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Optical solitons in fiber Bragg gratings with dispersive reflectivity for parabolic-nonlocal combo nonlinearity by extended trial function

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ABSTRACT

This paper displays bright and singular optical solitons in fiber Bragg gratings with dispersive reflectivity for parabolic–nonlocal combo nonlinearity. The extended trial function approach is the integration scheme adopted in this paper. The existence criteria for such solitons are indicated.

1. Introduction

Optical solitons in fiber Bragg gratings is an important area of study in the arena of mathematical photonics. The research works in this field has been going on for the past few decades. The study of solitons in fiber Bragg gratings with dispersive reflectivity has been conducted with three forms of nonlinearity, thus far. They are Kerr law, parabolic law and quadratic-cubic nonlinearity. Apart from numerical approaches, the analytical schemes that have been adopted, thus far, to address fiber Bragg gratings are method of undetermined coefficients and extended trial function method [1–15]. The current paper first proposes the model for fiber Bragg gratings with parabolic-nonlocal combo nonlinearity. After establishing the model, extended trial function approach is implemented to retrieve soliton solutions to the model. The existence criteria for such solitons, that naturally emerges from the scheme, are also displayed in the work. The details are penned down in the rest of the paper.

1.1. Governing model

The dimensionless form of the coupled nonlinear Schrödinger’s equation (NLSE) in fiber Bragg gratings, having parabolic-nonlocal combo nonlinearity, is given by [6,12]
\[ iq_1 + a_1 r x + (b_1 |q|^2 + c_1 |r|^2) r x q + (\xi_1 |q|^4 + \eta_1 |q|^2 |r|^2 + \zeta_1 |r|^4) q + i \alpha q_x + \beta q + \gamma = 0 \]  
(1)

\[ iq_2 + a_2 r x + (b_2 |q|^2 + c_2 |r|^2) r x r + (\xi_2 |r|^4 + \eta_2 |r|^2 |q|^2 + \zeta_2 |q|^4) r + i \alpha r_x + \beta q + \gamma = 0 \]  
(2)

In Eqs. (1) and (2), \( q(x, t) \) and \( r(x, t) \) represent forward and backward propagating waves respectively while \( a_j \) for \( j = 1, 2 \) are coefficients of dispersive reflectivity. Next, \( b_j \) are the coefficients of self-phase modulation (SPM); \( c_j \) represents the cross-phase modulation (XPM) for non-local nonlinearity. Next, \( \alpha_j \) represents inter-modal dispersion and finally \( \beta_j \) are detuning parameters. All of the coefficients are real valued constants and \( i = \sqrt{-1} \).

2. Preliminaries

In order for determining solutions to (1) and (2), the taken into consideration assumption is

\[ q(x, t) = P_1(\varepsilon)e^{i\phi(x, t)} \]  
(3)

\[ r(x, t) = P_2(\varepsilon)e^{i\phi(x, t)} \]  
(4)

where

\[ \varepsilon = x - vt \]  
(5)

and \( v \) stands for the soliton velocity. From the phase

\[ \phi(x, t) = -\kappa x + \omega t + \theta \]  
(6)

where \( \kappa, \omega \) and \( \theta \) give the soliton frequency, its wave number and phase center, respectively. The real and imaginary parts yield respectively

\[ (\alpha_j \kappa - \omega) P_j + (\beta_j - a_j \kappa^2) P_j + \xi_j P_j^3 + \eta_j P_j^3 + \zeta_j P_j^3 + 2b_j P_j^5 \]  
\[ + 2c_j P_j^5 + 2d_j P_j^5 + 2e_j P_j^5 = 0 \]  
(7)

and

\[ (v - \alpha_j) P_j^2 + 2a_j \kappa P_j^2 = 0 \]  
(8)

where \( j = 1, 2 \) and \( \tilde{j} = 3 - j \), after inserting (3) and (4) into (1) and (2). Next, the balancing principle causes

\[ P_j = P_j \]  
(9)

and thus (7) and (8) are reshaped as

\[ (\alpha_j \kappa + \beta_j - \omega - a_j \kappa^2) P_j + (\xi_j + \eta_j + \zeta_j) P_j^3 + 2(b_j + c_j) P_j^5 + 2d_j P_j^5 + 2e_j P_j^5 = 0 \]  
(10)

and

\[ (v - \alpha_j + 2a_j \kappa) P_j^2 = 0 \]  
(11)

respectively. From (11), one observes the the speed of soliton as

\[ v = \alpha_j - 2a_j \kappa \]  
(12)

and this speed brings about

\[ 2(a_j - a_\tilde{j}) \kappa - (\alpha_j - \alpha_\tilde{j}) = 0. \]  
(13)

