Optik - International Journal for Light and Electron Optics 199 (2019) 163338

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.com/locate/ijleo

Original research article

Optical solitons with Kudryashov's equation by *F*-expansion

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ARTICLE INFO

Keywords:
Solitons
Complexitons
Kudryashov's equation
F-expansion

ABSTRACT

The dynamics of solitons, with newly modeled Kudryashov's equation, is revealed in this paper by the aid of *F*-expansion scheme. Thus, bright, dark and singular solitons emerged from it as well as complexiton solutions are presented. The sustaining criteria for the solitons are also enumerated.

1. Introduction

A newly proposed law of refractive index for soliton propagation across inter-continental distances, is modeled with Kudrayashov's equation (KE) [11]. While this is a theoretical proposition, experimental results, with such a form of refractive index, are pending. There has been sustained success, thus far, with this model from theoretical perspective. While some preliminary results have been already reported [6,11], the current paper handles the governing equation with *F*-expansion scheme. As will be detailed, bright, dark and singular soliton solutions emerge from the scheme. Then, complexiton solutions also fall out of it. Additional solutions in terms of Weierstrass elliptic functions as well as periodic waves also becomes visible. These are all listed in order to gain a complete spectrum of solutions from this powerful scheme. The details of the methodology applied to KE are all pictured in the rest of the paper.

1.1. Governing model

The dimensionless form of Kudryashov's model is formulated as [6,11]

$$\mathrm{i}q_t + \mathrm{a}q_{xx} + (b_1\,|q|^{-2n} + b_2\,|q|^{-n} + b_3\,|q|^{n} + b_4\,|q|^{2n})q = 0. \tag{1}$$

In (1), the first term is the linear temporal evolution while a is the coefficient of group velocity dispersion. The next four terms are nonlinear and stem from the law of refractive index of an optical fiber and gives self-phase modulation to the model. When $b_1 = b_2 = 0$, the model collapses to dual-power law of refractive index, while if $b_1 = b_2 = b_3 = 0$, one recovers power law. Kudryashov's model thus stands as an extension to these known forms of refractive indices. Finally, the above cases with $n = 1$, are commonly referred to as parabolic law and the most fundamental Kerr law respectively.

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https://doi.org/10.1016/j.ijleo.2019.163338
Received 31 May 2019; Accepted 29 August 2019

1.2. Preliminaries

The starting assumption for extracting soliton solutions is assumed as [1–6,9–13,17–19]

$$q(x, t) = g(\varsigma)e^{i\phi(x,t)} \tag{2}$$

where

$$\varsigma = x - \mathrm{v\,t} \tag{3}$$

and v stands for the soliton velocity. From portion,

$$\phi = -\kappa x + \omega t + \theta_0 \tag{4}$$

where κ is the soliton frequency, ω is its wave number while θ_0 is the phase center. After inserting (2) into (1), real part causes

$$ag^{2n}g'' + b_1 g + b_2 g^{n+1} + b_3 g^{3n+1} + b_4 g^{4n+1} - (\omega + a\kappa^2)g^{2n+1} = 0 \tag{5}$$

while imaginary part is

$$(v + 2a\kappa)g^{2n}g' = 0. \tag{6}$$

From (6), velocity of the wave is

$$v = -2a\kappa. \tag{7}$$

The transformation

$$g = \varphi^{\frac{1}{n}} \tag{8}$$

is applied to Eq. (5) to reveal closed form solutions. So,

$$b_1 n^2 + b_2 n^2\varphi - n^2(\omega + a\kappa^2)\varphi^2 + b_3 n^2\varphi^3 + b_4 n^2\varphi^4 + a(1 - n)(\varphi')^2 + \mathrm{a\,n}\varphi\varphi'' = 0. \tag{9}$$

