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Highly dispersive optical solitons with cubic–quintic–septic law by exp-expansion

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ABSTRACT

This paper obtains highly dispersive singular optical solitons in cubic–quintic–septic nonlinear medium. The exp-function expansion is the applied integration algorithm. The solitons appear with their corresponding existence criteria.

1. Introduction

The theory of highly dispersive optical solitons was proposed earlier during 2019 and later on it became a popular model of study in optical fibers [1–9]. Such kind of solitons occur when, in addition to the familiar group velocity dispersion (GVD), inter-modal dispersion (IMD), third-order dispersion (3OD), fourth-order dispersion (4OD), fifth-order dispersion (5OD) and sixth-order dispersion (6OD) terms are included. In such a situation integration of the governing nonlinear Schrödinger's equation (NLSE) proposes challenge. It has been proved that the method of undetermined coefficients is an epic failure to secure soliton solutions to the corresponding NLSE [5]. Later on, the methods of extended trial function method and Jacobi's F-expansion scheme were successfully implemented. Lately, the exp-function scheme was applied to study highly dispersive optical solitons with quadratic–cubic nonlinearity [9]. In the past, this integration scheme, namely exp-function, was applied to study oblique resonant solitons, birefringent fibers and water waves [9–15]. This paper will implement the scheme to address highly dispersive optical solitons that come with cubic–quintic–septic (CQS) nonlinearity. The details are enumerated in the remainder of the paper after a quick introduction to the model.

1.1. Governing model

The dimensionless form of NLSE with CQS law in presence of dispersion terms of all orders is [1–9]:

$$iq_t + ia_1 q_x + a_2 q_{xx} + ia_3 q_{xxx} + a_4 q_{xxxx} + ia_5 q_{xxxxx} + a_6 q_{xxxxxx} + (b_1\ |q|^2 + b_2\ |q|^4 + b_3\ |q|^6)q = 0. \quad (1)$$

Here in (1), $q(x, t)$ represents soliton molecules and other forms of nonlinear waves where x and t are independent spatial and temporal variables respectively. The first term represents linear temporal evolution, while a_1, a_2, a_3, a_4, a_5 and a_6 are coefficients of IMD, GVD, 3OD, 4OD, 5OD and 6OD respectively. Next, b_1, b_2 and b_3 together comprise the CQS law. The complex-valued function q

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(x, t) is the wave profile that represents the dependent variable. The coefficients are all real-valued constants and $i = \sqrt{-1}$.

2. Preliminaries

To integrate the model, the starting point is selected as:

$$q(x, t) = g(\varsigma)e^{i\phi(x,t)} \tag{2}$$

where

$$\varsigma = x - \mathrm{v\,t} \tag{3}$$

with v indicates the speed of the wave. Next, $\phi(x, t)$ is phase component, defined as

$$\phi(x, t) = -\kappa x + \omega t + \theta \tag{4}$$

where κ is the soliton frequency, ω is its wave number and θ is the phase constant. After plugging (2) into (1) real part yields

$$\begin{aligned} &-(\omega + \kappa(-a_1 + \kappa(a_2 + \kappa(a_3 - \kappa(a_4 + a_5\kappa) + a_6\kappa^3))))g + b_1g^3 + b_2g^5 + b_3g^7 \\ &+ (a_2 + 3a_3\kappa - 6a_4\kappa^2 - 10a_5\kappa^3 + 15a_6\kappa^4)g'' + (a_4 + 5a_5\kappa - 15a_6\kappa^2)g^{(4)} + a_6g^{(6)} = 0. \end{aligned} \tag{5}$$

Next, the imaginary part is

$$(v - a_1 + 2a_2\kappa + 3a_3\kappa^2 - 4a_4\kappa^3 - 5a_5\kappa^4 + 6a_6\kappa^5)g + (-a_3 + 2\kappa(2a_4 + 5\kappa(a_5 - 2a_6\kappa)))g'' - (a_5 - 6a_6\kappa)g^{(4)} = 0. \tag{6}$$

From the last equation, the soliton speed falls out

$$v = a_1 - 2a_2\kappa - 8\kappa^3(a_4 + 2a_5\kappa) \tag{7}$$

together with the constraint conditions

$$a_5 = 6a_6\kappa \tag{8}$$

and

$$a_3 = \frac{4\kappa(3a_4 + 5a_5\kappa)}{3}. \tag{9}$$

Thus, the real part reduces to:

$$\begin{aligned} &-\kappa(6\omega - 6a_1\kappa + 6a_2\kappa^2 + 18a_4\kappa^4 + 35a_5\kappa^5)g + 6b_1\kappa g^3 + 6b_2\kappa g^5 + 6b_3\kappa g^7 \\ &+ 3\kappa(2a_2 + 12a_4\kappa^2 + 25a_5\kappa^3)g'' + 3\kappa(2a_4 + 5a_5\kappa)g^{(4)} + a_5g^{(6)} = 0. \end{aligned} \tag{10}$$