With the help of (13)

\[ a_j = a, \quad \alpha_j = \alpha \]  
(14)

and then

\[ v = \alpha - 2a \kappa. \]  
(15)

In view of these results, (1) and (2) are re-casted as

\[ iq_1 + ar x + (b_1 |q|^2 + c_1 |r|^2) r x q + (\xi_1 |q|^4 + \eta_1 |q|^2 |r|^2 + \zeta_1 |r|^4) q + iax q_x + \beta q + \gamma = 0 \]  
(16)

\[ ir_1 + ar x + (b_2 |r|^2 + c_2 |q|^2) r x r + (\xi_2 |r|^4 + \eta_2 |r|^2 |q|^2 + \zeta_2 |q|^4) r + iax r_x + \beta q + \gamma = 0 \]  
(17)

Therefore, real parts imply

\[ (\alpha \kappa + \beta_j - \omega - \alpha \kappa^2) P_j + (\xi_j + \eta_j + \zeta_j) P_j^3 + 2(b_j + c_j) P_j^5 + aP_j^5 + 2(b_j + c_j) P_j^5 P_j = 0. \]  
(18)
3. Extended trial function scheme

The discussion in this section will be on the investigation of soliton solutions to (1) and (2) employing extended trial function [5–7]. To kick off, the solution of (18) is selected in the form

\[ \sum_{i=0}^{n} \varphi_i \psi^i \quad (19) \]

where

\[ (\psi')^2 = \Gamma(\psi) = \Phi(\psi) = \frac{\mu_\psi \psi^2 + \cdots + \mu_1 \psi + \mu_0}{\chi_\psi \psi^2 + \cdots + \chi_0}. \quad (20) \]

Here \( \mu_0, \ldots, \mu_\psi; \chi_0, \ldots, \chi_\psi \) and \( \varphi_i, \ldots, \varphi_i \) are all constants and also \( \mu_0, \chi_0 \) and \( \varphi_i \) are non-zero. The reworded form of (20) is

\[ \pm (\epsilon - \epsilon_0) = \int \frac{d\psi}{\sqrt{\Gamma(\psi)}} = \int \sqrt{\frac{\Phi(\psi)}{\Gamma(\psi)}} d\psi. \quad (21) \]

The balance of the term \( P_j \) with the terms \( P_j \Phi(\psi)^2 \) or \( P_j^2 \Phi(\psi) \) seen in (18) generates

\[ \sigma - \rho - 2\varsigma = 2. \quad (22) \]

Considering \( \sigma = 4, \rho = 0 \) and \( \varsigma = 1 \),

\[ P_j = \varphi_0 \psi^0 + \varphi_1 \psi. \quad (23) \]

Plug (23) in (18) and then overcome the revealed system. Thus

\[ \mu_0 = \mu_0, \quad \mu_2 = \mu_2, \quad \varphi_0 = \varphi_0, \quad \varphi_1 = \varphi_1, \quad \chi_0 = \frac{12 \mu_2 (b + c_1)^2}{\epsilon_1 [a - 12 (\varphi_i^0)^2 (b + c_1)]}, \quad \mu_1 = \frac{2 \mu_2 \varphi_1^0 [a - 4 (\varphi_0^0)^2 (b + c_1)]}{\varphi_1^0 [a - 12 (\varphi_0^0)^2 (b + c_1)]}, \quad \mu_2 = \frac{8 \mu_2 \varphi_0^0 (\varphi_1^0) (b + c_1)}{12 (\varphi_0^0)^2 (b + c_1) - a'}, \quad \mu_3 = \frac{2 \mu_2 (\varphi_0^0)^2 (b + c_1) - a'}{12 (\varphi_0^0)^2 (b + c_1) - a}, \quad \omega = \frac{\epsilon_1 (2 \mu_2 (\varphi_0^0)^2 (b + c_1) [a - 12 (\varphi_0^0)^2 (b + c_1)] + a \mu_2 \mathcal{N}_j + 4 \mu_2 \mathcal{I}_j (b + c_1)^2)}{12 \mu_2 (b + c_1)^2}, \quad (24) \]