2. *F*-expansion scheme

The solution of (9) is supposed as [7,8,15,20]

$$\varphi(\varsigma) = \sum_{j=0}^{N} \delta_j F^j(\varsigma), \tag{10}$$

where δ_j are constants that needs to be identified and $F = F(\varsigma)$ ensures

$$(F')^2 = \mathrm{PF}^4 + \mathrm{QF}^2 + R, \tag{11}$$

together with constants P, Q and R. Balancing $(\varphi')^2$ or $\varphi\varphi''$ with φ^4 in (9) yields $N = 1$. Thus (10) becomes

$$\varphi(\varsigma) = \delta_0 + \delta_1 F(\varsigma). \tag{12}$$

Plugging (12) into (9), accumulating the coefficients of each power of F, and overcoming the obtained system yields

$$\begin{aligned}
b_1 &= -\frac{(n^2 - 1)(b_3^4 n^4(n + 1)^2 - 4ab_3^2 b_4 n^2(n + 1)(n + 2)^2 Q + 16a^2 b_4^2(n + 2)^4 \mathrm{P\,R})}{16 b_4^3 n^4(n + 2)^4},\\
b_2 &= -\frac{b_3(n^2 - n - 2)(b_3^2 n^2(n + 1) - 2ab_4(n + 2)^2 Q)}{4b_4^2 n^2(n + 2)^3},\\
\delta_0 &= -\frac{b_3(n + 1)}{2b_4(n + 2)},\quad \delta_1 = \pm\frac{\sqrt{-a(n + 1)P}}{\sqrt{b_4}\,n},\\
\omega &= \frac{2ab_4(n + 2)^2(Q - n^2\kappa^2) - 3b_3^2 n^2(n + 1)}{2b_4 n^2(n + 2)^2}.
\end{aligned} \tag{13}$$

Thus, the solution of Kudryashov's equation becomes the form

$$q(x, t) = \left\{-\frac{b_3(n + 1)}{2b_4(n + 2)} \pm \frac{\sqrt{-a(n + 1)P}}{\sqrt{b_4}\,n}F(\varsigma)\right\}^{\frac{1}{n}} \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n + 2)^2(Q - n^2\kappa^2) - 3b_3^2 n^2(n + 1)}{2b_4 n^2(n + 2)^2}\right)t + \theta_0\right\}\right]. \tag{14}$$

2.1. Jacobi's elliptic function solutions

By employing the solutions of the auxiliary equation (11), the solutions in terms of elliptic functions to the governing equation are listed as:

(1): $P = m^2,\quad Q = -(1 + m^2),\quad R = 1,\quad F(\varsigma) = \mathrm{sn}\,\varsigma,$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}\,n}\,\mathrm{sn}\,[x+2\kappa a\,t]\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t + \theta_0\right\}\right]. \tag{15}$$

(2): $P = -m^2, \quad Q = 2m^2 - 1, \quad R = 1 - m^2, \quad F(\varsigma) = \mathrm{cn}\,\varsigma,$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}\,n}\,\mathrm{cn}\,[x+2\kappa a\,t]\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t + \theta_0\right\}\right]. \tag{16}$$

(3): $P = 1, \quad Q = -(1+m^2), \quad R = m^2, \quad F(\varsigma) = \mathrm{ns}\,\varsigma,$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}\,n}\,\mathrm{ns}\,[x+2\kappa a\,t]\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t + \theta_0\right\}\right]. \tag{17}$$

(4): $P = 1, \quad Q = -(1+m^2), \quad R = m^2, \quad F(\varsigma) = \mathrm{dc}\,\varsigma,$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}\,n}\,\mathrm{dc}\,[x+2\kappa a\,t]\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t + \theta_0\right\}\right]. \tag{18}$$

(5): $P = 1 - m^2, \quad Q = 2 - m^2, \quad R = 1, \quad F(\varsigma) = \mathrm{sc}\,\varsigma,$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}\,n}\,\mathrm{sc}[x+2\kappa a\,t]\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t + \theta_0\right\}\right]. \tag{19}$$

(6): $P = 1, \quad Q = 2 - m^2, \quad R = 1 - m^2, \quad F(\varsigma) = \mathrm{cs}\,\varsigma,$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}\,n}\,\mathrm{cs}\,[x+2\kappa a\,t]\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t + \theta_0\right\}\right]. \tag{20}$$