3. $\exp(-\psi(\varsigma))$–expansion scheme

To recover highly dispersive optical solitons with CQS nonlinearity to the governing model by employing $\exp(-\psi(\varsigma))$-expansion approach [9–15], the starting assumption is:

$$g(\varsigma) = \sum_{j=1}^{N} \gamma_j(\exp[-\psi(\varsigma)])^j \tag{11}$$

where γ_j are constants to be detected later, such that $\gamma_N \neq 0$, and the function ψ satisfies

$$\psi'(\varsigma) = \exp[-\psi(\varsigma)] + \chi\exp[\psi(\varsigma)] + \vartheta. \tag{12}$$

Here, the solutions of Eq. (12) are presented as:

If $\chi \neq 0$ and $\vartheta^2 - 4\chi > 0$,

$$\psi(\varsigma) = \ln\left[-\frac{\sqrt{\vartheta^2 - 4\chi}\tanh\left(\frac{\sqrt{\vartheta^2 - 4\chi}}{2}(\varsigma + c)\right) + \vartheta}{2\chi}\right]. \tag{13}$$

For $\chi \neq 0$ and $\vartheta^2 - 4\chi < 0$,

$$\psi(\varsigma) = \ln\left[\frac{\sqrt{4\chi - \vartheta^2}\tan\left(\frac{\sqrt{4\chi - \vartheta^2}}{2}(\varsigma + c)\right) - \vartheta}{2\chi}\right]. \tag{14}$$

When $\chi = 0$, $\vartheta \neq 0$ and $\vartheta^2 - 4\chi > 0$,

$$\psi(\varsigma) = -\ln\left[\frac{\vartheta}{\exp(\vartheta(\varsigma + c)) - 1}\right]. \tag{15}$$

Whenever $\chi \neq 0$, $\vartheta \neq 0$ and $\vartheta^2 - 4\chi = 0$,

$$\psi(\varsigma) = \ln\left[-\frac{2(\vartheta(\varsigma + c) + 2)}{\vartheta^2(\varsigma + c)}\right]. \tag{16}$$

However, if $\chi = 0$, $\vartheta = 0$ and $\vartheta^2 - 4\chi = 0$,

$$\psi(\varsigma) = \ln[\varsigma + c]. \tag{17}$$

It needs to be noted that c is the integration constant. Balancing g^7 with $g^{(6)}$ in (10), it is possible in order to get that $N = 1$. So, (11) turns into

$$g(\varsigma) = \gamma_0 + \gamma_1 \exp[-\psi(\varsigma)] \tag{18}$$

and then the solution of (1) takes the form:

$$q(x, t) = \{\gamma_0 + \gamma_1 \exp[-\psi(\varsigma)]\} \exp[i(-\kappa x + \omega t + \theta)]. \tag{19}$$

Next, putting (18) into (10) and setting the coefficients of $\exp(-\psi(\varsigma))$ to zero brings about a set of equations