where

\[ \mathcal{N}_j = a - 2 (\varphi_0^0)^2 (b + c_1) \]

\[ \mathcal{I}_j = 3 \theta (a - a \theta) + 3 \theta^2 + (\varphi_0^0)^4 \epsilon_j \]

\[ \epsilon_j = \varsigma_j + \eta_j + \xi_j \quad (25) \]

and then (24) gives rise to

\[ \pm (\epsilon - \epsilon_0) = \frac{\sqrt{\chi_0}}{\mu_1} \int \frac{d\psi}{\sqrt{\Gamma(\psi)}} = \lambda \int \sqrt{\frac{\psi^4 + \mu_1 \psi^3 + \mu_2 \psi^2 + \mu_3 \psi + \mu_4}{\mu_4}}. \quad (26) \]

As a consequence, one attains traveling wave solutions for the model as:

For \( \Gamma(\psi) = (\psi - \theta_1)^4 \),

\[ q(x, t) = \left\{ \varphi_0 \psi^0 + \varphi_1 \psi^1 \pm \frac{\varphi_1 \lambda}{x + (2a/n - \sigma) t - \epsilon_0} \right\} \times \exp \left[ i \left\{ -\kappa x + \left( \frac{\epsilon_1 (2 \mu_2 (\varphi_0^0)^2 (b + c_1) [a - 12 (\varphi_0^0)^2 (b + c_1)] + a \mu_2 \mathcal{N}_j + 4 \mu_2 \mathcal{I}_j (b + c_1)^2)}{12 \mu_2 (b + c_1)^2} \right) t + \theta \right] \right] \quad (27) \]
Whenever \( \Gamma \) and however, when \( \Gamma = (\psi - \theta_1)^2(\psi - \theta_2) \) and \( \theta_2 > \theta_1 \),

\[
q(x, t) = \left\{ \varpi_0^{(1)} + \varpi_1^{(1)} \vartheta_1 + \frac{\varpi_0^{(1)} \vartheta_1 - \vartheta_2}{\exp \left[ \frac{1}{4} \vartheta_1 - \frac{\vartheta_2}{x + (2\alpha - \alpha)t - \varepsilon_0} \right] - 1} \right\} \times \exp \left\{ i \kappa \chi \left( \frac{1}{4} \vartheta_1 - \frac{\vartheta_2}{x + (2\alpha - \alpha)t - \varepsilon_0} \right) \right\}
\]

\[
\times \exp \left\{ i \kappa \chi \left( \frac{1}{4} \vartheta_1 - \frac{\vartheta_2}{x + (2\alpha - \alpha)t - \varepsilon_0} \right) \right\} \psi(\vartheta_1) \mid \vartheta_2 \right\} \frac{1}{12 \mu_x(b + c)^2} \right\} \theta + \theta \right\}.
\]

(28)

However, when \( \Gamma = (\psi - \theta_1)^2(\psi - \theta_2)^2 \),

\[
q(x, t) = \left\{ \varpi_0^{(1)} + \varpi_1^{(1)} \vartheta_1 + \frac{\varpi_0^{(1)} \vartheta_1 - \vartheta_2}{\exp \left[ \frac{1}{4} \vartheta_1 - \frac{\vartheta_2}{x + (2\alpha - \alpha)t - \varepsilon_0} \right] - 1} \right\} \times \exp \left\{ i \kappa \chi \left( \frac{1}{4} \vartheta_1 - \frac{\vartheta_2}{x + (2\alpha - \alpha)t - \varepsilon_0} \right) \right\}
\]

\[
\times \exp \left\{ i \kappa \chi \left( \frac{1}{4} \vartheta_1 - \frac{\vartheta_2}{x + (2\alpha - \alpha)t - \varepsilon_0} \right) \right\} \psi(\vartheta_1) \mid \vartheta_2 \right\} \frac{1}{12 \mu_x(b + c)^2} \right\} \theta + \theta \right\}.
\]