(7): $P = \frac{1}{4}, \quad Q = \frac{1-2m^2}{2}, \quad R = \frac{1}{4}, \quad F(\varsigma) = \mathrm{ns}\,\varsigma \pm \mathrm{cs}\,\varsigma,$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}\,n}(\mathrm{ns}\,[x+2\kappa a\,t] \pm \mathrm{cs}\,[x+2\kappa a\,t])\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t + \theta_0\right\}\right]. \tag{21}$$

(8): $P = \frac{1-m^2}{4}, \quad Q = \frac{1+m^2}{2}, \quad R = \frac{1-m^2}{4}, \quad F(\varsigma) = \mathrm{nc}\,\varsigma \pm \mathrm{sc}\,\varsigma,$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}\,n}(\mathrm{nc}\,[x+2\kappa a\,t] \pm \mathrm{sc}[x+2\kappa a\,t])\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t + \theta_0\right\}\right]. \tag{22}$$

(9): $P = \frac{m^2}{4}, \quad Q = \frac{m^2-2}{2}, \quad R = \frac{m^2}{4}, \quad F(\varsigma) = \mathrm{sn}\,\varsigma \pm i\,\mathrm{cn}\,\varsigma,$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}n}(\mathrm{sn}[x+2\kappa a\,t] \pm i\,\mathrm{cn}[x+2\kappa a\,t])\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right]. \tag{23}$$

(10): $P=\frac{m^2}{4}, \quad Q=\frac{m^2-2}{2}, \quad R=\frac{1}{4}, \quad F(\varsigma)=\frac{\mathrm{sn}\,\varsigma}{1\pm \mathrm{dn}\,\varsigma},$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}n}\frac{\mathrm{sn}[x+2\kappa a\,t]}{1\pm\mathrm{dn}[x+2\kappa a\,t]}\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right]. \tag{24}$$

(11): $P=-\frac{1}{4}, \quad Q=\frac{m^2+1}{2}, \quad R=\frac{(1-m^2)^2}{4}, \quad F(\varsigma)=m\,\mathrm{cn}\,\varsigma \pm \mathrm{dn}\,\varsigma,$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}n}(m\,\mathrm{cn}[x+2\kappa a\,t] \pm \mathrm{dn}[x+2\kappa a\,t])\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right]. \tag{25}$$

(12): $P=\frac{(1-m^2)^2}{4}, \quad Q=\frac{m^2+1}{2}, \quad R=\frac{1}{4}, \quad F(\varsigma)=\mathrm{ds}\,\varsigma \pm \mathrm{cs}\,\varsigma,$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}n}(\mathrm{ds}[x+2\kappa a\,t] \pm \mathrm{cs}[x+2\kappa a\,t])\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right]. \tag{26}$$

(13): $P>0, \quad Q<0, \quad R=\frac{m^2Q^2}{(1+m^2)^2P}, \quad F(\varsigma)=\sqrt{-\frac{m^2Q}{(1+m^2)P}}\,\mathrm{sn}\left(\sqrt{-\frac{Q}{1+m^2}}\varsigma\right),$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}n}\sqrt{-\frac{m^2Q}{(1+m^2)P}}\,\mathrm{sn}\left(\sqrt{-\frac{Q}{1+m^2}}[x+2\kappa a\,t]\right)\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right]. \tag{27}$$

(14): $P<0, \quad Q>0, \quad R=\frac{(1-m^2)Q^2}{(m^2-2)^2P}, \quad F(\varsigma)=\sqrt{-\frac{Q}{(2-m^2)P}}\,\mathrm{dn}\left(\sqrt{\frac{Q}{2-m^2}}\varsigma\right),$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}n}\sqrt{-\frac{Q}{(2-m^2)P}}\,\mathrm{dn}\left(\sqrt{\frac{Q}{2-m^2}}[x+2\kappa a\,t]\right)\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right]. \tag{28}$$

(15): $P=1, \quad Q=m^2+2, \quad R=1-2m^2+m^4, \quad F(\varsigma)=\frac{\mathrm{dn}\,\varsigma\,\mathrm{cn}\,\varsigma}{\mathrm{sn}\,\varsigma},$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}n}\frac{\mathrm{dn}[x+2\kappa a\,t]\,\mathrm{cn}[x+2\kappa a\,t]}{\mathrm{sn}[x+2\kappa a\,t]}\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right]. \tag{29}$$

