0\lambda - \frac{6aB^2\lambda}{3A\beta_0 + b(1+\lambda)}\left[\frac{(m^4 - m^2 - h_2^2 + 1)\text{ns}^2(\xi)}{(1 + m^2 + h_2)\text{ns}^2(\xi) - 3m^2}\right]e^{i(-kx+\omega t+\theta)}, \tag{51}$$

or

$$q(x, t) = \beta_0 - \frac{6aB^2}{3A\beta_0 + b(1+\lambda)}\left[\frac{(m^4 - m^2 - h_2^2 + 1)\text{dc}^2(\xi)}{(1 + m^2 + h_2)\text{dc}^2(\xi) - 3m^2}\right]e^{i(-kx+\omega t+\theta)}, \tag{52}$$

$$r(x, t) = \beta_0\lambda - \frac{6aB^2\lambda}{3A\beta_0 + b(1+\lambda)}\left[\frac{(m^4 - m^2 - h_2^2 + 1)\text{dc}^2(\xi)}{(1 + m^2 + h_2)\text{dc}^2(\xi) - 3m^2}\right]e^{i(-kx+\omega t+\theta)}, \tag{53}$$

under the same constraint condition (35). In particular, if $m \to 1$, then we have the singular soliton solution:

$$q(x, t) = \beta_0 - \frac{6aB^2}{3A\beta_0 + b(1+\lambda)}\left[\frac{(h_2^2 - 1)\coth^2(\xi)}{3 - (h_2 + 2)\coth^2(\xi)}\right]e^{i(-kx+\omega t+\theta)}, \tag{54}$$

$$r(x, t) = \beta_0\lambda - \frac{6aB^2\lambda}{3A\beta_0 + b(1+\lambda)}\left[\frac{(h_2^2 - 1)\coth^2(\xi)}{3 - (h_2 + 2)\coth^2(\xi)}\right]e^{i(-kx+\omega t+\theta)}, \tag{55}$$

under the same constraint condition (38).

(5) If $l_0 = -m^2$, $l_2 = 2m^2 - 1$, $l_4 = 1 - m^2$, $0 < m < 1$, then $U(\xi) = \text{nc}(\xi)$. In this case, we have the Jacobi elliptic function solution of Eqs.(2) and (3):

$$q(x, t) = \beta_0 - \frac{6aB^2}{3A\beta_0 + b(1+\lambda)}\left[\frac{(m^4 - m^2 - h_2^2 + 1)\text{nc}^2(\xi)}{-(2m^2 - 1 - h_2)\text{nc}^2(\xi) + 3m^2}\right]e^{i(-kx+\omega t+\theta)}, \tag{56}$$

$$r(x,t)=\beta_0\lambda-\frac{6aB^2\lambda}{3A\beta_0+b(1+\lambda)}\left[\frac{(m^4-m^2-h_2^2+1)\mathrm{nc}^2(\xi)}{-(2m^2-1-h_2)\mathrm{nc}^2(\xi)+3m^2}\right]e^{i(-\mathrm{kx}+\omega t+\theta)},\tag{57}$$

under the constraint condition

$$h_4^2(2m^2-1-h_2)[9m^2(1-m^2)+(2m^2-1-h_2)(4m^2-2+h_2)]+3h_6[3m^2(1-m^2)+(2m^2-1)^2-h_2^2]^2=0.\tag{58}$$

In particular, if $m\to1$, then we have the bright soliton solution:

$$q(x,t)=\beta_0-\frac{6aB^2}{3A\beta_0+b(1+\lambda)}\left[\frac{(1-h_2^2)}{3\mathrm{sech}^2(\xi)-(1-h_2)}\right]e^{i(-\mathrm{kx}+\omega t+\theta)},\tag{59}$$

$$r(x,t)=\beta_0\lambda-\frac{6aB^2\lambda}{3A\beta_0+b(1+\lambda)}\left[\frac{(1-h_2^2)}{3\mathrm{sech}^2(\xi)-(1-h_2)}\right]e^{i(-\mathrm{kx}+\omega t+\theta)},\tag{60}$$