(29)

and

\[
q(x, t) = \left\{ \varpi_0^{(1)} + \varpi_1^{(1)} \vartheta_1 + \frac{\varpi_0^{(1)} \vartheta_1 - \vartheta_2}{\exp \left[ \frac{1}{4} \vartheta_1 - \frac{\vartheta_2}{x + (2\alpha - \alpha)t - \varepsilon_0} \right] - 1} \right\} \times \exp \left\{ i \kappa \chi \left( \frac{1}{4} \vartheta_1 - \frac{\vartheta_2}{x + (2\alpha - \alpha)t - \varepsilon_0} \right) \right\}
\]

\[
\times \exp \left\{ i \kappa \chi \left( \frac{1}{4} \vartheta_1 - \frac{\vartheta_2}{x + (2\alpha - \alpha)t - \varepsilon_0} \right) \right\} \psi(\vartheta_1) \mid \vartheta_2 \right\} \frac{1}{12 \mu_x(b + c)^2} \right\} \theta + \theta \right\}.
\]

(30)

Whenever \( \Gamma = (\psi - \theta_1)^2(\psi - \theta_2)(\psi - \theta_3) \) and \( \theta_1 > \theta_2 > \theta_3 \),

\[
q(x, t) = \left\{ \varpi_0^{(1)} + \varpi_1^{(1)} \vartheta_1 + \frac{\varpi_0^{(1)} \vartheta_1 - \vartheta_2}{\exp \left[ \frac{1}{4} \vartheta_1 - \frac{\vartheta_2}{x + (2\alpha - \alpha)t - \varepsilon_0} \right] - 1} \right\} \times \exp \left\{ i \kappa \chi \left( \frac{1}{4} \vartheta_1 - \frac{\vartheta_2}{x + (2\alpha - \alpha)t - \varepsilon_0} \right) \right\}
\]

\[
\times \exp \left\{ i \kappa \chi \left( \frac{1}{4} \vartheta_1 - \frac{\vartheta_2}{x + (2\alpha - \alpha)t - \varepsilon_0} \right) \right\} \psi(\vartheta_1) \mid \vartheta_2 \right\} \frac{1}{12 \mu_x(b + c)^2} \right\} \theta + \theta \right\}.
\]

(31)
\[
    r(x,t) = \left\{ \frac{\varpi_0^{(1)}(\vartheta_1 - \vartheta_2) - 2\varpi_0^{(2)}(\vartheta_1 - \vartheta_3)(\vartheta_1 - \vartheta_4)}{2\vartheta_1 - \vartheta_2 - \vartheta_3 + (\vartheta_1 - \vartheta_2)\cosh \left[ \frac{\vartheta_1 - \vartheta_2}{4} (x + (2\alpha - \alpha') t) \right]} \right\} \\
    \times \exp \left\{ -i\kappa x + \left[ \epsilon_j(2\mu_0(\varpi_0^{(1)})^2(b_j + c_j)\left[ a - 12(\varpi_0^{(1)})^2(b_j + c_j) + a\mu_j N_j \right] + 4\mu_j \lambda_j(b_j + c_j)^2 \right] t + \theta \right\}.
\]

On the other hand, if \( \Gamma(\psi) = (\psi - \theta_1)(\psi - \theta_2)(\psi - \theta_3)(\psi - \theta_4) \) and \( \theta_1 > \theta_2 > \theta_3 > \theta_4 \),

\[
    q(x,t) = \left\{ \frac{\varpi_0^{(1)} + \varpi_0^{(2)}\vartheta_1 + \varpi_0^{(3)}(\vartheta_1 - \vartheta_3)(\vartheta_1 - \vartheta_4)}{\vartheta_1 - \vartheta_2 + (\vartheta_1 - \vartheta_4)\sin^2 \left[ \frac{\vartheta_1 - \vartheta_2}{4} (x + (2\alpha - \alpha') t) - \theta_0, k \right]} \right\} \\
    \times \exp \left\{ -i\kappa x + \left[ \epsilon_j(2\mu_0(\varpi_0^{(1)})^2(b_j + c_j)\left[ a - 12(\varpi_0^{(1)})^2(b_j + c_j) + a\mu_j N_j \right] + 4\mu_j \lambda_j(b_j + c_j)^2 \right] t + \theta \right\}.
\]