$$\begin{aligned}
&720a_5\gamma_1 + 6\gamma_1^7 b_3\kappa = 0,\\
&2520a_5\gamma_1\vartheta + 42\gamma_0\gamma_1^6 b_3\kappa = 0,\\
&360a_5\gamma_1\kappa^2 + 144a_4\gamma_1\kappa + 1680a_5\gamma_1\chi + 3360a_5\gamma_1\vartheta^2 + 126\gamma_0^2\gamma_1^5 b_3\kappa + 6\gamma_1^5 b_2\kappa = 0,\\
&900a_5\gamma_1\kappa^2\vartheta + 360a_4\gamma_1\kappa\vartheta + 4200a_5\gamma_1\chi\vartheta + 2100a_5\gamma_1\vartheta^3 + 210\gamma_0^3\gamma_1^4 b_3\kappa + 30\gamma_0\gamma_1^4 b_2\kappa = 0,\\
&150a_5\gamma_1\kappa^4 + 72a_4\gamma_1\kappa^3 + 600a_5\gamma_1\kappa^2\chi + 750a_5\gamma_1\kappa^2\vartheta^2 + 240a_4\gamma_1\kappa\chi + 300a_4\gamma_1\kappa\vartheta^2\\
&\quad + 12a_2\gamma_1\kappa + 1232a_5\gamma_1\chi^2 + 3584a_5\gamma_1\chi\vartheta^2 + 602a_5\gamma_1\vartheta^4 + 210\gamma_0^4\gamma_1^3 b_3\kappa + 60\gamma_0^2\gamma_1^3 b_2\kappa + 6\gamma_1^3 b_1\kappa = 0,\\
&225a_5\gamma_1\kappa^4\vartheta + 108a_4\gamma_1\kappa^3\vartheta + 900a_5\gamma_1\kappa^2\chi\vartheta + 225a_5\gamma_1\kappa^2\vartheta^3 + 360a_4\gamma_1\kappa\chi\vartheta + 90a_4\gamma_1\kappa\vartheta^3\\
&\quad + 18a_2\gamma_1\kappa\vartheta + 1848a_5\gamma_1\chi^2\vartheta + 1176a_5\gamma_1\chi\vartheta^3 + 63a_5\gamma_1\vartheta^5 + 126\gamma_0^5\gamma_1^2 b_3\kappa + 60\gamma_0^3\gamma_1^2 b_2\kappa + 18\gamma_0\gamma_1^2 b_1\kappa = 0,\\
&-35a_5\gamma_1\kappa^6 - 18a_4\gamma_1\kappa^5 + 150a_5\gamma_1\kappa^4\chi + 75a_5\gamma_1\kappa^4\vartheta^2 + 72a_4\gamma_1\kappa^3\chi + 36a_4\gamma_1\kappa^3\vartheta^2 - 6a_2\gamma_1\kappa^3 + 240a_5\gamma_1\kappa^2\chi^2\\
&\quad + 330a_5\gamma_1\kappa^2\chi\vartheta^2 + 15a_5\gamma_1\kappa^2\vartheta^4 + 6a_1\gamma_1\kappa^2 + 96a_4\gamma_1\kappa\chi^2 + 132a_4\gamma_1\kappa\chi\vartheta^2 + 12a_2\gamma_1\kappa\chi + 6a_4\gamma_1\kappa\vartheta^4 + 6a_2\gamma_1\kappa\vartheta^2\\
&\quad + 272a_5\gamma_1\chi^3 + 720a_5\gamma_1\chi^2\vartheta^2 + 114a_5\gamma_1\chi\vartheta^4 + a_5\gamma_1\vartheta^6 - 6\gamma_1\kappa\omega + 42\gamma_0^6\gamma_1 b_3\kappa + 30\gamma_0^4\gamma_1 b_2\kappa + 18\gamma_0^2\gamma_1 b_1\kappa = 0,\\
&-35a_5\gamma_0\kappa^6 - 18a_4\gamma_0\kappa^5 + 75a_5\gamma_1\kappa^4\chi\vartheta + 36a_4\gamma_1\kappa^3\chi\vartheta - 6a_2\gamma_0\kappa^3 + 120a_5\gamma_1\kappa^2\chi^2\vartheta + 15a_5\gamma_1\kappa^2\chi\vartheta^3\\
&\quad + 6a_1\gamma_0\kappa^2 + 48a_4\gamma_1\kappa\chi^2\vartheta + 6a_4\gamma_1\kappa\chi\vartheta^3 + 6a_2\gamma_1\kappa\chi\vartheta + 136a_5\gamma_1\chi^3\vartheta + 52a_5\gamma_1\chi^2\vartheta^3 + a_5\gamma_1\chi\vartheta^5\\
&\quad - 6\gamma_0\kappa\omega + 6\gamma_0^7 b_3\kappa + 6\gamma_0^5 b_2\kappa + 6\gamma_0^3 b_1\kappa = 0,
\end{aligned} \tag{20}$$

from which

$$\begin{aligned}
\gamma_0 &= \gamma_0,\\
\gamma_1 &= \frac{2\gamma_0}{\vartheta},\\
\omega &= \frac{24a_1\kappa^2\vartheta^2(6\kappa^2 + 20\chi - 5\vartheta^2) - 12a_2\kappa\vartheta^2\ell_2 + a_5\vartheta^2\ell_4 + 48b_1\gamma_0^2\kappa\ell_3}{24\kappa\vartheta^2(6\kappa^2 + 20\chi - 5\vartheta^2)},\\
a_4 &= -\frac{\vartheta^2(12a_2\kappa + a_5(150\kappa^4 + 150\kappa^2\ell_1 + 77\ell_1^2)) + 24b_1\gamma_0^2\kappa}{12\kappa\vartheta^2(6\kappa^2 + 20\chi - 5\vartheta^2)},\\
b_2 &= \frac{\vartheta^4(6a_2\kappa - a_5(15\kappa^4 + 105\kappa^2\ell_1 + 49\ell_1^2)) + 12b_1\gamma_0^2\kappa\vartheta^2}{4\gamma_0^4\kappa(6\kappa^2 + 20\chi - 5\vartheta^2)},\\
b_3 &= -\frac{15a_5\vartheta^6}{8\gamma_0^6\kappa},
\end{aligned} \tag{21}$$

where

$$\begin{aligned}
\ell_1 &= 4\chi - \vartheta^2\\
\ell_2 &= 6\kappa^4 + 10\kappa^2\ell_1 - 3\ell_1^2\\
\ell_3 &= 3\kappa^4 - 3\kappa^2\ell_1 - \ell_1^2\\
\ell_4 &= 60\kappa^8 + 200\kappa^6\ell_1 + 372\kappa^4\ell_1^2 - 360\kappa^2\ell_1^3 - 69\ell_1^4.
\end{aligned} \tag{22}$$