(16): $P=\frac{A^2(m-1)^2}{4}, \quad Q=\frac{m^2+6m+1}{2}, \quad R=\frac{(m-1)^2}{4A^2}, \quad F(\varsigma)=\frac{\mathrm{dn}\,\varsigma\,\mathrm{cn}\,\varsigma}{A(1+\mathrm{sn}\,\varsigma)(1+m\,\mathrm{sn}\,\varsigma)},$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}n}\frac{\mathrm{dn}[x+2\kappa a\,t]\,\mathrm{cn}[x+2\kappa a\,t]}{A(1+\mathrm{sn}[x+2\kappa a\,t])(1+m\,\mathrm{sn}[x+2\kappa a\,t])}\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right]. \tag{30}$$

(17): $P=-\frac{4}{m},\quad Q=6m-m^2-1,\quad R=-2m^3+m^4+m^2,\qquad F(\varsigma)=\frac{m\,\mathrm{cn}\,\varsigma\,\mathrm{dn}\,\varsigma}{m\,\mathrm{sn}^2\varsigma+1},$

$$q(x,\,t)\ =\ \left\{-\frac{b_3(n+1)}{2b_4(n+2)}\pm\frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}\,n}\frac{m\,\mathrm{cn}\,[x+2\kappa \mathrm{a\,t}]\,\mathrm{dn}\,[x+2\kappa \mathrm{a\,t}]}{m\,\mathrm{sn}^2\,[x+2\kappa \mathrm{a\,t}]+1}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x+\left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right].\tag{31}$$

(18): $P=\frac{1}{4},\quad Q=\frac{1-2m^2}{2},\quad R=\frac{1}{4},\quad F(\varsigma)=\frac{\mathrm{sn}\,\varsigma}{1\pm\,\mathrm{cn}\,\varsigma},$

$$q(x,\,t)\ =\ \left\{-\frac{b_3(n+1)}{2b_4(n+2)}\pm\frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}\,n}\frac{\mathrm{sn}\,[x+2\kappa \mathrm{a\,t}]}{1\pm\,\mathrm{cn}\,[x+2\kappa \mathrm{a\,t}]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x+\left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right].\tag{32}$$

(19): $P=\frac{1-m^2}{4},\quad Q=\frac{1+m^2}{2},\quad R=\frac{1-m^2}{4},\quad F(\varsigma)=\frac{\mathrm{cn}\,\varsigma}{1\pm\,\mathrm{sn}\,\varsigma},$

$$q(x,\,t)\ =\ \left\{-\frac{b_3(n+1)}{2b_4(n+2)}\pm\frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}\,n}\frac{\mathrm{cn}\,[x+2\kappa \mathrm{a\,t}]}{1\pm\,\mathrm{sn}\,[x+2\kappa \mathrm{a\,t}]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x+\left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right].\tag{33}$$

(20): $P=\frac{2-m^2-2m_1}{4},\quad Q=\frac{m^2-6m_1-2}{2},\quad R=\frac{2-m^2-2m_1}{4},\qquad F(\varsigma)=\frac{m^2\,\mathrm{sn}\,\varsigma\,\mathrm{cn}\,\varsigma}{\mathrm{sn}^2\varsigma+(1+m_1)\,\mathrm{dn}\,\varsigma-1-m_1},$

$$q(x,\,t)\ =\ \left\{-\frac{b_3(n+1)}{2b_4(n+2)}\pm\frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}\,n}\frac{m^2\,\mathrm{sn}\,[x+2\kappa \mathrm{a\,t}]\,\mathrm{cn}\,[x+2\kappa \mathrm{a\,t}]}{\mathrm{sn}^2\,[x+2\kappa \mathrm{a\,t}]+(1+m_1)\,\mathrm{dn}\,[x+2\kappa \mathrm{a\,t}]-1-m_1}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x+\left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right],\tag{34}$$

where $m_1=\sqrt{1-m^2}$.