\[
    r(x,t) = \left\{ \frac{\varpi_0^{(1)} + \varpi_0^{(2)}\vartheta_1 + \varpi_0^{(3)}(\vartheta_1 - \vartheta_3)(\vartheta_1 - \vartheta_4)}{\vartheta_1 - \vartheta_2 + (\vartheta_1 - \vartheta_4)\sin^2 \left[ \frac{\vartheta_1 - \vartheta_2}{4} (x + (2\alpha - \alpha') t) - \theta_0, k \right]} \right\} \\
    \times \exp \left\{ -i\kappa x + \left[ \epsilon_j(2\mu_0(\varpi_0^{(1)})^2(b_j + c_j)\left[ a - 12(\varpi_0^{(1)})^2(b_j + c_j) + a\mu_j N_j \right] + 4\mu_j \lambda_j(b_j + c_j)^2 \right] t + \theta \right\}.
\]

Here modulus \( k \) is represented as below:

\[
    k^2 = \frac{(\vartheta_2 - \vartheta_1)(\vartheta_1 - \vartheta_3)}{(\vartheta_1 - \vartheta_2)(\vartheta_1 - \vartheta_4)}
\]

and \( \theta_0 \) for \( j = 1, 2, 3, 4 \) are the roots of

\[
    \Gamma(\psi) = 0.
\]

Supposing

\[
    \epsilon_0 = \varpi_0^{(1)} + \varpi_0^{(2)}\vartheta_1 = \varpi_0^{(2)} + \varpi_0^{(3)}\vartheta_1 = 0
\]

the solutions indicated in \( (27)-(36) \) can be converted into plane wave solutions

\[
    q(x,t) = \pm \frac{\varpi_0^{(1)}\lambda}{x + (2\alpha - \alpha') t} \\
    \times \exp \left\{ -i\kappa x + \left[ \epsilon_j(2\mu_0(\varpi_0^{(1)})^2(b_j + c_j)\left[ a - 12(\varpi_0^{(1)})^2(b_j + c_j) + a\mu_j N_j \right] + 4\mu_j \lambda_j(b_j + c_j)^2 \right] t + \theta \right\}.
\]

\[
    r(x,t) = \pm \frac{\varpi_0^{(2)}\lambda}{x + (2\alpha - \alpha') t} \\
    \times \exp \left\{ -i\kappa x + \left[ \epsilon_j(2\mu_0(\varpi_0^{(1)})^2(b_j + c_j)\left[ a - 12(\varpi_0^{(1)})^2(b_j + c_j) + a\mu_j N_j \right] + 4\mu_j \lambda_j(b_j + c_j)^2 \right] t + \theta \right\}.
\]

\[
    q(x,t) = \pm \frac{4\varpi_0^{(1)}\lambda^2(\vartheta_2 - \vartheta_1)}{4\lambda^2 - [(\vartheta_1 - \vartheta_3)(x + (2\alpha - \alpha') t)]^2} \\
    \times \exp \left\{ -i\kappa x + \left[ \epsilon_j(2\mu_0(\varpi_0^{(1)})^2(b_j + c_j)\left[ a - 12(\varpi_0^{(1)})^2(b_j + c_j) + a\mu_j N_j \right] + 4\mu_j \lambda_j(b_j + c_j)^2 \right] t + \theta \right\}.
\]

\[
    r(x,t) = \pm \frac{4\varpi_0^{(2)}\lambda^2(\vartheta_2 - \vartheta_1)}{4\lambda^2 - [(\vartheta_1 - \vartheta_3)(x + (2\alpha - \alpha') t)]^2} \\
    \times \exp \left\{ -i\kappa x + \left[ \epsilon_j(2\mu_0(\varpi_0^{(1)})^2(b_j + c_j)\left[ a - 12(\varpi_0^{(1)})^2(b_j + c_j) + a\mu_j N_j \right] + 4\mu_j \lambda_j(b_j + c_j)^2 \right] t + \theta \right\}.
\]
singular soliton solutions

\[
q(x, t) = \left\{ \begin{array}{l}
\frac{\omega_1(\theta_1 - \theta_2)}{2} \left(1 \mp \coth \left[ \frac{\theta_1 - \theta_2}{2\lambda} (x + (2ax - \alpha)t) \right] \right)
\end{array} \right\} 
\times \exp \left[ \left\{ -\kappa x + \left( \frac{\ell_j(\omega_1(\theta_1 - \theta_2))r_i + c_j}{12\omega_2(b_i + c_j)^2} \right) \right\} \right]
\]  