under the constraint condition

$$h_4^2(h_2+2)+3h_6(h_2+1)^2=0.\tag{61}$$

(6) If $l_0=-1$, $l_2=\ 2-m^2$, $l_4=-(1-m^2)$, $0<m<1$, then $U(\xi)=\mathrm{nd}(\xi)$. In this case, we have the Jacobi elliptic function solution of Eqs. (2) and (3):

$$q(x,t)=\beta_0-\frac{6aB^2}{3A\beta_0+b(1+\lambda)}\left[\frac{(m^4-m^2-h_2^2+1)\mathrm{nd}^2(\xi)}{-(2-m^2-h_2)\mathrm{nd}^2(\xi)+3}\right]e^{i(-\mathrm{kx}+\omega t+\theta)},\tag{62}$$

$$r(x,t)=\beta_0\lambda-\frac{6aB^2\lambda}{3A\beta_0+b(1+\lambda)}\left[\frac{(m^4-m^2-h_2^2+1)\mathrm{nd}^2(\xi)}{-(2-m^2-h_2)\mathrm{nd}^2(\xi)+3}\right]e^{i(-\mathrm{kx}+\omega t+\theta)},\tag{63}$$

under the same constraint condition (49). In particular, if $m\to1$, then we have the same bright soliton solutions (59) and (60).

(7) If $l_0=1$, $l_2=2-m^2$, $l_4=1-m^2$, $0<m<1$, then $U(\xi)=\mathrm{sc}(\xi)$. In this case, we have the Jacobi elliptic function solution of Eqs.(2) and (3):

$$q(x,t)=\beta_0+\frac{6aB^2}{3A\beta_0+b(1+\lambda)}\left[\frac{(m^4-m^2-h_2^2+1)\mathrm{sc}^2(\xi)}{(2-m^2-h_2)\mathrm{sc}^2(\xi)+3}\right]e^{i(-\mathrm{kx}+\omega t+\theta)},\tag{64}$$

$$r(x,t)=\beta_0\lambda+\frac{6aB^2\lambda}{3A\beta_0+b(1+\lambda)}\left[\frac{(m^4-m^2-h_2^2+1)\mathrm{sc}^2(\xi)}{(2-m^2-h_2)\mathrm{sc}^2(\xi)+3}\right]e^{i(-\mathrm{kx}+\omega t+\theta)},\tag{65}$$

under the same constraint condition (49). In particular, if $m\to1$, then we have the singular soliton solution:

$$q(x,t)=\beta_0+\frac{6aB^2}{3A\beta_0+b(1+\lambda)}\left[\frac{(1-h_2^2)}{3\mathrm{csch}^2(\xi)+(1-h_2)}\right]e^{i(-\mathrm{kx}+\omega t+\theta)},\tag{66}$$

$$r(x,t)=\beta_0\lambda+\frac{6aB^2\lambda}{3A\beta_0+b(1+\lambda)}\left[\frac{(1-h_2^2)}{3\mathrm{csch}^2(\xi)+(1-h_2)}\right]e^{i(-\mathrm{kx}+\omega t+\theta)},\tag{67}$$

under the same constraint condition (61), while if $m\to0$, then we have the periodic solution:

$$q(x,t)=\beta_0-\frac{6aB^2}{3A\beta_0+b(1+\lambda)}\left[\frac{(h_2^2-1)\tan^2(\xi)}{3-(h_2-2)\tan^2(\xi)}\right]e^{i(-\mathrm{kx}+\omega t+\theta)},\tag{68}$$

$$r(x,t)=\beta_0\lambda-\frac{6aB^2\lambda}{3A\beta_0+b(1+\lambda)}\left[\frac{(h_2^2-1)\tan^2(\xi)}{3-(h_2-2)\tan^2(\xi)}\right]e^{i(-\mathrm{kx}+\omega t+\theta)},\tag{69}$$

under the constraint condition

$$-h_4^2(h_2-2)+3h_6(h_2-1)^2=0.\tag{70}$$

(8) If $l_0=1$, $l_2=\ 2m^2\ -1$, $l_4=-m^2(1-m^2)$, $0<m<1$, then $U(\xi)=\mathrm{sd}(\xi)$. In this case, we have the Jacobi elliptic function solution of Eqs.(2) and (3):