(21): $P=\frac{2-m^2+2m_1}{4},\quad Q=\frac{m^2+6m_1-2}{2},\quad R=\frac{2-m^2+2m_1}{4},\qquad F(\varsigma)=\frac{m^2\,\mathrm{sn}\,\varsigma\,\mathrm{cn}\,\varsigma}{\mathrm{sn}^2\varsigma+(-1+m_1)\,\mathrm{dn}\,\varsigma-1-m_1},$

$$q(x,\,t)\ =\ \left\{-\frac{b_3(n+1)}{2b_4(n+2)}\pm\frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}\,n}\frac{m^2\,\mathrm{sn}\,[x+2\kappa \mathrm{a\,t}]\,\mathrm{cn}\,[x+2\kappa \mathrm{a\,t}]}{\mathrm{sn}^2\,[x+2\kappa \mathrm{a\,t}]+(-1+m_1)\,\mathrm{dn}\,[x+2\kappa \mathrm{a\,t}]-1-m_1}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x+\left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right].\tag{35}$$

(22): $P=\frac{C^2m^4-(B^2+C^2)m^2+B^2}{4},\quad Q=\frac{m^2+1}{2},\quad R=\frac{m^2-1}{4(C^2m^2-B^2)},\qquad F(\varsigma)=\frac{\sqrt{\frac{B^2-C^2}{B^2-C^2m^2}}+\mathrm{sn}\,\varsigma}{B\,\mathrm{cn}\,\varsigma+C\,\mathrm{dn}\,\varsigma},$

$$q(x,\,t)\ =\ \left\{-\frac{b_3(n+1)}{2b_4(n+2)}\pm\frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}\,n}\frac{\sqrt{\frac{B^2-C^2}{B^2-C^2m^2}}\ +\ \mathrm{sn}\,[x+2\kappa \mathrm{a\,t}]}{B\,\mathrm{cn}\,[x+2\kappa \mathrm{a\,t}]+C\,\mathrm{dn}\,[x+2\kappa \mathrm{a\,t}]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x+\left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right].\tag{36}$$

(23): $P=\frac{B^2+C^2m^2}{4},\quad Q=\frac{1-2m^2}{2},\quad R=\frac{1}{4(B^2+C^2m^2)},\qquad F(\varsigma)=\frac{\sqrt{\frac{C^2m^2+B^2-C^2}{B^2+C^2m^2}}+\mathrm{cn}\,\varsigma}{B\,\mathrm{sn}\,\varsigma+C\,\mathrm{dn}\,\varsigma},$

$$q(x,\,t)\ =\ \left\{-\frac{b_3(n+1)}{2b_4(n+2)}\pm\frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}\,n}\frac{\sqrt{\frac{C^2m^2+B^2-C^2}{B^2+C^2m^2}}\ +\ \mathrm{cn}\,[x+2\kappa \mathrm{a\,t}]}{B\,\mathrm{sn}\,[x+2\kappa \mathrm{a\,t}]+C\,\mathrm{dn}\,[x+2\kappa \mathrm{a\,t}]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x+\left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right].\tag{37}$$

(24): $P=\frac{B^2+C^2}{4},\quad Q=\frac{m^2-2}{2},\quad R=\frac{m^4}{4(B^2+C^2)},\quad F(\varsigma)=\frac{\sqrt{\frac{B^2+C^2-C^2m^2}{B^2+C^2}}+\mathrm{dn}\,\varsigma}{B\,\mathrm{sn}\,\varsigma+C\,\mathrm{cn}\,\varsigma},$

$$q(x,t)=\left\{-\frac{b_3(n+1)}{2b_4(n+2)}\pm\frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}\,n}\frac{\sqrt{\frac{B^2+C^2-C^2m^2}{B^2+C^2}}+\mathrm{dn}\,[x+2\kappa\mathrm{a}\,\mathrm{t}]}{B\,\mathrm{sn}\,[x+2\kappa\mathrm{a}\,\mathrm{t}]+C\,\mathrm{cn}\,[x+2\kappa\mathrm{a}\,\mathrm{t}]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x+\left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right].\tag{38}$$

2.2. Weierstrass' elliptic function solutions

Weierstrass' elliptic function $\wp$ is introduced as [14]:

$$\wp(z;\omega_1,\omega_2)=\frac{1}{z^2}+\sum_{\{m,n\}\neq\{0,0\}}\left(\frac{1}{(z+m\omega_1+n\omega_2)^2}-\frac{1}{(m\omega_1+n\omega_2)^2}\right).\tag{39}$$