(46)

\[
r(x, t) = \left\{ \begin{array}{l}
\frac{\omega_1(\theta_1 - \theta_2)}{2} \left(1 \mp \coth \left[ \frac{\theta_1 - \theta_2}{2\lambda} (x + (2ax - \alpha)t) \right] \right)
\end{array} \right\} 
\times \exp \left[ \left\{ -\kappa x + \left( \frac{\ell_j(\omega_1(\theta_1 - \theta_2))r_i + c_j}{12\omega_2(b_i + c_j)^2} \right) \right\} \right]
\]  

(47)

and bright soliton solutions

\[
q(x, t) = \left\{ \begin{array}{l}
\frac{D}{F + \cosh[H(x + (2ax - \alpha)t)]}
\end{array} \right\} 
\times \exp \left[ \left\{ -\kappa x + \left( \frac{\ell_j(\omega_1(\theta_1 - \theta_2))r_i + c_j}{12\omega_2(b_i + c_j)^2} \right) \right\} \right]
\]  

(48)

\[
r(x, t) = \left\{ \begin{array}{l}
\frac{D}{F + \cosh[H(x + (2ax - \alpha)t)]}
\end{array} \right\} 
\times \exp \left[ \left\{ -\kappa x + \left( \frac{\ell_j(\omega_1(\theta_1 - \theta_2))r_i + c_j}{12\omega_2(b_i + c_j)^2} \right) \right\} \right]
\]  

(49)

Here

\[
D = \frac{2\omega_1(\theta_1 - \theta_2)(\theta_1 - \theta_2)}{\theta_1 - \theta_2}
\]  

(50)

\[
\bar{D} = \frac{2\omega_1(\theta_1 - \theta_2)(\theta_1 - \theta_2)}{\theta_1 - \theta_2}
\]  

(51)

\[
H = \frac{\sqrt{\theta_1 - \theta_2} (\theta_1 - \theta_2)}{\lambda}
\]  

(52)

\[
F = \frac{2\theta_1 - \theta_2 - \theta_1}{\theta_1 - \theta_2}
\]  

(53)

where \( D \) and \( \bar{D} \) stand for the amplitude of the solitons, while \( H \) implies the inverse width. These solitons exist for \( \omega_1 < 0 \) and \( \omega_2 > 0 \). On the one hand, for

\[
\epsilon_0 = \omega_1^{(1)}; \epsilon_0^2 = \omega_2^{(2)}; \theta_2 = 0
\]  

(54)

elliptic sn function solutions change into

\[
q(x, t) = \left\{ \begin{array}{l}
\frac{D_1}{F_1 + \sn^2[H_j(x + (2ax - \alpha)t)], (\frac{\theta_1 - \theta_2)(\theta_1 - \theta_2)}{\theta_1 - \theta_2}}
\end{array} \right\} 
\times \exp \left[ \left\{ -\kappa x + \left( \frac{\ell_j(\omega_1(\theta_1 - \theta_2))r_i + c_j}{12\omega_2(b_i + c_j)^2} \right) \right\} \right]
\]  

(55)

\[
r(x, t) = \left\{ \begin{array}{l}
\frac{D_1}{F_1 + \sn^2[H_j(x + (2ax - \alpha)t)], (\frac{\theta_1 - \theta_2)(\theta_1 - \theta_2)}{\theta_1 - \theta_2}}
\end{array} \right\} 
\times \exp \left[ \left\{ -\kappa x + \left( \frac{\ell_j(\omega_1(\theta_1 - \theta_2))r_i + c_j}{12\omega_2(b_i + c_j)^2} \right) \right\} \right]
\]  