$$q(x,t)=\beta_0+\frac{6aB^2}{3A\beta_0+b(1+\lambda)}\left[\frac{(m^4-m^2-h_2^2+1)\mathrm{sd}^2(\xi)}{(2m^2-1-h_2)\mathrm{sd}^2(\xi)+3}\right]e^{i(-\mathrm{kx}+\omega t+\theta)},\tag{71}$$

$$r(x,t)=\beta_0\lambda+\frac{6aB^2\lambda}{3A\beta_0+b(1+\lambda)}\left[\frac{(m^4-m^2-h_2^2+1)\mathrm{sd}^2(\xi)}{(2m^2-1-h_2)\mathrm{sd}^2(\xi)+3}\right]e^{i(-\mathrm{kx}+\omega t+\theta)},\tag{72}$$

under the same constraint condition (58). In particular, if $m\to1$, then we have the same singular soliton solutions (66) and (67), while if $m \to 0$, then we have the same periodic solutions (39) and (40).

(9) If $l_0 = 1 - m^2$, $l_2 = 2 - m^2$, $l_4 = 1$, $0 < m < 1$, then $U(\xi) = \mathrm{cs}(\xi)$. In this case, we have the Jacobi elliptic function solution of Eqs.(2) and (3):

$$q(x, t) = \beta_0 + \frac{6aB^2}{3A\beta_0 + b(1+\lambda)}\left[\frac{(m^4 - m^2 - h_2^2 + 1)\mathrm{cs}^2(\xi)}{(2 - m^2 - h_2)\mathrm{cs}^2(\xi) + 3(1 - m^2)}\right]e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{73}$$

$$r(x, t) = \beta_0\lambda - \frac{6aB^2\lambda}{3A\beta_0 + b(1+\lambda)}\left[\frac{(m^4 - m^2 - h_2^2 + 1)\mathrm{cs}^2(\xi)}{(2 - m^2 - h_2)\mathrm{cs}^2(\xi) + 3(1 - m^2)}\right]e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{74}$$

under the same constraint condition (49). In particular, if $m \to 0$, then we have the periodic solution:

$$q(x, t) = \beta_0 - \frac{6aB^2}{3A\beta_0 + b(1+\lambda)}\left[\frac{(h_2^2 - 1)\cot^2(\xi)}{3 - (h_2 - 2)\cot^2(\xi)}\right]e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{75}$$

$$r(x, t) = \beta_0\lambda - \frac{6aB^2\lambda}{3A\beta_0 + b(1+\lambda)}\left[\frac{(h_2^2 - 1)\cot^2(\xi)}{3 - (h_2 - 2)\cot^2(\xi)}\right]e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{76}$$

under the same constraint condition (70).

(10) If $l_0 = -m^2(1 - m^2)$, $l_2 = 2m^2 - 1$, $l_4 = 1$, $0 < m < 1$, then $U(\xi) = \mathrm{ds}(\xi)$. In this case, we have the following Jacobi elliptic function solution of Eqs.(2) and (3):

$$q(x, t) = \beta_0 - \frac{6aB^2}{3A\beta_0 + b(1+\lambda)}\left[\frac{(m^4 - m^2 - h_2^2 + 1)\mathrm{ds}^2(\xi)}{-(2m^2 - 1 - h_2)\mathrm{ds}^2(\xi) + 3m^2(1 - m^2)}\right]e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{77}$$

$$r(x, t) = \beta_0\lambda - \frac{6aB^2\lambda}{3A\beta_0 + b(1+\lambda)}\left[\frac{(m^4 - m^2 - h_2^2 + 1)\mathrm{ds}^2(\xi)}{-(2m^2 - 1 - h_2)\mathrm{ds}^2(\xi) + 3m^2(1 - m^2)}\right]e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{78}$$

under the same constraint condition (58).