As a result, one reveals singular soliton solution

$$
\begin{aligned}
q(x,t) &= \gamma_0\left[1-\frac{4\chi}{\vartheta^2+\vartheta\sqrt{\vartheta^2-4\chi}\tanh\left[\frac{\sqrt{\vartheta^2-4\chi}\,(x-\{a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa)\}t+c)}{2}\right]}\right]\\
&\times\exp\left[i\left\{-\kappa x+\left(\frac{24a_1\kappa^2\vartheta^2(6\kappa^2+20\chi-5\vartheta^2)-12a_2\kappa\vartheta^2\ell_2+a_5\vartheta^2\ell_4+48b_1\gamma_0^2\kappa\ell_3}{24\kappa\vartheta^2(6\kappa^2+20\chi-5\vartheta^2)}\right)t+\theta\right\}\right]
\end{aligned}
\tag{23}
$$

periodic solution

$$
\begin{aligned}
q(x,t) &= \gamma_0\left[1-\frac{4\chi}{\vartheta^2-\vartheta\sqrt{4\chi-\vartheta^2}\tan\left[\frac{\sqrt{4\chi-\vartheta^2}\,(x-\{a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa)\}t+c)}{2}\right]}\right]\\
&\times\exp\left[i\left\{-\kappa x+\left(\frac{24a_1\kappa^2\vartheta^2(6\kappa^2+20\chi-5\vartheta^2)-12a_2\kappa\vartheta^2\ell_2+a_5\vartheta^2\ell_4+48b_1\gamma_0^2\kappa\ell_3}{24\kappa\vartheta^2(6\kappa^2+20\chi-5\vartheta^2)}\right)t+\theta\right\}\right]
\end{aligned}
\tag{24}
$$

singular 1-soliton solution of second type

$$
\begin{aligned}
q(x,t) &= \gamma_0\coth\left[\frac{\vartheta(x-\{a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa)\}t+c)}{2}\right]\\
&\times\exp\left[i\left\{-\kappa x+\left(\frac{\vartheta^2(12\kappa(2a_1(6\kappa^3-5\kappa\vartheta^2)+a_2(-6\kappa^4+10\kappa^2\vartheta^2+3\vartheta^4))+a_5\ell_6)+48b_1\gamma_0^2\kappa\ell_5}{24\vartheta^2(6\kappa^3-5\kappa\vartheta^2)}\right)t+\theta\right\}\right]
\end{aligned}
\tag{25}
$$

where

$$
\begin{aligned}
&\ell_5=3\kappa^4+3\kappa^2\vartheta^2-\vartheta^4\\
&\ell_6=60\kappa^8-200\kappa^6\vartheta^2+372\kappa^4\vartheta^4+360\kappa^2\vartheta^6-69\vartheta^8
\end{aligned}
\tag{26}
$$

and finally plane wave solution

$$
\begin{aligned}
q(x,t) &= \frac{2\gamma_0}{\vartheta(x-\{a_1-2a_2\kappa-8\kappa^3(a_4+2a_5\kappa)\}t+c)+2}\\
&\times\exp\left[i\left\{-\kappa x+\left(\frac{\kappa\vartheta^2(-5a_5\kappa^6+6a_2\kappa^3-2a_1(6\kappa^2+20\chi-5\vartheta^2))-12b_1\gamma_0^2\kappa^4}{2\vartheta^2(5(\vartheta^2-4\chi)-6\kappa^2)}\right)t+\theta\right\}\right].
\end{aligned}
\tag{27}
$$

4. Conclusions

The exp-function integration scheme has been successfully applied to NLSE with six dispersion terms and CQS nonlinearity. The algorithm yielded singular solitons of both types. It is evident that the scheme has it limitations. It fails to retrieve bright and dark optical solitons and their combinations thereof. Later on, additional integration schemes will be implemented to secure such soliton solutions to the model. Some of these schemes are Lie symmetry analysis, Kudryashov's method, Riccati–Bernoulli's sub-ODE method and several others. It is expected that these algorithms individually or collectively lead to the display of a complete spectrum of soliton solutions.

Conflict of interest

The authors also declare that there is no conflict of interest.

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