By means of the solutions of (11) presented in [16], Weierstrass elliptic function solutions to (1) are:

(25): $g_2=\frac{4(Q^2-3\mathrm{PR})}{3},\quad g_3=\frac{4Q(-2Q^2+9\mathrm{PR})}{27},\quad F(\varsigma)=\sqrt{\frac{1}{P}\left[\wp(\varsigma;g_2,g_3)-\frac{1}{3}Q\right]},$

$$q(x,t)=\left\{-\frac{b_3(n+1)}{2b_4(n+2)}\pm\frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}\,n}\sqrt{\frac{1}{P}\left[\wp([x+2\kappa\mathrm{a}\,\mathrm{t}];g_2,g_3)-\frac{1}{3}Q\right]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x+\left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right].\tag{40}$$

(26): $g_2=\frac{4(Q^2-3\mathrm{PR})}{3},\quad g_3=\frac{4Q(-2Q^2+9\mathrm{PR})}{27},\quad F(\varsigma)=\sqrt{\frac{3R}{3\wp(\varsigma;g_2,g_3)-Q}},$

$$q(x,t)=\left\{-\frac{b_3(n+1)}{2b_4(n+2)}\pm\frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}\,n}\sqrt{\frac{3R}{3\wp([x+2\kappa\mathrm{a}\,\mathrm{t}];g_2,g_3)-Q}}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x+\left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right].\tag{41}$$

(27): $g_2=-\frac{5\mathrm{QD}+4Q^2+33\mathrm{PQR}}{12},\quad g_3=\frac{21Q^2D-63\mathrm{PRD}+20Q^3-27\mathrm{PQR}}{216},$
$F(\varsigma)=\frac{\sqrt{12R\wp(\varsigma;g_2,g_3)+2R(2Q+D)}}{12\wp(\varsigma;g_2,g_3)+D},$

$$q(x,t)=\left\{-\frac{b_3(n+1)}{2b_4(n+2)}\pm\frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}\,n}\frac{\sqrt{12R\wp([x+2\kappa\mathrm{a}\,\mathrm{t}];g_2,g_3)+2R(2Q+D)}}{12\wp([x+2\kappa\mathrm{a}\,\mathrm{t}];g_2,g_3)+D}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x+\left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right].\tag{42}$$

(28): $g_2=\frac{Q^2+12\mathrm{PR}}{12},\quad g_3=\frac{Q(36\mathrm{PR}-Q^2)}{216},\quad F(\varsigma)=\frac{\sqrt{R}\,[6\wp(\varsigma;g_2,g_3)+Q]}{3\wp'(\varsigma;g_2,g_3)},$

$$q(x,t)=\left\{-\frac{b_3(n+1)}{2b_4(n+2)}\pm\frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}\,n}\frac{\sqrt{R}\,[6\wp([x+2\kappa\mathrm{a}\,\mathrm{t}];g_2,g_3)+Q]}{3\wp'([x+2\kappa\mathrm{a}\,\mathrm{t}];g_2,g_3)}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x+\left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right].\tag{43}$$

(29): $g_2=\frac{Q^2+12\mathrm{PR}}{12},\quad g_3=\frac{Q(36\mathrm{PR}-Q^2)}{216},\quad F(\varsigma)=\frac{3\wp'(\varsigma;g_2,g_3)}{\sqrt{P}\,[6\wp(\varsigma;g_2,g_3)+Q]},$

$$q(x,t)=\left\{-\frac{b_3(n+1)}{2b_4(n+2)}\pm\frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}\,n}\frac{3\wp'([x+2\kappa\mathrm{a}\,\mathrm{t}];g_2,g_3)}{\sqrt{P}\,[6\wp([x+2\kappa\mathrm{a}\,\mathrm{t}];g_2,g_3)+Q]}\right\}^{\frac{1}{n}}\times\exp\left[i\left\{-\kappa x+\left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right].\tag{44}$$