(11) If $l_0 = \frac{1-m^2}{4}$, $l_2 = \frac{1+m^2}{2}$, $l_4 = \frac{1-m^2}{4}$, $0 < m < 1$, then $U(\xi) = \mathrm{nc}(\xi) \pm \mathrm{sc}(\xi)$ or $U(\xi) = \frac{\mathrm{cn}(\xi)}{1 \pm \mathrm{sn}(\xi)}$. In this case, we have the Jacobi elliptic function solutions of Eqs. (2) and (3):

$$q(x, t) = \beta_0 + \frac{6aB^2}{3A\beta_0 + b(1+\lambda)}\left[\frac{(m^4 + 14m^2 - 16h_2^2 + 1)[\mathrm{nc}(\xi) \pm \mathrm{sc}(\xi)]^2}{8(1 + m^2 - 2h_2)[\mathrm{nc}(\xi) \pm \mathrm{sc}(\xi)]^2 + 12(1 - m^2)}\right]e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{79}$$

$$r(x, t) = \beta_0\lambda + \frac{6aB^2\lambda}{3A\alpha\beta_0 + b(1+\lambda)}\left[\frac{(m^4 + 14m^{62} - 16h_2^2 + 1)[\mathrm{nc}(\xi) \pm \mathrm{sc}(\xi)]^2}{8(1 + m^2 - 2h_2)[\mathrm{nc}(\xi) \pm \mathrm{sc}(\xi)]^2 + 12(1 - m^2)}\right]e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{80}$$

or

$$q(x, t) = \beta_0 + \frac{6aB^2}{3A\beta_0 + b(1+\lambda)}\left[\frac{(m^4 + 14m^2 - 16h_2^2 + 1)\mathrm{cn}^2(\xi)}{8(1 + m^2 - 2h_2)\mathrm{cn}^2(\xi) + 12(1 - m^2)[1 \pm \mathrm{sn}(\xi)]^2}\right]e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{81}$$

$$r(x, t) = \beta_0\lambda + \frac{6aB^2\lambda}{3A\beta_0 + b(1+\lambda)}\left[\frac{(m^4 + 14m^2 - 16h_2^2 + 1)\mathrm{cn}^2(\xi)}{8(1 + m^2 - 2h_2)\mathrm{cn}^2(\xi) + 12(1 - m^2)[1 \pm \mathrm{sn}(\xi)]^2}\right]e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{82}$$

under the constraint condition

$$8h_4^2(1 + m^2 - 2h_2)[9(1 - m^2)^2 - 8(1 + m^2 - 2h_2)(1 + m^2 + h_2)] + 3h_6[3(1 - m^2)^2 - 4(1 + m^2)^2 + 16h_2^2]^2 = 0. \tag{83}$$

In particular, if $m \to 0$, then we have the periodic solution:

$$q(x, t) = \beta_0 + \frac{6aB^2}{4[3A\beta_0 + b(1+\lambda)]}\left[\frac{(1 - 16h_2^2)[\sec(\xi) \pm \tan(\xi)]^2}{2(1 - 2h_2)[\sec(\xi) \pm \tan(\xi)]^2 + 3}\right]e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{84}$$

$$r(x, t) = \beta_0\lambda + \frac{6aB^2\lambda}{4[3A\beta_0 + b(1+\lambda)]}\left[\frac{(1 - 16h_2^2)[\sec(\xi) \pm \tan(\xi)]^2}{2(1 - 2h_2)[\sec(\xi) \pm \tan(\xi)]^2 + 3}\right]e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{85}$$

or

$$q(x, t) = \beta_0 + \frac{6aB^2}{4[3A\beta_0 + b(1+\lambda)]}\left[\frac{(1 - 16h_2^2)}{2(1 - 2h_2) + 3(\sec(\xi) \pm \tan(\xi))^2}\right]e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{86}$$

$$r(x, t) = \beta_0\lambda + \frac{6aB^2\lambda}{4[3A\beta_0 + b(1+\lambda)]}\left[\frac{(1 - 16h_2^2)}{2(1 - 2h_2) + 3(\sec(\xi) \pm \tan(\xi))^2}\right]e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{87}$$

under the constraint condition:

$$8h_4^2(1-2h_2)+3h_6(1-4h_2)^2=0. \tag{88}$$

(12) If $l_0=\frac{-(1-m^2)^2}{4}$, $l_2=\frac{1+m^2}{2}$, $l_4=\frac{-1}{4}$, $0<m<1$, then $U(\xi)=m\mathrm{cn}(\xi)\pm\mathrm{dn}(\xi)$. In this case, we have the Jacobi elliptic function solutions of Eqs.(2) and (3):

$$q(x,t)=\beta_0-\frac{6\mathrm{aB}^2}{3A\beta_0+b(1+\lambda)}\left(\frac{[m\mathrm{cn}(\xi)\pm\mathrm{dn}(\xi)]^2}{-8(1+m^2-2h_2)[m\mathrm{cn}(\xi)\pm\mathrm{dn}(\xi)]^2+12(1-m^2)^2}\right)e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{89}$$

$$r(x,t)=\beta_0\lambda-\frac{6\mathrm{aB}^2\lambda}{3A\beta_0+b(1+\lambda)}\left(\frac{[m\mathrm{cn}(\xi)\pm\mathrm{dn}(\xi)]^2}{-8(1+m^2-2h_2)[m\mathrm{cn}(\xi)\pm\mathrm{dn}(\xi)]^2+12(1-m^2)^2}\right)e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{90}$$

under the same constraint condition (83).

(13) If $l_0=\frac{1}{4}$, $l_2=\frac{1-2m^2}{2}$, $l_4=\frac{1}{4}$, $0<m<1$, then $U(\xi)=\frac{\mathrm{sn}(\xi)}{1\pm\mathrm{cn}(\xi)}$. In this case, we have the Jacobi elliptic function solutions of Eqs.(2) and (3):

$$q(x,t)=\beta_0+\frac{3\mathrm{aB}^2}{2[3A\beta_0+b(1+\lambda)]}\left[\frac{(16m^4-16m^2-16h_2^2+1)\mathrm{sn}^2(\xi)}{2(1-2m^2-2h_2)\mathrm{sn}^2(\xi)+3[1\pm\mathrm{cn}(\xi)]^2}\right]e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{91}$$

$$r(x,t)=\beta_0\lambda+\frac{3\mathrm{aB}^2\lambda}{2[3A\beta_0+b(1+\lambda)]}\left[\frac{(16m^4-16m^2-16h_2^2+1)\mathrm{sn}^2(\xi)}{2(1-2m^2-2h_2)\mathrm{sn}^2(\xi)+3[1\pm\mathrm{cn}(\xi)]^2}\right]e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{92}$$

under the constraint condition

$$8h_4^2(1-2m^2-2h_2)[9-8(1-2m^2-2h_2)(1-2m^2+h_2)]+3h_6[3-4(1-2m^2)^2+16h_2^2]^2=0. \tag{93}$$

In particular, if $m\to 1$, then we have the soliton solution:

$$q(x,t)=\beta_0-\frac{3\mathrm{aB}^2}{2[3A\beta_0+b(1+\lambda)]}\left[\frac{(1-16h_2^2)\tanh^2(\xi)}{2(1+2h_2)\tanh^2(\xi)-3[1\pm\mathrm{sech}^2(\xi)]}\right]e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{94}$$

$$r(x,t)=\beta_0\lambda-\frac{3\mathrm{aB}^2\lambda}{2[3A\beta_0+b(1+\lambda)]}\left[\frac{(1-16h_2^2)\tanh^2(\xi)}{2(1+2h_2)\tanh^2(\xi)-3[1\pm\mathrm{sech}^2(\xi)]}\right]e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{95}$$

under the constraint condition

$$-8h_4^2(1+2h_2)+3h_6(4h_2+1)^2=0, \tag{96}$$

while if $m\to 0$, then we have the periodic solution:

$$q(x,t)=\beta_0+\frac{3\mathrm{aB}^2}{2[3A\beta_0+b(1+\lambda)]}\left[\frac{(1-16h_2^2)}{2(1-2h_2)+3[\csc(\xi)\pm\cot(\xi)]^2}\right]e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{97}$$