(56)

where
\[ \mathcal{D}_1 = \frac{\omega_1'(\theta_1 - \theta_2)(\theta_1 - \theta_3)}{\theta_1 - \theta_4} \]
\[ \mathcal{D}_2 = \frac{\omega_2'(\theta_1 - \theta_2)(\theta_1 - \theta_3)}{\theta_1 - \theta_4} \]
\[ \mathcal{F}_1 = \frac{\theta_1 - \theta_3}{\theta_1 - \theta_4} \]
\[ \mathcal{H}_j = \frac{(-1)^j \sqrt{(\theta_1 - \theta_2)(\theta_2 - \theta_4)}}{2\lambda} \quad \text{for} \ j = 1, 2. \]

**Remark-1:** For \( k \to 1 \), singular optical solitons occur

\[ q(x, t) = \frac{\mathcal{D}_1}{\mathcal{F}_1 + \tanh^2[\mathcal{H}_j(x + (2\pi - \alpha)t)]} \times \exp \left\{ i \left[ -kx + \left( \frac{\ell(2\mu_0 (\omega^{(l)})(b_1 + c_1)[a - 12(\omega^{(l)})(b_1 + c_1)] + a\mu_2 N_j + 4\mu_2 \xi_2 (b_1 + c_1)^2)}{12\mu_2 (b_1 + c_1)^2} \right) \right] \right\} \]
\[ r(x, t) = \frac{\mathcal{D}_1}{\mathcal{F}_1 + \tanh^2[\mathcal{H}_j(x + (2\pi - \alpha)t)]} \times \exp \left\{ i \left[ -kx + \left( \frac{\ell(2\mu_0 (\omega^{(l)})(b_1 + c_1)[a - 12(\omega^{(l)})(b_1 + c_1)] + a\mu_2 N_j + 4\mu_2 \xi_2 (b_1 + c_1)^2)}{12\mu_2 (b_1 + c_1)^2} \right) \right] \right\} \]
for \( \theta_3 = \theta_4 \).

**Remark-2:** However, if \( k \to 0 \), periodic solutions are

\[ q(x, t) = \frac{\mathcal{D}_1}{\mathcal{F}_1 + \sin^2[\mathcal{H}_j(x + (2\pi - \alpha)t)]} \times \exp \left\{ i \left[ -kx + \left( \frac{\ell(2\mu_0 (\omega^{(l)})(b_1 + c_1)[a - 12(\omega^{(l)})(b_1 + c_1)] + a\mu_2 N_j + 4\mu_2 \xi_2 (b_1 + c_1)^2)}{12\mu_2 (b_1 + c_1)^2} \right) \right] \right\} \]
\[ r(x, t) = \frac{\mathcal{D}_1}{\mathcal{F}_1 + \sin^2[\mathcal{H}_j(x + (2\pi - \alpha)t)]} \times \exp \left\{ i \left[ -kx + \left( \frac{\ell(2\mu_0 (\omega^{(l)})(b_1 + c_1)[a - 12(\omega^{(l)})(b_1 + c_1)] + a\mu_2 N_j + 4\mu_2 \xi_2 (b_1 + c_1)^2)}{12\mu_2 (b_1 + c_1)^2} \right) \right] \right\} \]
for \( \theta_3 = \theta_4 \).

4. **Conclusions**

The paper studied fiber Bragg gratings with dispersive reflectivity for parabolic-nonlocal combo nonlinearity by the aid of extended trial function scheme. Bright and singular soliton solutions to the governing coupled NLSE emerged from the scheme. These solitons have their corresponding constraint conditions, for their existence, and they are also displayed. The model proposed in this paper is new and the corresponding results are appearing for the first time in this paper. A visible limitation of this approach is that the scheme fails to secure dark soliton solutions to the model. Nevertheless, the results stand on a strong footing to move further along in this avenue. Later, this model along with others for fiber Bragg gratings will be handled using additional algorithms. These would include Lie symmetry analysis, Kudryashov’s method and several others. Thus, additional solutions including the much needed dark solitons, would presumably appear for the model. Such results are all currently awaited.

**Conflicts of interest**

The authors also declare that there is no conflict of interest.

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