(30): $R=\frac{5Q^2}{36P},\quad g_2=\frac{2Q^2}{9},\quad g_3=\frac{Q^3}{54},\quad F(\varsigma)=\frac{Q\sqrt{-15Q/2P}\,\wp(\varsigma;g_2,g_3)}{3\wp(\varsigma;g_2,g_3)+Q},$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}\,n}\frac{Q\sqrt{-15Q/2P}\,\wp([x+2\kappa a\,t];g_2,g_3)}{3\wp([x+2\kappa a\,t];g_2,g_3)+Q}\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right]. \tag{45}$$

2.3. Solitons and other solutions

Bright, dark and singular solitons and complexitons arise when the modulus $m \to 1$ as:

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)}}{\sqrt{b_4}\,n}\tanh[x+2\kappa a\,t]\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\frac{2ab_4(n+2)^2(2+n^2\kappa^2)+3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right], \tag{46}$$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{a(n+1)}}{\sqrt{b_4}\,n}\,\mathrm{sech}\,[x+2\kappa a\,t]\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n+2)^2(1-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right], \tag{47}$$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)}}{\sqrt{b_4}\,n}\coth[x+2\kappa a\,t]\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\frac{2ab_4(n+2)^2(2+n^2\kappa^2)+3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right], \tag{48}$$

Solutions (46)–(48) respectively stands for dark, bright and singular solitons.

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)}}{\sqrt{b_4}\,n}\,\mathrm{csch}\,[x+2\kappa a\,t]\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n+2)^2(1-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right], \tag{49}$$

Solution (49) is the second form of singular soliton solution.

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)}}{2\sqrt{b_4}\,n}(\coth[x+2\kappa a\,t] \pm \mathrm{csch}\,[x+2\kappa a\,t])\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\frac{ab_4(n+2)^2(1+2n^2\kappa^2)+3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right], \tag{50}$$

Solution (50) also is singular solitons.

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)}}{2\sqrt{b_4}\,n}(\tanh[x+2\kappa a\,t] \pm i\,\mathrm{sech}\,[x+2\kappa a\,t])\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\frac{ab_4(n+2)^2(1+2n^2\kappa^2)+3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right], \tag{51}$$

Solution (51) denotes complexiton solutions.

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}\,n}\sqrt{-\frac{Q}{2P}}\tanh\left(\sqrt{-\frac{Q}{2}}[x+2\kappa a\,t]\right)\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t+\theta_0\right\}\right], \tag{52}$$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)P}}{\sqrt{b_4}\,n}\sqrt{-\frac{Q}{P}}\ \mathrm{sech}\left(\sqrt{Q}\,[x+2\kappa a\,t]\right)\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n+2)^2(Q-n^2\kappa^2) - 3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t + \theta_0\right\}\right], \tag{53}$$

Again (52) and (53) respectively stand for dark and bright solitons. (52) is valid provided $Q < 0$ and $P > 0$, while (53) will be valid for $Q > 0$ and $P < 0$.

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{2\sqrt{-a(n+1)}}{\sqrt{b_4}\,n}\ \mathrm{csch}\,2[x+2\kappa a\,t]\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n+2)^2(3-n^2\kappa^2) - 3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t + \theta_0\right\}\right], \tag{54}$$

Eq. (54) represents singular solitons.

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{2\sqrt{a(n+1)}}{\sqrt{b_4}\,n}\ \mathrm{sech}\,2[x+2\kappa a\,t]\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n+2)^2(4-n^2\kappa^2) - 3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t + \theta_0\right\}\right], \tag{55}$$

Eq. (55) is another form of bright solitons.

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)}}{2\sqrt{b_4}\,n}\coth\left[\frac{x+2\kappa a\,t}{2}\right]\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\frac{ab_4(n+2)^2(1+2n^2\kappa^2) + 3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t + \theta_0\right\}\right], \tag{56}$$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \mp \frac{\sqrt{-a(n+1)}}{2\sqrt{b_4}\,n}\tanh\left[\frac{x+2\kappa a\,t}{2}\right]\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\frac{ab_4(n+2)^2(1+2n^2\kappa^2) + 3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t + \theta_0\right\}\right], \tag{57}$$

Finally, (56) and (57) are singular and dark solitons respectively.