$$r(x,t)=\beta_0\lambda+\frac{3\mathrm{aB}^2\lambda}{2[3A\beta_0+b(1+\lambda)]}\left[\frac{(1-16h_2^2)}{2(1-2h_2)+3[\csc(\xi)\pm\cot(\xi)]^2}\right]e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{98}$$

under the constraint condition

$$-8h_4^2(1-2h_2)+3h_6(4h_2-1)^2=0. \tag{99}$$

(14) If $l_0=\frac{1}{4}$, $l_2=\frac{1+m^2}{2}$, $l_4=\frac{(1-m^2)^2}{4}$, $0<m<1$, then $U(\xi)=\frac{\mathrm{sn}(\xi)}{\mathrm{cn}(\xi)\pm\mathrm{dn}(\xi)}$. In this case, we have the Jacobi elliptic function solutions of Eqs. (2) and (3):

$$q(x,t)=\beta_0+\frac{3\mathrm{aB}^2}{2[3A\beta_0+b(1+\lambda)]}\left[\frac{(m^4+14m^2-16h_2^2+1)\mathrm{sn}^2(\xi)}{2(1+m^2-2h_2)\mathrm{sn}^2(\xi)+3[\mathrm{cn}(\xi)\pm\mathrm{dn}(\xi)]^2}\right]e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{100}$$

$$r(x,t)=\beta_0\lambda+\frac{3\mathrm{aB}^2\lambda}{2[3A\beta_0+b(1+\lambda)]}\left[\frac{(m^4+14m^2-16h_2^2+1)\mathrm{sn}^2(\xi)}{2(1+m^2-2h_2)\mathrm{sn}^2(\xi)+3[\mathrm{cn}(\xi)\pm\mathrm{dn}(\xi)]^2}\right]e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{101}$$

under the same constraint condition (83). In particular, if $m\to 1$, then we have the solitary solution:

$$q(x,t)=\beta_0+\frac{6\mathrm{aB}^2}{3A\beta_0+b(1+\lambda)}\left(\frac{(1-h_2^2)\tanh^2(\xi)}{(1-h_2)\tanh^2(\xi)+3\mathrm{sech}^2(\xi)}\right)e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{102}$$

$$r(x,t)=\beta_0\lambda+\frac{6\mathrm{aB}^2\lambda}{3A\beta_0+b(1+\lambda)}\left(\frac{(1-h_2^2)\tanh^2(\xi)}{(1-h_2)\tanh^2(\xi)+3\mathrm{sech}^2(\xi)}\right)e^{i(-\mathrm{kx}+\omega t+\theta)}, \tag{103}$$

under the same constraint condition (38), while if $m\to 0$, then we have the periodic solution:

$$q(x,t)=\beta_0+\frac{3aB^2}{2[3A\beta_0+b(1+\lambda)]}\left[\frac{(1-16h_2^2)\tan^2(\xi)}{2(1-2h_2)\tan^2(\xi)+3[1\pm\sec^2(\xi)]}\right]e^{i(-kx+\omega t+\theta)}, \tag{104}$$

$$r(x,t)=\beta_0\lambda+\frac{3aB^2\lambda}{2[3A\beta_0+b(1+\lambda)]}\left[\frac{(1-16h_2^2)\tan^2(\xi)}{2(1-2h_2)\tan^2(\xi)+3[1\pm\sec^2(\xi)]}\right]e^{i(-kx+\omega t+\theta)}, \tag{105}$$

under the same constraint condition (88).

Finally, there are a lot other Jacobi's elliptic functions solutions to Eqs. (2) and (3) which are omitted here.

4. Conclusions

The newly developed ϕ^6–model expansion method has been successfully applied in this paper for locating several new exact solutions including Jacobi elliptic solutions, solitons and other solutions to the coupled system of NLSE with QC nonlinearity that models birefringence. The soliton solutions that are recovered in this paper are being reported for the first time. The effect of 4WM was taken into account and consequently phase–matching condition has been implemented to secure these soliton solutions. Our results show extreme promise with the methodology applied to such a model. The results will, in future, be applied to additional optoelectronic devices as well as optoelectronic phenomena and these include DWDM networks, highly dispersive solitons, meta–optics and other such devices. The results of those research activities are currently awaited, but not for long!!

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