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)}}{2\sqrt{b_4}\,n}\,\frac{\sqrt{B^2} + \sqrt{B^2+C^2}\ \mathrm{sech}[x+2\kappa a\,t]}{B\tanh[x+2\kappa a\,t] + C\,\mathrm{sech}[x+2\kappa a\,t]}\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\frac{ab_4(n+2)^2(1+2n^2\kappa^2) + 3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t + \theta_0\right\}\right]. \tag{58}$$

2.4. Trigonometric function solutions

Trigonometric functions solutions and the combinations of these are achieved when $m \to 0$ as:

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)}}{\sqrt{b_4}\,n}\csc[x+2\kappa a\,t]\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\frac{2ab_4(n+2)^2(1+n^2\kappa^2) + 3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t + \theta_0\right\}\right], \tag{59}$$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)}}{\sqrt{b_4}\,n}\sec[x+2\kappa a\,t]\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\frac{2ab_4(n+2)^2(1+n^2\kappa^2) + 3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t + \theta_0\right\}\right], \tag{60}$$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)}}{\sqrt{b_4}n}\tan[x+2\kappa a\,t]\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n+2)^2(2-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t + \theta_0\right\}\right], \tag{61}$$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)}}{\sqrt{b_4}n}\cot[x+2\kappa a\,t]\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{2ab_4(n+2)^2(2-n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t + \theta_0\right\}\right], \tag{62}$$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)}}{2\sqrt{b_4}n}(\csc[x+2\kappa a\,t] \pm \cot[x+2\kappa a\,t])\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{ab_4(n+2)^2(1-2n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t + \theta_0\right\}\right], \tag{63}$$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)}}{2\sqrt{b_4}n}(\sec[x+2\kappa a\,t] \pm \tan[x+2\kappa a\,t])\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{ab_4(n+2)^2(1-2n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t + \theta_0\right\}\right], \tag{64}$$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)}}{2\sqrt{b_4}n}\frac{\sqrt{B^2-C^2}+\sqrt{B^2}\sin[x+2\kappa a\,t]}{B\cos[x+2\kappa a\,t]+C}\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{ab_4(n+2)^2(1-2n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t + \theta_0\right\}\right], \tag{65}$$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)}}{2\sqrt{b_4}n}\frac{\sqrt{B^2-C^2}+\sqrt{B^2}\cos[x+2\kappa a\,t]}{B\sin[x+2\kappa a\,t]+C}\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x + \left(\frac{ab_4(n+2)^2(1-2n^2\kappa^2)-3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t + \theta_0\right\}\right], \tag{66}$$

$$q(x,t) = \left\{-\frac{b_3(n+1)}{2b_4(n+2)} \pm \frac{\sqrt{-a(n+1)(B^2+C^2)}}{\sqrt{b_4}n(B\sin[x+2\kappa a\,t]+C\cos[x+2\kappa a\,t])}\right\}^{\frac{1}{n}} \times \exp\left[i\left\{-\kappa x - \left(\frac{2ab_4(n+2)^2(1+n^2\kappa^2)+3b_3^2n^2(n+1)}{2b_4n^2(n+2)^2}\right)t + \theta_0\right\}\right]. \tag{67}$$

3. Conclusions

Today's paper structured the soliton solutions to Kudryashov's equation that models soliton propagation through optical fibers for newly proposed refractive index format. *F*-expansion scheme revealed bright, dark and singular soliton solutions along with complexiton solutions that are all listed in the work. The existence and sustainability criteria for the solutions paint a complete picture to the governing model. The results of this paper thus form the foundation stone to move further along with this newly proposed model. One move is to extend the model to additional optoelectronic devices such as optical couplers, magneto-optic waveguides, meta-optics and many such. Later, several additional features, such as soliton fission, will be handled. Moreover, some more integration schemes will provide additional perspective to the model. One such scheme is Lie symmetry analysis. Then, of course, there are several more, not to mention here. These results will be gradually streaming.

Conflict of interest

The authors also declare that there is no conflict of interest.

Acknowledgements

The research work of the fifth author NPRP 11S-1126-170033 was supported by the grant NPRP 8-028-1-001 from QNRF and he is thankful for it.

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